Envelope Level Crossing Rate and Average Fade Duration in Mobile-to-mobile Fading Channels

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Abstract—A three-dimensional (3-D) analytical model for mobile-to-mobile communications is presented. From the analytical model, the envelope level crossing rate and average fade duration are derived for a 3-D non-isotropic scattering environment. The obtained analytical results are compared with measured data. The close agreement between the analytical and empirical curves confirms the utility of the proposed model.

I. INTRODUCTION

Several emerging wireless communication systems such as mobile ad-hoc wireless networks, intelligent transportation systems, and relay-based cellular networks require direct transmission between mobile terminals. Such mobile-to-mobile (M-to-M) communication systems are equipped with low elevation antennas and have both the transmitter and receiver in motion. To successfully design M-to-M systems, it is necessary to have a detailed knowledge of the M-to-M multipath fading channel and its statistical properties. Early studies of single-input single-output (SISO) M-to-M Rayleigh fading channels are reported in [1]. They showed that the received envelope of M-to-M channels is Rayleigh faded under non-line-of-sight (NLoS) conditions, but the statistical properties differ from conventional fixed-to-mobile cellular radio channels. They also proposed the reference model for two-dimensional (2-D) SISO M-to-M Rayleigh fading channels. Simulation models for 2-D SISO M-to-M channels are proposed in [2]-[4]. Channel measurements for SISO narrowband and wideband M-to-M communications are reported in [5], [6]. The reference models for 2-D narrowband multiple-input multiple-output (MIMO) M-to-M channels are proposed in [7], [8]. To appropriately model an urban environment, we have recently proposed the three-dimensional (3-D) reference models for narrowband and wideband MIMO M-to-M multipath fading channels [9], [10]. We have also derived the first-order statistics for the proposed models.

To assess system characteristics such as handoff, velocities of the transmitter and receiver, and fading rate, accurate characterization of the second-order statistics, such as envelope level crossing rate (LCR) and average fade duration (AFD), is necessary. To derive the LCR and AFD, we first construct a 3-D analytical model that employs the “two-cylinders” model proposed in [9] and generates the complex faded envelope as a superposition of the line-of-sight (LoS), single-bounced, and double-bounced rays (as proposed in [10]). The parametric nature of the model makes it adaptable to a variety of propagation environments. From the analytical model, we derive the LCR and AFD for a 3-D non-isotropic scattering environment. Finally, we compare the analytical results for the LCR and AFD with the measured data in [5] and with our measured data. The close agreement between the analytical and empirical curves confirms the utility of the proposed model.

The remainder of the paper is organized as follows. Section II presents a 3-D analytical model used to derive the LCR and AFD. Section III derives the LCR and AFD of the the complex faded envelope for 3-D non-isotropic scattering. Section IV compares analytical and measurement results to verify theoretical derivations. Finally, Section V provides some concluding remarks.

II. AN ANALYTICAL MODEL FOR MOBILE-TO-MOBILE CHANNELS

This section briefly describes an analytical model for narrowband SISO M-to-M multipath fading channels. It is assumed that both the transmitter (T x) and receiver (R x) are in motion and equipped with low elevation antennas. The radio propagation occurs in outdoor micro- and macro-cells, which are characterized by 3-D scattering with line-of-sight (LoS) or non-line-of-sight (NLoS) propagation conditions between the T x and R x.

Fig. 1 shows a 3-D geometrical model that is a simplified version of our “two-cylinder” model [9]. From the 3-D geometrical model, we observe that waves from the T x antenna traverse directly to the R x antenna or they are single- or double-bounced before arriving at the R x antenna. Hence, the received complex faded envelope of the link AT - AR can be written as a superposition of the LoS, single-bounced, and double-bounced rays [10], i.e.,

\[ h(t) = h^{SBT}(t) + h^{SBR}(t) + h^{DB}(t) + h^{LoS}(t), \]  \hspace{1cm} (1)

where the single-bounced components of the received complex faded envelope are, respectively,
on the scatterers $S^{(m)}_T$ and $S^{(n)}_R$, whereas $\alpha_R^{(m)}$ and $\alpha_R^{(n)}$ are the azimuth angles of arrival (AAoA) of the waves scattered from $S^{(m)}_T$ and $S^{(n)}_R$, respectively. Similarly, the symbols $\beta_T^{(m)}$ and $\beta_T^{(n)}$ denote the elevation angles of departure (EAoD), whereas the symbols $\beta_R^{(m)}$ and $\beta_R^{(n)}$ denote the elevation angles of arrival (EAoA), respectively. Finally, the symbols $\beta_T^{LoS}$ and $\beta_R^{LoS}$ denote the EAoD and the EAoA of the LoS paths, respectively. It is assumed that the angles of departure $\alpha_T^{(m)}$ and $\alpha_T^{(n)}$ independent from the angles of departure $\alpha_R^{(m)}$ and $\alpha_R^{(n)}$ [11]. On the other hand, single-bounced rays have the angles of arrival $\alpha_R^{(m)}$ and $\alpha_R^{(n)}$ independent from the angles of arrival $\alpha_T^{(m)}$ and $\alpha_T^{(n)}$.

III. LCR and AFD in M-to-M Narrowband Fading Channels

Assuming a 3-D non-isotropic scattering environment, we now derive the LCR and the AFD of the complex faded envelope described in (1). The LCR at a specified level $R$, $L(R)$, is defined as the rate at which the signal envelope crosses level $R$ in the positive going direction. When a LoS component is present, the LCR can be written as [12]

$$L(R) = \frac{2R\sqrt{K+1}}{\pi^{3/2}} \sqrt{\frac{b_2}{b_0}} e^{-K-(K+1)R^2} \times \left[ e^{-\left(\chi \sin \theta\right)^2} + \sqrt{\pi} \chi \sin \theta \operatorname{erf}(\chi \sin \theta) \right] d\theta,$$

where $\operatorname{cosh}(\cdot)$ is the hyperbolic cosine function, $\operatorname{erf}(\cdot)$ is the error function, and the parameter $\chi$ is equal to $\sqrt{Kb_2^2/(b_0b_2-b_1^2)}$. Finally, parameters $b_0$, $b_1$, and $b_2$ are defined as [13]

$$b_0 \triangleq \operatorname{E}[h'_R(t)^2] = \operatorname{E}[h'_R(t)^2],$$

$$b_1 \triangleq \operatorname{E}[h'_R(t) \hat{h}_R(t)] = \operatorname{E}[h'_R(t) \hat{h}_R(t)],$$

$$b_2 \triangleq \operatorname{E}[h'_R(t)^2] = \operatorname{E}[h'_R(t)^2],$$

where $h'_R(t)$ and $h'_R(t)$ denote the in-phase and quadrature component of the complex faded envelope $h(t) = h^{SBT}(t) + h^{SR}(t) + h^{DB}(t)$, $\operatorname{E}(\cdot)$ denotes the statistical expectation operator, and $h'_R(t)$ and $h'_R(t)$ denote the first derivative of $h'_R(t)$ and $h'_R(t)$ with respect to time $t$. The LCR for NLoS conditions can be obtained from (6) by setting $K = 0$. 

![Fig. 1. The 3-D geometrical model for SISO M-to-M channel.](image-url)
Using the analytical model described in Section II, we now derive the closed-form expressions for the parameters $b_0$, $b_1$, and $b_2$. Since the number of local scatterers in the reference model described in Section II is infinite, the discrete AAs, $\alpha_T^{(m)}$, EAOs, $\beta_R^{(n)}$, $\nu$, $\tau$, and $K$ can be replaced with continuous random variables $\alpha_T$, $\beta_T$, and $\beta_R$ with joint probability density functions (pdfs) $f(\alpha_T, \beta_T)$ and $f(\alpha_R, \beta_R)$, respectively. We assume that the azimuth and elevation angles are independent of each other, and thus, the joint pdf of $f(\alpha_T, \beta_T)$ and $f(\alpha_R, \beta_R)$ can be decomposed to $f(\alpha_T)f(\beta_T)$ and $f(\alpha_R)f(\beta_R)$, respectively. This assumption is based on experimental data in [14]. By substituting (1) into (7), the parameter $b_0$ becomes

$$b_0 = b_{0SBT} + b_{0SBR} + b_{0DB} = \frac{1}{K+1},$$

where $b_{0SBT}$, $b_{0SBR}$, and $b_{0DB}$ are, respectively,

$$b_{0SBT} = \frac{\eta_T}{K+1} \int_{-\beta_T - \pi}^{\beta_T - \pi} f(\alpha_T)f(\beta_T) d\alpha_T d\beta_T = \frac{\eta_T}{K+1},$$

$$b_{0SBR} = \frac{\eta_R}{K+1} \int_{-\beta_R - \pi}^{\beta_R - \pi} f(\alpha_R)f(\beta_R) d\alpha_R d\beta_R = \frac{\eta_R}{K+1},$$

$$b_{0DB} = \frac{\eta_T}{K+1} \frac{1}{2} \left[ \int_{-\beta_T - \pi}^{\beta_T - \pi} f(\alpha_T)f(\beta_T) d\alpha_T d\beta_T \right] \times \int_{-\beta_R - \pi}^{\beta_R - \pi} f(\alpha_R)f(\beta_R) d\alpha_R d\beta_R = \frac{\eta_T \eta_R}{K+1},$$

and $\beta_T$ and $\beta_R$ are the maximum elevation angles of the scatterers around the $T_x$ and $R_x$, respectively. Similarly, by substituting (1) into (8) and (9), the parameters $b_1$ and $b_2$ become

$$b_n = b_{nSBT} + b_{nSBR} + b_{nDB},$$

where $n \in \{1, 2\}$ and $b_{nSBT}$, $b_{nSBR}$, and $b_{nDB}$ are, respectively,

$$b_{nSBT} = \frac{\eta_T}{K+1} (2\pi)^n \int_{-\beta_T - \pi}^{\beta_T - \pi} \int_{-\beta_T - \pi}^{\beta_T - \pi} f(\alpha_T)f(\beta_T) \left[ f_{T_{max}} \cos(\alpha_T - \beta_T) + f_{R_{max}} \right] d\alpha_T d\beta_T,$$

$$b_{nSBR} = \frac{\eta_R}{K+1} (2\pi)^n \int_{-\beta_R - \pi}^{\beta_R - \pi} \int_{-\beta_R - \pi}^{\beta_R - \pi} f(\alpha_R)f(\beta_R) \left[ f_{R_{max}} \cos(\alpha_R - \beta_R) + f_{T_{max}} \right] d\alpha_R d\beta_R,$$

$$b_{nDB} = \frac{\eta_T}{K+1} (2\pi)^n \int_{-\beta_T - \pi}^{\beta_T - \pi} \int_{-\beta_R - \pi}^{\beta_R - \pi} f(\alpha_R)f(\beta_R) \left[ f_{R_{max}} \cos(\alpha_R - \beta_R) + f_{T_{max}} \right] d\alpha_R d\beta_R,$$

where $\eta_T$ and $\eta_R$ are the number of scatterers in the $T_x$ and $R_x$ directions, respectively.

Several different scatterer distributions, such as uniform, von Mises, Gaussian, and Laplacian, are used in prior work to characterize the random azimuth angles $\alpha_T$ and $\alpha_R$. In this paper, we use the von Mises pdf because it approximates many of the previously mentioned distributions and leads to closed-form solutions for many useful situations. The von Mises pdf is defined as $f(\theta) = \exp[k \cos(\theta - \mu)]/2\pi I_0(k)$ ([15], where $\theta \in [-\pi, \pi]$, $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind, $\mu \in [-\pi, \pi]$ is the mean angle at which the scatterers are distributed in the $x-y$ plane, and $k$ controls the spread of scatterers around the mean. Prior work uses several different scatterer distributions, such as uniform, cosine, and Gaussian, to characterize the random elevation angles $\beta_T$ and $\beta_R$. Here, we use the pdf

$$f(\varphi) = \left\{ \begin{array}{ll} \frac{\pi}{\varphi_m} \cos \left( \frac{\pi \varphi_m}{2} \right), & |\varphi| \leq \varphi_m \leq \frac{\pi}{2}, \\
0, & \text{otherwise} \end{array} \right.$$

because it matches well the experimental data in [14]. Parameter $\varphi_m$ is the absolute value of the maximum elevation angle.

By denoting the von Mises pdf for the $T_x$ and $R_x$ azimuth angles as $f(\alpha_T) = \exp[k \cos(\alpha_T - \mu_T)]/(2\pi I_0(k_T))$ and $f(\alpha_R) = \exp[k \cos(\alpha_R - \mu_R)]/(2\pi I_0(k_R))$, respectively, by denoting the pdf for the $T_x$ and $R_x$ elevation angles as $f(\beta_T) = \pi \cos(\pi \theta_T)/(2\pi f_{\beta_T})$ and $f(\beta_R) = \pi \cos(\pi \theta_R)/(2\pi f_{\beta_R})$, respectively, using trigonometric transformations, and the equality $f(\varphi) = e^{\varphi + \varphi_0} \cos(\theta)$, we have

$$f(\varphi) = \left\{ \begin{array}{ll} \frac{\pi}{\varphi_m} \cos \left( \frac{\pi \varphi_m}{2} \right), & |\varphi| \leq \varphi_m \leq \frac{\pi}{2}, \\
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$$f(\varphi) = \left\{ \begin{array}{ll} \frac{\pi}{\varphi_m} \cos \left( \frac{\pi \varphi_m}{2} \right), & |\varphi| \leq \varphi_m \leq \frac{\pi}{2}, \\
0, & \text{otherwise} \end{array} \right.$$
This section also uses the data that were collected on the highway in Germany [5]. The narrowband measurements were performed at 5.2 GHz and the maximum Doppler frequencies were \( f_{\text{D,max}} = f_{\text{T,max}} = 500 \) Hz. Both, the \( T_x \) and \( R_x \) were equipped with the omnidirectional antennas. The distance between the \( T_x \) and \( R_x \) was approximately \( D = 300 \) m. The moving directions and the antenna elevations are not specified in [5]. Here, we assume that they are \( \gamma_T = \gamma_R = \Delta_H = 0 \).

Figs. 2(a) and 3(a) compare the analytical LCR and AFD derived in Section III with the measured LCR and AFD obtained from our data, respectively. The analytical LCR and AFD are obtained using the parameters \( K = 2.43, \mu_T = 12.5^\circ, k_T = 20.2, \beta_{I,m} = 10^\circ, \Delta_T = \Delta_R = 0.033, \mu_R = 153.4^\circ, k_R = 18.5, \beta_{R,m} = 5^\circ, \eta_T = 0.043, \eta_R = 0.037, \) and \( \eta_{TR} = 0.92 \). The Ricean parameter \( K \) is estimated using the method in [20], and the other parameters are estimated using the method in [21]. The parameters \( D, \gamma_T, \gamma_R, \Delta_H, f_{\text{T,max}}, \) and \( f_{\text{R,max}} \) are selected as in the measurement setup described above.

Figs. 2(b) and 3(b) compare the analytical LCR and AFD with the measured LCR and AFD taken from Fig. 8 of [5]. The analytical LCR and AFD are obtained using the parameters \( K = 1.73, \mu_T = 31.2^\circ, k_T = 18.2, \beta_{I,m} = 10^\circ, \Delta_T = \Delta_R = 0.6, \mu_R = 216.3^\circ, k_R = 10.6, \beta_{R,m} = 5^\circ, \eta_T = 0.36, \eta_R = 0.22, \) and \( \eta_{TR} = 0.42 \). The parameters are manually chosen to match the curves in Fig. 8 of [5].

Figs. 2 and 3 show the close agreement between the theoretical and empirical LCR and AFD. These results confirm the utility of the proposed model. From the results we can observe that, in the urban area, the double-bounced rays bear more energy than the single-bounced rays, whereas, on the highway, the single-bounced rays are prevalent.

V. Conclusions

In this paper, the 3-D analytical model for SISO M-to-M communications is proposed. From the analytical model, the LCR and AFD are derived for a 3-D non-isotropic scattering environment. It is shown that the analytical results for the LCR and AFD compare very well with the measured data.

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References


Fig. 2. The analytical and measured LCR in an urban environment (a) and on a highway (b).

Fig. 3. The analytical and measured AFD in an urban environment (a) and on a highway (b).


