ECE 4601
ASSIGNMENT NO. 8: PRACTICE PROBLEMS

DATE ASSIGNED: Thursday, December 4, 2003
DO BEFORE: Tuesday, December 9, 2003. Solutions will be sent to you via email.
REMINDER: These are practice problems only, do not turn in your solutions.
PROJECT: Project reports are due at 6 pm Dec. 10, 2003. You may email your report but please, do not send me a Microsoft document. Only PDF will be accepted. Late penalties apply.
FINAL EXAM: Thursday, December 11, 2:50pm – 5:40 pm, open books and notes, cumulative.

PROBLEM 8.1. Let x_{max} denote the positive value of x that maximizes the function $f(x) = x^{1/x}$. Find x_{max} to 6 decimals of precision.

PROBLEM 8.2. Suppose the relationship between the symbol-rate samples $\{r_k\}$ at the receiver and the transmitted symbols $\{a_k\}$ is given by:

$$r_k = \sum_{n=0}^{\infty} h_n a_{k-n} + noise\,,$$

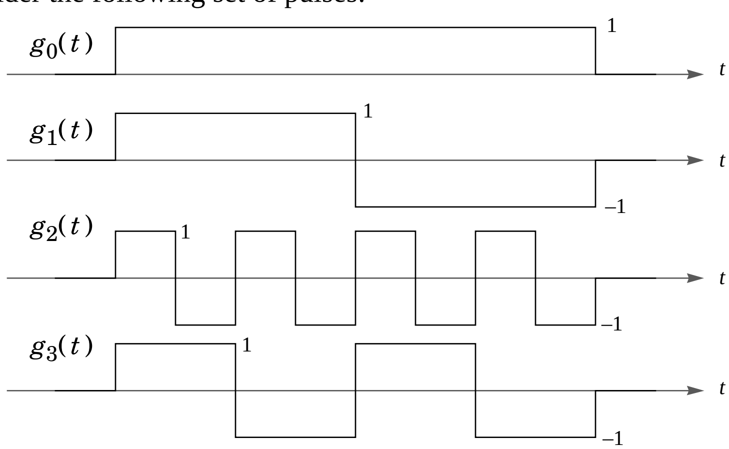
where the ISI impulse response is $h_k = 5^{-k} u_k$ (where u_k is the unit step).

(a) Find $H(z)$, the Z-transform of h_k.

(b) Specify the impulse response c_k of a zero-forcing linear equalizer.

(c) Specify the impyulse responses f_k and b_k of the forward and feedback filters, respectively, of a zero-forcing decision-gfeedback equalizer.

PROBLEM 8.3. Consider the following set of pulses:



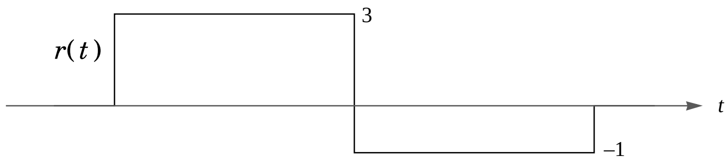
(a) Is this set *orthogonal*? (In orther words, is the correlation $\int_{-\infty}^{\infty} g_m(t) g_n(t) dt$ zero when $m \neq n$?)

(b) Suppose the above pulse set is used to create an OPAM signal of the following form:

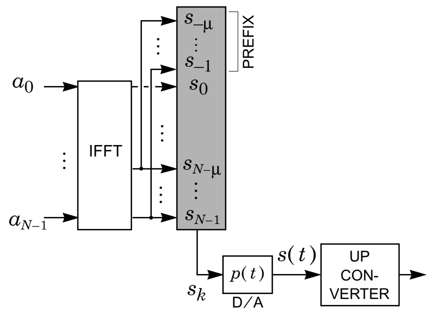
$$s(t) = \sum_{n=0}^{N-1} a_n g_n(t)\,,$$

with $N = 4$, where the alphabet is $\mathcal{A} = \{0, 1, 2, 3, 4, 5, 6, 7\}$.
Sketch $s(t)$ when $a_0 = 0$, $a_1 = 1$, $a_2 = 2$, and $a_3 = 3$.

(c) Find the minimum-distance decision $\hat{a}_0$ about the zero-th symbol when the received signal is as sketched below:



PROBLEM 8.4. Consider the following block diagram of an OFDM transmitter with a cyclic prefix:



By definition of the D/A converter, the compex envelope of the transmitted signal is:

$$s(t) = \sum_{k=-\mu}^{N-1} s_k \, p(t - kT/N) ,$$

where $p(t)$ is defined by (6.85) of the text.

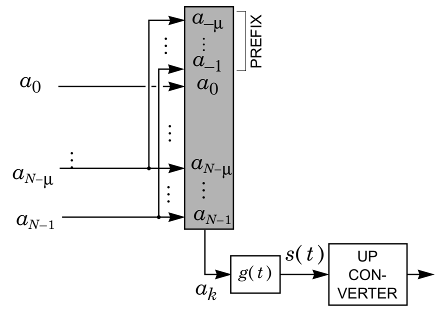
(a) Does there exist a set of pulses $\{g_n(t)\}$ such that $s(t)$ can be decomposed using the standard OPAM decomposition, as follows?

$$s(t) = \sum_{n=0}^{N-1} a_n \, g_n(t) .$$

(b) If the answer to (a) is Yes, find a simplified expression for the n-th pulse $g_n(t)$.

(c) If the answer to (a) is Yes, are the pulses $\{g_n(t)\}$ orthogonal? Prove your answer.

PROBLEM 8.5. A cyclic prefix can be beneficial even when OFDM is not used. For example, consider the following conventional QAM transmitter that prepends a cyclic prefix of length μ to each block of N QAM symbols $\{a_0, \ldots a_{N-1}\}$:



When the transmit pulse shape $g(t)$ has zero excess bandwidth, this is essentially the same as the OFDM transmitter of Prob. 8.4, but without the IFFT block. Therefore, to convey a block of N QAM symbols $\{a_0, \ldots a_{N-1}\}$, the complex envelope of the transmitted signal is:

$$s(t) = \sum_{k=-\mu}^{N-1} a_k g(t - kT) .$$

Explain how the receiver can perform linear equalization without any time-domain filtering, but instead using only an FFT block, an IFFT block, and some scalar gains.

PROBLEM 8.6. Consider the binary PAM modulation scheme $s(t) = ag(t)$, where the binary alphabet is $a \in \mathcal{A} = \{5\sqrt{E}, 6\sqrt{E}\}$. Assume $g(t)$ has unit energy.

(a) Find the normalized bandwidth requirement of this modulation scheme.

(b) Find the maximal spectral efficiency (in b/s/Hz) of this modulation scheme.

(c) Find the required value of E_b / N_0, in dB, in order to achieve a probability of error of 10^{-6}.

(d) Compare the power efficiency and bandwidth efficiency of this modulation scheme with binary antipodal signaling.