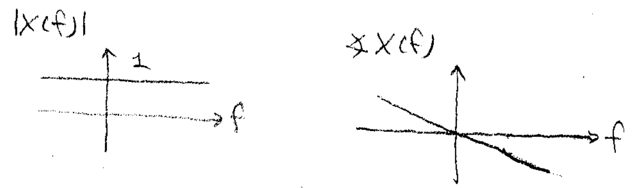
H.W #1 Solutions

Prob 1.1

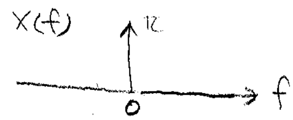
(a) $x(t) = \delta(t-0.1)$

$\boxed{X(f)} = e^{-j2\pi f 0.1} = \boxed{e^{-j0.2\pi f}}$



(b) $x(t) = \pi$

$\boxed{X(f)} = \boxed{\pi\,\delta(f)}$



(c) $x(t) = \dfrac{\sin(4\pi t)}{(4\pi t)} = \text{sinc}(4t)$

since $\text{rect}(t/\tau) \leftrightarrow \tau\,\text{sinc}(f\tau)$.

by duality of Fourier transform,

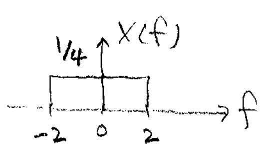
$\tau\,\text{sinc}(t\tau) \leftrightarrow \text{rect}(-f/\tau) = \text{rect}(f/\tau)$

let $\tau = 4$

$\boxed{X(f)} = \mathcal{F}\{\text{sinc}(t\tau)\}\big|_{\tau=4}$

$= \frac{1}{\tau}\,\text{rect}(f/\tau)\big|_{\tau=4}$

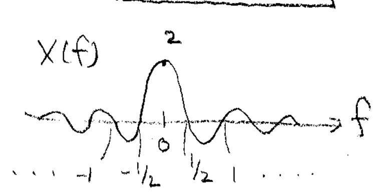
$= \boxed{\frac{1}{4}\,\text{rect}(f/4)}$



Def. $\text{sinc}(x) \triangleq \dfrac{\sin\pi x}{\pi x}$

(d) $x(t) = u(t+1) - u(t-1) = \text{rect}(t/2)$

$\boxed{X(f)} = \boxed{2\cdot\text{sinc}(2f)} = \dfrac{\sin 2\pi f}{\pi f}$



(e) $x(t) = 8\cos(8\pi t - \frac{\pi}{4})$

$= 8\cos(8\pi(t - \frac{1}{32}))$

Let $g(t) = \cos(8\pi t)$.

then $x(t) = 8\,g(t-t_0)$, $t_0 = 1/32$.

since

$g(t) = \cos(8\pi t) \leftrightarrow G(f)$

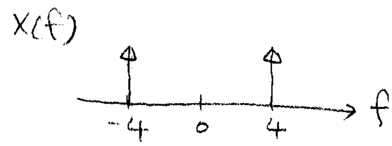
$= \frac{1}{2}[\delta(f+4) + \delta(f-4)]$

and

$g(t-t_0) \leftrightarrow G(f)\,e^{-j2\pi f t_0}$,

$\boxed{X(f)} = 8\cdot\frac{1}{2}\,e^{-j2\pi f\cdot\frac{1}{32}}\,[\delta(f+4) + \delta(f-4)]$

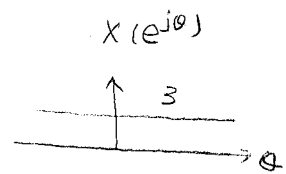
$= \boxed{4\,e^{-j\frac{\pi}{16}f}\,[\delta(f+4) + \delta(f-4)]}$



(f) $x_k = 3\,\delta_{k+3}$

$\boxed{X(e^{j\theta})} = \sum_{k=-\infty}^{\infty} x_k\,e^{-jk\theta}$

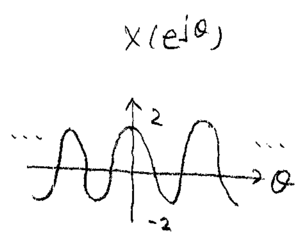
$= \boxed{3\cdot e^{j3\theta}}$



(g) $x_k = \delta_{k-1} + \delta_{k+1}$.

$\boxed{X(e^{j\theta})} = e^{j\theta} + e^{-j\theta}$

$= \boxed{2\cos(\theta)}$



Prob 1.2

(a) $X(f) = u(f+220) - u(f-220)$

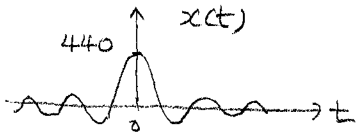
$$= \text{rect}(f/440)$$

Since as in Prob 1.1.(c),

$\tau \,\text{sinc}(t \cdot \tau) \leftrightarrow \text{rect}(f/\tau)$

Take $\tau = 440$.

$\therefore \boxed{x(t)} = \boxed{440 \,\text{sinc}(440t)}$

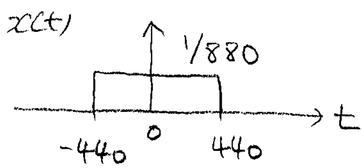


(b) $X(f) = \dfrac{\sin(880\pi f)}{880\pi f} = \text{sinc}(880 f)$

Since $\text{rect}(t/\tau) \leftrightarrow \tau \,\text{sinc}(f \cdot \tau)$

$\boxed{x(t)} = \frac{1}{\tau} \text{rect}(t/\tau) \Big|_{\tau = 880}$

$= \boxed{\frac{1}{880} \text{rect}(t/880)}$



(c) $X(f) = \delta(f+9)$

$\boxed{x(t)} = e^{-j2\pi \cdot 9 \cdot t} = \boxed{e^{-j18\pi t}}$

(d) $X(e^{j\theta}) = e^{j4\theta}$ $\quad \because \delta[n-n_0] \Leftrightarrow e^{-j\theta n_0}$

$\boxed{x_n} = \boxed{\delta[n+4]}$

(e) $X(e^{j\theta}) = 5\cos(5\theta) = \frac{5}{2}[e^{j5\theta} + e^{-j5\theta}]$

$\boxed{x_n} = \boxed{\frac{5}{2}[\delta[n+5] + \delta[n-5]]}$

Prob 1.3 Text 2.3

(a)

Let $v(t) = x(t) * g(t) \leftrightarrow V(f) = X(f)G(f)$

$w_k = h_k * v_k$, $v_k = v(kT)$

$W(e^{j2\pi fT}) = H(e^{j2\pi fT})\, V(e^{j2\pi fT})$

$$V(e^{j2\pi fT}) = \frac{1}{T}\sum_{m=-\infty}^{\infty} V(f-\frac{m}{T})$$

$$= \frac{1}{T}\sum_{m=-\infty}^{\infty} X(f-\frac{m}{T})\,G(f-\frac{m}{T})$$

$$\boxed{Y(f)} = W(e^{j2\pi fT})\cdot F(f)$$

$$= \boxed{\left[\frac{1}{T}\sum_{m=-\infty}^{\infty} X(f-\frac{m}{T})\,G(f-\frac{m}{T})\right]\cdot H(e^{j2\pi fT})\cdot F(f)}$$

(b)

System $T\{\}$ is linear iff

① Additivity

$$T\{x_1 + x_2\} = T\{x_1\} + T\{x_2\} = y_1 + y_2$$

and

② homogeneity

$$T\{ax\} = aT\{x\} = ay$$

hold.

① $T\{X_1(f) + X_2(f)\}$

$$= \frac{1}{T}\sum_m [X_1(f-\frac{m}{T}) + X_2(f-\frac{m}{T})]\,G(f-\frac{m}{T})\,H(e^{j2\pi fT})\,F(f)$$

$$= \frac{1}{T}\sum_m X_1(f-\frac{m}{T})\,G(f-\frac{m}{T})\,H(e^{j2\pi fT})\,F(f)$$

$$+ \frac{1}{T}\sum_m X_2(f-\frac{m}{T})\,G(f-\frac{m}{T})\,H(e^{j2\pi fT})\,F(f)$$

$$= T\{X_1(f)\} + T\{X_2(f)\} = Y_1(f) + Y_2(f)$$

② $T\{aX(f)\}$

$$= \frac{1}{T}\sum_m aX(f-\frac{m}{T})\,G(f-\frac{m}{T})\,H(e^{j2\pi fT})\cdot F(f)$$

$$= a\cdot \frac{1}{T}\sum_m X(f-\frac{m}{T})\,G(f-\frac{m}{T})\,H(e^{j2\pi fT})\,F(f)$$

$$= aT\{X(f)\}$$

$$= aY(f).$$

By ① & ② the system is linear.

(c) System $T\{\}$ is time-invariant

$\Leftrightarrow$ If $T\{x(t)\} = y(t)$

then $T\{x(t-t_0)\} = y(t-t_0)$

or in the freq. domain

$$T\{X(f)e^{-j2\pi f t_0}\} = Y(f)e^{-j2\pi f t_0}.$$

From (b)-② and letting $a = e^{-j2\pi f t_0}$,

the system is time-invariant

if $G(f)$, $H(e^{j2\pi fT})$, $F(f)$ does NOT depend on time.

Prob 1.4 (Text P2.4)

① $\int_{-\infty}^{\infty} |x(t)|^2 dt$

$$= \int_{-\infty}^{\infty} \left| \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \right|^2 dt$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(f) X^*(s) \underbrace{\int_{-\infty}^{\infty} e^{j2\pi(f-s)t} dt}\, df\, ds$$

since $\int_{-\infty}^{\infty} e^{j2\pi(f-s)t} dt = \delta(f-s)$

$$= \int_{-\infty}^{\infty} |X(f)|^2 df \quad //$$

② $\sum_{k=-\infty}^{\infty} |x_k|^2$

$$= \sum_{k=-\infty}^{\infty} \left| \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta}) e^{j\theta k} d\theta \right|^2$$

$$= \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} X(e^{j\theta_1}) X^*(e^{j\theta_2}) \underbrace{\sum_{k=-\infty}^{\infty} e^{j(\theta_1-\theta_2)k}}\, d\theta_1 d\theta_2$$

since $\sum_{k=-\infty}^{\infty} e^{j(\theta_1-\theta_2)k} = 2\pi\delta(\theta_1-\theta_2)$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\theta})|^2 d\theta \quad //$$

Prob 1.5 (Text p.2.5)

① $\int_{-\infty}^{\infty} x(t) y^*(t) dt$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \right) \left(\int_{-\infty}^{\infty} Y^*(s) e^{-j2\pi s t} ds \right) dt$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(f) Y^*(s) \underbrace{\int_{-\infty}^{\infty} e^{j2\pi(f-s)t} dt}\, df\, ds$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(f) Y^*(s)\, \delta(f-s)\, df\, ds$$

$$= \int_{-\infty}^{\infty} X(f) Y^*(f) df \quad //$$

② $\sum_{k=-\infty}^{\infty} x_k y_k^*$

$$= \sum_{k=-\infty}^{\infty} \left[\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta_1}) e^{j\theta_1 k} d\theta_1 \right] \left[\frac{1}{2\pi} \int_{-\pi}^{\pi} Y^*(e^{j\theta_2}) e^{-j\theta_2 k} d\theta_2 \right]$$

$$= \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} X(e^{j\theta_1}) Y^*(e^{j\theta_2}) \cdot \sum_{k=-\infty}^{\infty} e^{j(\theta_1-\theta_2)k} \cdot d\theta_1 d\theta_2$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} X(e^{j\theta_1}) Y^*(e^{j\theta_2})\, \delta(\theta_1-\theta_2)\, d\theta_1 d\theta_2$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta}) Y^*(e^{j\theta}) d\theta \quad //$$