Space-Division Relay: A High-Rate Cooperation Scheme for Fading Multiple-Access Channels

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***Abstract* — We propose a new cooperation protocol for the fading multiple-access channel called *space-division relay (SDR)*. It is similar to the protocol of Laneman, Tse and Wornell (LTW) [1], except that the relays use space-division instead of time-division multiplexing. The result is a nonorthogonal cooperation protocol with a higher rate that nevertheless achieves the full diversity of the two-user Rayleigh-fading cooperative multiple-access channel, a result that is proven in this paper. We examine the SDR protocol with two types of relays: the amplify-and-forward (AF) relay and the amplify/decode-and-forward (ADF) relay. We derive the outage probability of the proposed scheme with AF relays. We present numerical results for the two-user channel at 1 bps/Hz which show that SDR-ADF outperforms all previously reported cooperative schemes. In particular, in terms of the SNR required to achieve a 10^{-3} outage probability, the SDR-AF protocol outperforms the LTW-AF protocol by 1.9 dB, while the SDR-ADF protocol outperforms another non-orthogonal protocol called NAF [4] by 1 dB, falling only 2.3 dB short of the ideal cooperation bound.**

I. Introduction

This paper concerns the cooperative multiple-access channel, where two or more *users* wish to send independent messages to a common *destination*, and where these users cooperate by occasionally acting as relays for each other. The users adhere to a half-duplex constraint, preventing them from transmitting and receiving at the same time. By sharing their antennas and signal processing resources, the users together create a "virtual transmit array" [1]-[9] that provides each user with an additional diversity against fading, thereby increasing the reliability of communication.

A noncooperative multiple-access strategy like SDMA can achieve a high rate but with low diversity. In contrast, because user cooperation necessitates that the users spend some fraction of time listening to other users and acting as relays, a cooperative strategy will have a lower rate and a higher diversity [1]. There is a fundamental tradeoff in cooperative systems between rate and diversity. The technical challenge is to design a protocol that achieves full cooperation diversity while keeping the rate loss to a minimum.

Cooperative multiple-access protocols can be classified as either *orthogonal* or *nonorthogonal*. Orthogonal protocols, such as the LTW protocol [1], are those in which different users are constrained to transmit in nonoverlapping time or frequency subchannels, thereby avoiding interference. These protocols have the advantage of simple decoding, but suffer from low rates due to the orthogonality constraint, and consequently result in high outage probabilities. Nonorthogonal protocols, such as the NAF protocol [4], allow simultaneous transmission among users. This enables a higher rate at the expense of higher decoding complexity.

This paper focuses primarily on a three-node multiple-access system, with two users sending independent information to a common destination. We propose a new nonorthogonal cooperation protocol called the *space-division relay (SDR)* protocol. It modifies the LTW protocol [1] to use space-division multiplexing instead of time-division multiplexing for the relays. We show that SDR achieves the best outage performance among all previously reported orthogonal and nonorthogonal cooperation protocols for the two-user Rayleigh-fading multiple-access channel.

This paper is organized as follows. In Section II, we describe the channel model and system assumptions. In Section III, we describe the SDR protocol. In Section IV, we derive the outage probability and diversity order of the SDR protocol. In Section V, we present some numerical results. In Section VI we present our conclusions.

II. Channel Model

We consider a Rayleigh-fading multiple-access channel with two users communicating with a common destination. Each of the three nodes is equipped with a single antenna. To simplify our presentation we consider a completely symmetric scenario: (1) both users have an identical target spectral efficiency of R bps/Hz; (2) both have an identical average energy of E per signalling interval; and (3) the average path loss between users is identical to the average path loss from each user to the destination. Asymmetry in any of these variables is easily incorporated into the system design without affecting the design principle.

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Let h_i denote the channel gain between the i-th user and the destination, and let h_{12} denote the channel gain between the two users. The channels are assumed to be linear and flat fading over the signal bandwidth. Also, the channels are assumed to be quasistatic, so that the channel response is constant over a *frame* consisting of T symbol periods, and it changes to an independent value from one frame to the next. The channel coefficients $\{h_1, h_2, h_{12}\}$ are i.i.d. circularly symmetric complex Gaussian random variables with zero mean and unit variance. The additive noise at each receiving terminal is independent circularly symmetric Gaussian random variable with zero mean and variance N_0. Under these assumptions, the SNR of each user at any receiver is $S = E/N_0$.

We assume that the users are frame-synchronized. We further assume that the destination knows all of channel coefficients $\{h_1, h_2, h_{12}\}$, whereas the users know only h_{12}.

III. The Space-Division Relay Protocol

An illustration of the proposed SDR protocol is shown in Fig. 1. The static fading frame is divided into three equal-sized *blocks* of duration $T/3$ signaling intervals. During the first block, the first user (U_1) transmits its information, while the second user (U_2) and the destination (D) each listen to the transmission. During the second block, U_2 transmits its own independent information, while U_1 and D listen to the transmission. This completes the direct transmission part of the cooperation protocol for one frame.

During the third block, both users relay the received packets from the other user *simultaneously*, in a space-division multiple access fashion, so that the destination receives a linear combination of these two transmissions. Initially, we assume that the nodes use the amplify-and-forward (AF) relaying technique [1].

The motivation of the SDR protocol is to increase the rate compared to the LTW protocol by relaxing the orthogonality

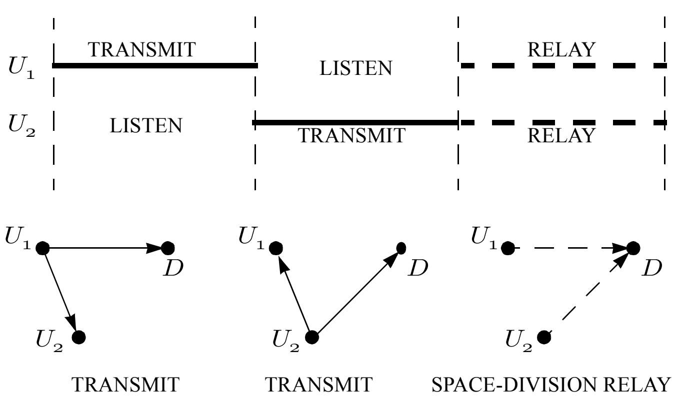

Fig. 1. Illustration of the space-division relay cooperation protocol.

constraint. Specifically, whereas the rate of each user in the LTW protocol is $1/4$, the rate of each user in the SDR protocol is $1/3$. Nevertheless we will see that SDR still achieves full cooperative diversity.

The cooperation scheme can be summarized as follows. During the first block, the first user transmits $\{x_1(1), \ldots x_1(T/3)\}$ with average symbol energy $E_t = E[|x_1(i)|^2]$, while the second user listens. The received samples at D and U_2 are given by

$$\begin{aligned} y_1(i) &= h_1 x_1(i) + n_1(i), \\ y_{12}(i) &= h_{12} x_1(i) + n_3(i), \end{aligned} \tag{1}$$

for $i \in \{1, 2, \ldots T/3\}$. During the second block, the first user listens, while the second user transmits its own information symbols $\{x_2(1), \ldots x_2(T/3)\}$. The samples received by D and U_1 are given by

$$\begin{aligned} y_2(i) &= h_2 x_2(i) + n_2(i), \\ y_{21}(i) &= h_{12} x_2(i) + n_4(i), \end{aligned} \tag{2}$$

for $i \in \{1, 2, \ldots T/3\}$.

The third block is the relay phase. During the third block in the case of AF relays, user 1 transmits $\{\alpha y_{21}(1), \ldots \alpha y_{21}(T/3)\}$ while user 2 simultaneously transmits $\{\alpha y_{12}(1), \ldots \alpha y_{12}(T/3)\}$, where α is the amplification factor:

$$\alpha = \sqrt{\frac{1}{|h_{12}|^2 + \frac{N_0}{E_t}}}. \tag{3}$$

The samples received by the destination during the third block are thus

$$y_3(i) = h_1 \alpha y_{21}(i) + h_2 \alpha y_{12}(i) + n_5(i), \tag{4}$$

for $i \in \{1, 2, \ldots T/3\}$. Since each node is silent $1/3$ of the time, the average power constraint is satisfied by choosing $E_t = 3E/2$.

Instead of AF, the nodes could also use the amplify/decode-and-forward (ADF) relay technique [6]. An ADF relay will use its knowledge of the channel coefficients to make a decision to either act as an AF relay or a decode-and-forward relay. Specifically, if the interuser channel is not in outage, i.e., if $\log_2(1 + 1.5|h_{12}|^2 E) > 3R$, then each user can perfectly decode the other's information, and hence forward a clean version to the destination. The ADF relay will thus act as a decode-and-forward relay. On the other hand, in the case of an outage, it would be counter-productive to forward erroneously decoded information, so the ADF relay simply amplifies and forwards the samples instead. This hybrid relay strategy was shown to be better than both AF and DF [6].

In the following section, we derive an expression for the outage probability and the diversity order of the SDR protocol with AF relays.

IV. Outage Analysis

In this section, we derive the outage probability of the SDR cooperation protocol with AF relays. In the SDR protocol, the observations at the destination consist of three received blocks Y_1, Y_2, and Y_3, where $Y_i = [y_i(1), y_i(2)\ldots y_i(T/3)]$, corresponding to the two blocks X_1 and X_2 transmitted by the two users, where $X_i = [x_i(1), x_i(2)\ldots x_i(T/3)]$. The discrete, memoryless multiple-access channel created by the SDR protocol is then:

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \mathbf{H}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \mathbf{AN}, \tag{5}$$

where the (i, j)-th element of the noise matrix $\mathbf{N}$ is $n_i(j)$, and where the matrices $\mathbf{H}$ and $\mathbf{A}$ given by

$$\mathbf{H} = \begin{bmatrix} h_1 & 0 \\ 0 & h_2 \\ \alpha h_2 h_{12} & \alpha h_1 h_{12} \end{bmatrix},\ \mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \alpha h_1 & \alpha h_2 \end{bmatrix}. \tag{6}$$

Also, let $\mathbf{H}_1$ and $\mathbf{H}_2$ denote the first and second columns of $\mathbf{H}$. The outage event for this multiple access system is the union of the following three events [11]:

$$\begin{aligned} \mathcal{O}_1&:\ C_{1|2} = \max_{p(x)} \frac{1}{T} I(X_1; Y|\ X_2) < R \\ \mathcal{O}_2&:\ C_{2|1} = \max_{p(x)} \frac{1}{T} I(X_2; Y|\ X_1) < R \\ \mathcal{O}_{12}&:\ C_{12} = \max_{p(x)} \frac{1}{2T} I(X_1, X_2; Y) < R, \end{aligned} \tag{7}$$

where $p(x)$ is the joint probability density function of X_1 and X_2. The outage probability of thus:

$$\begin{aligned} P_o &= \Pr[\mathcal{O}_1 \cup \mathcal{O}_2 \cup \mathcal{O}_{12}] \\ &= \Pr[\min\{C_{1|2}, C_{2|1}, C_{12}\} < R]. \end{aligned} \tag{8}$$

These quantities can be shown to be maximized when the input alphabet at each source follows an independent Gaussian distribution. Upon maximization, we get

$$\begin{aligned} C_{1|2} &= \frac{1}{3}\log_2\det(\mathbf{I} + \frac{3}{2} S\mathbf{H}_1{}^*(\mathbf{AA}^*)^{-1}\mathbf{H}_1) \\ C_{2|1} &= \frac{1}{3}\log_2\det(\mathbf{I} + \frac{3}{2} S\mathbf{H}_2{}^*(\mathbf{AA}^*)^{-1}\mathbf{H}_2) \\ C_{12} &= \frac{1}{6}\log_2\det(\mathbf{I} + \frac{3}{2} S\mathbf{H}^*(\mathbf{AA}^*)^{-1}\mathbf{H}). \end{aligned} \tag{9}$$

Intuitively, the factor $1/3$ represents the fact that the sources transmit new information only $1/3$ of the total time. The expression for $C_{1|2}$ further simplifies to:

$$\begin{aligned} C_{1|2} &= \frac{1}{3}\log_2\det(1 + \frac{3}{2} S\mathbf{H}_1{}^*(\mathbf{AA}^*)^{-1}\mathbf{H}_1) & (10) \\ &= \frac{1}{3}\log_2\left(1 + \frac{3}{2}S|h_1|^2 + \frac{3S|\alpha|^2|h_2|^2|h_{12}|^2}{2\Delta'}\right), & (11) \end{aligned}$$

where $\Delta' = 1 + |\alpha|^2(|h_1|^2 + |h_2|^2)$. Substituting for α and simplifying, we get

$$C_{1|2} = \frac{1}{3}\log_2\left(1 + \frac{3}{2}S\left(|h_1|^2 + \frac{3S|h_2|^2|h_{12}|^2}{\Delta}\right)\right), \tag{12}$$

where $\Delta = (2 + 3S(|h_2|^2 + |h_1|^2 + |h_{12}|^2))$. The expression for $C_{2|1}$ can be obtained by exchanging h_1 and h_2 in the above expression, and C_{12} can be evaluated by similar substitutions. Thus, the outage probability of the SDR protocol can be evaluated by substituting (9) and (12) into (8). We now briefly discuss the rate and diversity aspects of the SDR scheme.

Definition 1. The *rate* μ of a cooperative multiple-access protocol is the average number of information symbols transmitted by each user per signalling interval.

For a two-user CMA system, TDMA has rate $1/2$, while SDMA has rate 1. The LTW protocol has rate $1/4$, whereas the SDR protocol has rate $1/3$.

Definition 2. The *diversity order* d of a cooperative system is defined as

$$d = \lim_{S \to \infty} \frac{-\log P_o(S, R)}{\log S}, \tag{13}$$

where $P_o(S, R)$ is the outage probability of the scheme. For a two-user CMA system with one antenna at each node, TDMA and SDMA achieve a diversity order of just one, whereas the LTW protocol achieves the *full* diversity ($d = 2$).

We next show that the SDR protocol with AF achieves full diversity. The diversity order of the SDR protocol can be computed as follows. Let $P_1 = \Pr[\mathcal{O}_1]$, $P_2 = \Pr[\mathcal{O}_2]$, and $P_{12} = \Pr[\mathcal{O}_{12}]$. Let the diversity order corresponding to P_1, P_2 and P_{12} be d_1, d_2 and d_{12}, respectively. Note that the assumption that h_1 and h_2 are statistically identical implies that $d_1 = d_2$. The outage probability can be bounded using the union bound as

$$P_\delta \le P_o \le P_1 + P_2 + P_{12}, \tag{14}$$

where P_δ is either of P_1, P_2 or P_{12}. We state the following theorem on the diversity order of SDR.

Theorem 1. The SDR protocol for a two-user cooperative multiple-access channel with one antenna at each node achieves the full diversity order of $d = 2$.

Proof: (Sketch) Using the bound in (14), it is easy to show that $d_{\text{SDR}} = \min\{d_1, d_2, d_{12}\}$. Using the inequality $I(X_1, X_2; Y) \geq I(X_1; Y \mid X_2)$, we see that $2C_{12} \geq C_{1|2}$, implying that $d_{12} \geq d_1$. Using (12), the probability P_1 is given by

$$P_1 = \Pr\left[|h_1|^2 + \frac{3S|h_2|^2|h_{12}|^2}{\Delta} < \frac{2(2^{3R}-1)}{3S}\right]. \qquad (15)$$

Using the fact that $|h_1|^2$, $|h_2|^2$ and $|h_{12}|^2$ are i.i.d. exponentially distributed random variables, and employing transformation of random variables, it can be shown that P_1 decays as S^{-2} for large values of S, thus completing the proof.

The outage probability of SDR with ADF can be derived in a similar fashion and can be shown to be strictly less than SDR with AF. Consequently, SDR with ADF also achieves full diversity.

In terms of the diversity-multiplexing framework of Zheng and Tse [10], the diversity-multiplexing tradeoff of each user in the two-user SDR protocol can be shown to be

$$d(\rho) = 2(1 - 3\rho), \quad \text{for } 0 \leq \rho \leq 1/3, \qquad (16)$$

where the multiplexing gain ρ of each user is at most $1/3$.

V. Numerical Results

In this section, we present numerical results for a Rayleigh-fading cooperative multiple-access system with two users and a single destination, each equipped with one antenna. Each user has a target spectral efficiency of $R = 1$ bps/Hz, and each has the same average SNR. To achieve this target spectral efficiency, the LTW protocol needs a user to transmit information at 4 bps/Hz during its active transmissions, while SDR and NAF require the user to transmit at 3 bps/Hz and 2 bps/Hz respectively when active.

In Fig. 2, we compare several candidate schemes by plotting the outage probability versus SNR. Traditional multiple access schemes such as TDMA and SDMA perform well at low SNR, but their performance suffers from a lack of diversity at high SNR. In contrast, the benefits of cooperative diversity (LTW and SDR) at high SNR are clearly evident. At an outage probability of 10^{-3}, SDR with AF outperforms LTW with AF by 1.9 dB. A similar result (not shown) is observed with ADF relays as well. Also shown in the figure (labeled co-located bound) is the outage probability of a 2×1 MISO channel, which serves as a lower bound on the outage

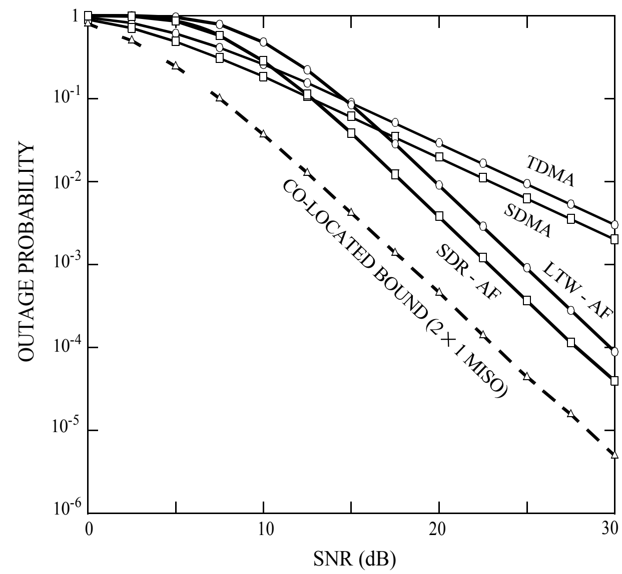

Fig. 2. Comparison of outage probabilities of various multiple access schemes for a 2-user system, with $R = 1$ bps/Hz.

probability of any CMA scheme, although it may not be achievable. We see from Fig. 2 that SDR with AF falls 4.7 dB short of the MISO bound.

In Fig. 3, we compare the performance of three nonorthogonal cooperation protocols: SDR with AF, SDR with ADF, and the NAF protocol. We see that SDR with AF is 1.4 dB worse than NAF. However, SDR with ADF outperforms NAF by 1 dB. Therefore, SDR-ADF achieves the best outage performance among all previously reported cooperation protocols, falling only 2.3 dB short of the MISO bound. For $R = 2$ bps/Hz, SDR with ADF outperforms NAF by 1.2 dB and LTW with ADF by 4.5 dB at an outage probability of 10^{-3}.

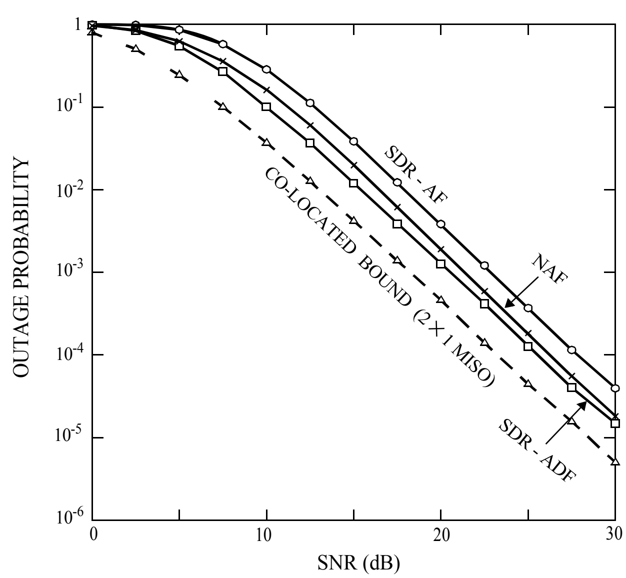

Fig. 3. Comparison of outage probabilities of non-orthogonal cooperation schemes for a 2-user system, with $R = 1$ bps/Hz.

Both SDR and NAF achieve full diversity and the rate of NAF $(1/2)$ is higher than that of SDR $(1/3)$. However, the NAF protocol suffers a power penalty because of the need for each user to share its energy between current and past symbols of its own and the other user, a drawback not captured by the definition of rate. Moreover, the sequential nature of NAF makes it incompatible with the ADF strategy. This explains the inferior performance of NAF when compared to SDR-ADF, despite its higher rate.

We note that the relative performance of these multiple-access strategies depends strongly on the target spectral efficiency and SNR. Since the rate of SDR is higher than that of LTW, the SNR improvement over LTW increases as the target spectral efficiency increases. For the same reason, NAF outperforms SDR at a sufficiently high spectral efficiency, beyond about 4 bps/Hz. However, for $R > 3.5$ bps/Hz, it turns out that SDMA requires even less SNR. Overall, of the multiple-access strategies compared in this paper, the best outage performance can be achieved by switching between SDR and SDMA as the spectral efficiency and SNR vary.

Though nonorthogonal protocols outperform orthogonal protocols, they typically have a higher decoding complexity. Roughly, in terms of the alphabet size M, the decoding complexity of LTW scales as $O(M)$, whereas the decoding complexity of SDR and NAF scales as $O(M^2)$.

The SDR protocol is even more advantageous when there are more than 2 users. SDR can be extended to a multiple-access system with N users by employing space-division multiplexing over the transmission phase as well as the relay phase. For example, in Fig. 4 we illustrate how the SDR protocol applies to the case of $N = 3$ users. From the figure we see that the rate of each user is $1/2$. In general, the rate of the SDR protocol for an N-user system is $(N-1)/(N+1)$. Interestingly, the rate of the SDR protocol per user grows with N. In stark contrast, the rate for LTW is $1/N^2$, while the rate for NAF is $1/N$. Despite the high rate, the SDR protocol is sufficient to ensure good diversity performance. For example, on the 3-user Rayleigh-fading multiple access channel, SDR-AF outperforms the corresponding LTW-AF protocol by 7.1 dB.

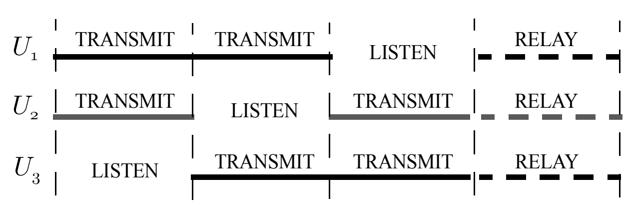

Fig. 4. Extension of the SDR protocol to $N = 3$ users.

VI. Conclusions

We proposed a new cooperative multiple-access strategy called *space-division relay (SDR)*. We introduced space-division relay as a simple nonorthogonal cooperation protocol that achieves the full cooperative diversity. SDR uses space-division multiplexing during its relay phase to achieve a higher transmission rate. We investigated SDR with both amplify-and-forward and amplify/decode-and-forward relays. We showed that the high rate of SDR-AF enables it to outperform the LTW-AF protocol by 1.9 dB at an outage probability of 10^{-3} at a target spectral efficiency of 1 bps/Hz. We also showed that SDR-ADF outperforms NAF by 1 dB. We also observe that SDR-ADF achieves the best outage performance among all previously reported protocols, falling only 2.3 dB short of the ideal cooperation bound.

VII. References

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