Optimal Training Symbol Placement for Frequency Acquisition on Magnetic Recording Channels

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I. SUMMARY

The modified Cramér-Rao bound (CRB) on frequency estimation error variance for magnetic recording channels is derived as a function of training symbol locations, and it is shown that placing the known symbols half at the beginning and half at the end of the sector minimizes the CRB. Simulation results show a 1.5 dB gain in loss-of-lock rate over having all known symbols at the start of the sector.

II. MOTIVATION AND METHOD

The constant demand for higher data densities in digital magnetic recording has led to lower operating signal-to-noise ratio (SNR) than ever before. In addition, smaller form-factors lead to higher variations in other system parameters like the motor speed and the eccentricity of the magnetic disk. These demands make the symbol frequency acquisition performance of the detector critical to the overall drive performance. In current hard drives, frequency is acquired by placing a set of known symbols, usually the so-called 2T pattern which is a periodic pattern formed by repetitions of $[1, 1, -1, -1]$, at the start of each sector and running a trained phase-locked loop (PLL) in the detector.

The conventional strategy of placing all the known symbols at the start of the sector is not necessarily optimal. In this paper, arbitrary locations are allowed for the training symbols. The Cramér-Rao bound (CRB) on frequency estimation error variance is used as a tool to evaluate different placement strategies. This analysis applies to a broad class of channels with inter-symbol interference and frequency offset, and magnetic recording channels occur as special cases. For the general problem, the CRB is intractable due to the presence of both known (training) and unknown (data) symbols. Using a looser bound called the modified CRB (MCRB) [1] leads to similar intractability. We were able to solve this problem in two special cases. The first case assumes that only the training symbols are written, *i.e.*, the unknown data symbols are absent. With this rather unrealistic assumption, the MCRB is derived as a function of the training symbol placement strategy. The second case assumes the presence of both the training and data symbols, but the channel model is simplified. A linear model for the timing observations is used, where the known symbols lead to reduced measurement noise in the timing estimator. With the known and the unknown symbols thus accounted for, the CRB for this linearized model is derived as a function of the placement strategy. Finally, the placement strategy that minimizes the bounds in these cases is derived.

III. OPTIMAL TRAINING LOCATIONS

Consider the pulse-amplitude modulated system of Figure 1, with the waveform $y(t)$ given by:

$$y(t) = \sum_{l=0}^{N-1} a_l h(t - lT - \tau_l) + n(t), \qquad (1)$$

where T is the bit period, $a_l \in \{\pm 1\}$ are the N data symbols of which K are training symbols, $h(t)$ is the channel impulse response, $n(t)$ is additive white Gaussian noise, and τ_l is the unknown timing offset for the l^{th} symbol. The timing offsets are given by $\tau_k = \tau_0 + k\Delta T$, where τ_0 is the initial timing offset, $\Delta T = T - T'$ is the symbol duration offset, and T' is the receiver's nominal estimate of T.

The relative importance of estimating the two parameters ΔT and τ_0 is quantified by the mean squared error (MSE) defined by

$$\text{MSE} = \frac{1}{N} \sum_{k=0}^{N-1} E[(\tau_k - \hat{\tau}_k)^2], \qquad (2)$$

where $\{\hat{\tau}_k\}$ are the detector's estimates of $\{\tau_k\}$. For both the cases of Section II, the CRB on frequency estimation and the MSE are minimized by the split-preamble strategy shown in Figure 2. The intuition behind the split-preamble strategy is that the observations need to be as far apart as possible while estimating the slope of a straight line.

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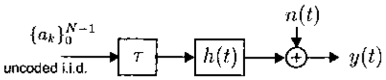

Fig. 1. System block diagram under consideration.

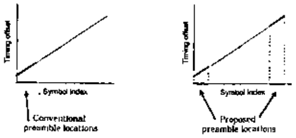

Fig. 2. Proposed training symbol placement strategy.

Three schemes are compared for performance with respect to estimating ΔT: (i) all the known symbols at the start of the packet with error variance V_1; (ii) the split-preamble arrangement with V_2; and (iii) sprinkle the known symbols uniformly throughout the packet with V_3. Figure 3 plots the normalized error variance coefficient $C = \frac{V_{\Delta T} E_{h'}}{\sigma_n^2}$ for the three strategies for $N = 4000$, K ranging from 1 to N, and assuming only training symbols being written. Here, $E_{h'}$ is the energy in the derivative of the channel impulse response, $V_{\Delta T}$ is the CRB on estimating ΔT for the particular placement scheme, and σ_n^2 is the measurement noise of the timing estimator. To get $C = 2 \times 10^{-9}$, for example, the optimal scheme needs 86 symbols, the uniform scheme needs 248 symbols, whereas the conventional scheme needs 1588 symbols of the 4000 to be known! The MSE performance comparison of the three schemes leads to similar conclusions, and is omitted for brevity. Also, the conclusions are similar with the linearized model and both training and data symbols present.

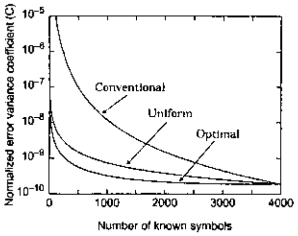

Fig. 3. Split-preamble arrangement outperforms conventional one.

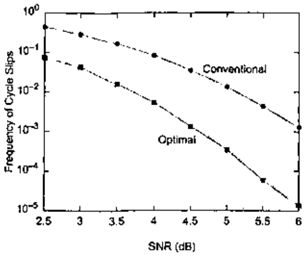

Fig. 4. Splitting the preamble reduces the occurrence of cycle slips.

An added advantage with the split-preamble method is that it is well equipped to handle cycle-slips, as shown by the simulation results in Figure 4 for a PR-IV channel [2] with $N = 5000$, $K = 120$, $\tau_0 = 0$ and $\Delta T = 0.002T$. For the conventional detector, a trained PLL is used for acquisition. With the split-preamble, a trained PLL is used during the first half of the preamble. The PLL is then switched to decision-directed mode till the end of the sector, and the PLL decisions from the end of the sector are correlated with the second half of the preamble. Using timing estimates from the two halves, a straight-line interpolation is performed to correct cycle slips that might have occurred during tracking. The PLL gain parameters are chosen to minimize the number of cycle slips for the conventional system at an SNR of 5 dB. The split-preamble acquisition scheme reduced the number of residual cycle slips by a factor of 10 to 100 depending on the SNR. In terms of SNR, at a cycle slip rate of 10^{-3}, it leads to an improvement of around 1.5 dB.

REFERENCES

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