# A Simplified Decision-Feedback Tree-Search Receiver for the Partial Erasure Channel

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***Abstract* — Non-linear distortion in the form of partial erasure is a primary impediment at high magnetic recording densities. This paper simplifies a decision-feedback tree-search receiver for combating partial erasure. Utilizing an internal flag and a pre-computed threshold lookup table, the proposed receiver requires no multiplications for its metric computation.**

## I. Introduction

The partial erasure channel was proposed in [1] to model non-linear intersymbol interference (ISI), a primary impediment at high magnetic recording densities that is caused by nonlinear interactions between neighboring transitions. Also proposed in [1] were receivers for mitigating nonlinear ISI, such as the non-linear partial-response maximum-likelihood (NPRML) receiver and the decision-feedback tree-search (DFTS) receiver.

Although significantly less complicated than the NPRML receiver, the DFTS receiver is substantially more complex than a conventional decision-feedback equalizer (DFE). In this paper, we simplify the DFTS receiver by transforming the metric computation of the tree search into a simple table lookup. While retaining the same performance as the DFTS receiver, the resulting complexity is comparable to that of the DFE.

In Section II we review the partial erasure channel model of [1]. In Section III we review the DFTS receiver of [1]. In Section IV we propose a simplified DFTS receiver, and in Section V we conclude by comparing the performance and complexity of the proposed receiver to that of a table-lookup-based DFE, the NPRML, and the original DFTS receiver.

## II. Channel Model

Fig. 1 illustrates the partial erasure channel model of [1]. The NRZI modulation encodes binary data $x_k \in \{0, 1\}$ into a binary sequence $a_k \in \{-1, 1\}$. The inductive readback process converts the sequence $a_k$ into a ternary output transition sequence $b_k \in \{-2, 0, 2\}$, where $b_k = a_k - a_{k-1}$.

At high densities, the effective readback amplitude of $b_k$ can be partially erased by neighboring transitions. Unlike linear ISI, non-linear ISI involves a significantly shorter symbol span and is usually a function of only a few adjacent transitions. The partial erasure channel model proposed in [1] is a simple and effective model for non-linear ISI. It is based on the concept of effective transition width reduction, and it assumes that the partial erasure effect is identical for both right and left transitions. As illustrated in Fig. 1, the transition $b_k$ is partially erased by a factor $r_k$ before being passed through the channel. The data-dependent amplitude $r_k$ depends on the previous and upcoming transitions and is given by:

$$r_k = \gamma^{w(b_{k-1}) + w(b_{k+1})}, \tag{1}$$

where $\gamma \in (0, 1)$ is the erasure parameter, and $w(\cdot)$ denotes the Hamming weight. Thus, $r_k$ is either 1, $\gamma$, or $\gamma^2$, depending on whether or not $b_{k-1}$ and $b_{k+1}$ are non-zero.

In Fig. 1, the linear filter $H(D)$ models the linear ISI channel, and $n_k'$ is additive white Gaussian noise with variance $N_0$. The channel output $w_k$ can be expressed as:

$$w_k = \sum_i h_i r_{k-i} b_{k-i} + n_k'. \tag{2}$$

The receiver uses a linear equalizer $F(D)$ to shape $H(D)$ into a target channel response. For example, $F(D)$ may be chosen so that the target response $C(D) = H(D)F(D)$ is either a PR4 channel or a minimum-phase channel. Regardless of $F(D)$, partial erasure causes the overall system to be non-causal. The equalizer output $z_k$ is given by:

$$z_k = r_k b_k + c_1 r_{k-1} b_{k-1} + v_k + n_k, \tag{3}$$

where $v_k = \sum_{i=2}^{\infty} c_i r_{k-i} b_{k-i}$ represents the residual ISI due to symbols of delay greater than one, $n_k$ is the filtered additive noise, and $\{c_i\}$ are the linear ISI coefficients of the equalized channel.

## III. The Decision-Feedback Tree-Search Receiver

The DFTS receiver of [1] is illustrated in Fig. 2. It has the same form as the well-known fixed-delay tree-search with decision feedback (FDTS-DF) receiver [3]. At time $k$, the

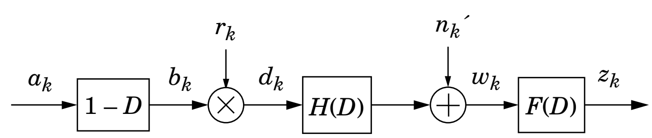


Fig. 1. The partial erasure channel of [1].

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receiver uses past decisions $\{\hat{b}_{k-1}, \hat{b}_{k-2}, \ldots\}$ to form an estimate $\hat{v}_k$ of the residual ISI, and then subtracts this estimate from $z_k$ using decision feedback, yielding $p_k = z_k - \hat{v}_k$. If past decisions within the ISI span are correct, then $p_k$ reduces to $p_k = S(b_{k-2}, b_{k-1}, b_k, b_{k+1}) + n_k$, where from (3):

$$S = r_k b_k + c_1 r_{k-1} b_{k-1}$$
$$= A\gamma^{w(b_{k+1})} b_k + B\gamma^{w(b_k)}$$
$$= \begin{cases} B & \text{if } b_k = 0 \\ Ab_k + \gamma B & \text{if } b_k = \pm 2, b_{k+1} = 0 \\ A\gamma b_k + \gamma B & \text{if } b_k = \pm 2, b_{k+1} = \pm 2 \end{cases} , \quad (4)$$

where $A = \gamma^{w(b_{k-1})}$ and $B = c_1 \gamma^{w(b_{k-2})} b_{k-1}$ are introduced to simplify notation. Observe that $A$ and $B$ depend on past decisions only.

Given $\hat{b}_{k-1}$ and $\hat{b}_{k-2}$, the tree search of [1] chooses the $b_k$ of the pair $(b_k, b_{k+1})$ that minimizes $| p_k - S |^2$. In other words, it chooses $b_k$ to minimize the minimum of the two values $| p_k - S(\hat{b}_{k-2}, \hat{b}_{k-1}, b_k, 0) |^2$ and $| p_k - S(\hat{b}_{k-2}, \hat{b}_{k-1}, b_k, \pm 2) |^2$.

## IV. A Simplified DFTS Receiver

Let $y_k = p_k - B = \tilde{S}(b_{k-2}, b_{k-1}, b_k, b_{k+1}) + n_k$, where from (4) we see that $\tilde{S} = S - B$ is given by:

$$\tilde{S} = \begin{cases} 0 & \text{if } b_k = 0 \\ Ab_k - (1-\gamma)B & \text{if } b_k = \pm 2, b_{k+1} = 0 \\ A\gamma b_k - (1-\gamma)B & \text{if } b_k = \pm 2, b_{k+1} = \pm 2 \end{cases} . \quad (5)$$

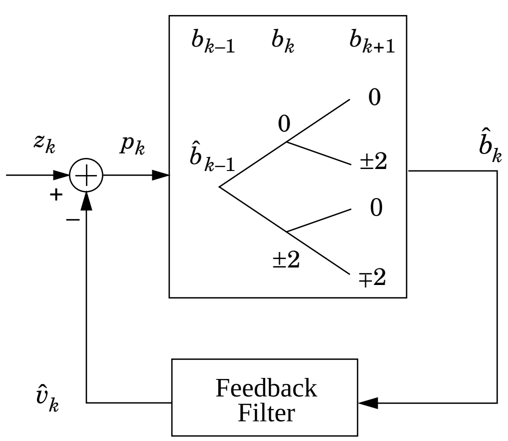


Fig. 2. The DFTS receiver of [1].

Thus, choosing $b_k$ to minimize $| p_k - S |^2$ is equivalent to choosing $b_k$ to minimize $| y_k - \tilde{S} |^2$. Because $\gamma \in (0, 1)$, and because the ISI coefficient $c_1$ is generally equalized to be positive at high densities (e.g., $c_1 = 1$ for PR4 and $c_1 = 1.65$ when an all-pass filter is used to equalize a Lorentzian channel at $pw_{50}/T = 2$ into a minimum-phase channel), the last term $\tilde{S}(b_{k-2}, b_{k-1}, \pm 2, \pm 2)$ is always closer to zero than is $\tilde{S}(b_{k-2}, b_{k-1}, \pm 2, 0)$. Hence, the DFTS can equivalently classify $b_k$ as either zero or non-zero by comparing $y_k$ to a threshold of $\tau = \frac{1}{2}\tilde{S}(\hat{b}_{k-2}, \hat{b}_{k-1}, \pm 2, \pm 2)$.

Recall that the inductive readback process constrains each non-zero transition $b_k$ to be opposite in sign from the previous non-zero transition. Let $\sigma \in \{-1, 1\}$ be an internal flag denoting the expected sign of the next transition. (For example, if $\hat{b}_{k-1}$ is nonzero then $\sigma = -\frac{1}{2}\hat{b}_{k-1}$.) In this case, the threshold can be rewritten as $\tau = A\sigma\gamma - (1-\gamma)B/2$. Thus, we see that the threshold depends only on the previous two decisions $\hat{b}_{k-1}$ and $\hat{b}_{k-2}$ (through $A$ and $B$) and on the expected sign $\sigma$ of the next transition. These data dependent threshold levels can be pre-computed and stored in a table. As shown in Table I, the lookup table has only three entries, and the sign of the threshold is controlled by the sign flag $\sigma$.

A simple logic circuit may be implemented for selecting the appropriate threshold from the table. The relatively small-sized threshold table can be adapted easily for a time-varying channel. The resulting simplified DFTS receiver is illustrated in Fig. 3.

Because the proposed receiver and the original DFTS are functionally equivalent, they will encounter the same error propagation problem, if any. However, because of the NRZI input modulation used, a transition detection error does not induce serious error propagation, as verified by the computer simulation results presented below.

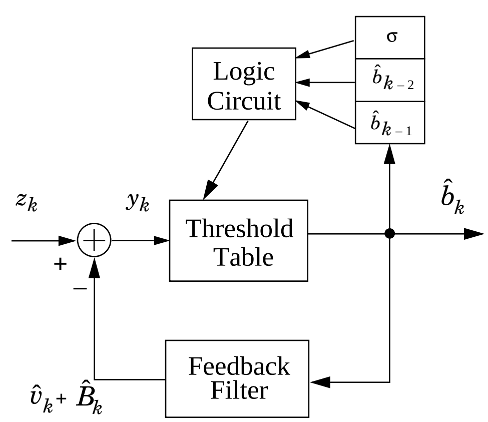


Fig. 3. The simplified DFTS receiver.

## V. DISCUSSION

We now compare the complexity of the NPRML receiver, the DFTS receiver, and a DFE receiver. We assume the DFE combats causal partial erasure only by assuming $r_k = 1$, $r_{k-1} = \gamma^{w(b_{k-2})}$, and $r_{k-j} = \gamma^{w(b_{k-j+1}) + w(b_{k-j-1})}$ for $j > 1$. To reduce complexity, the feedback filters of the DFTS and DFE receivers can be replaced by RAM lookup tables, thus eliminating computation in the feedback path. The complexity of the NPRML receiver depends radically on the target response.

Consider first a PR4 target response. In this case, the equalizer output in (3) is $z_k = r_k b_k + r_{k-1} b_{k-1} + n_k$, and a full-state Viterbi receiver (NPRML) with modified branch metrics requires eleven states in order to accommodate the memory due to both linear and non-linear ISI. For the same channel, the original DFTS receiver requires two multiplications, one addition, one table lookup, and one comparison per bit decision, whereas the simplified DFTS receiver requires no multiplications, one addition, two table lookups, and one comparison per bit decision. The DFE requires no multiplications, one addition, one table lookup, and one comparison per bit decision.

Consider next a minimum-phase target response shaped by an all-pass equalizer. The target impulse response can be quite long, making a full-state ML receiver infeasible. On the other hand, the complexity of the MP-DFTS and MP-DFE receivers is comparable to that for the PR4 case, the only difference being an increase in size of the feedback lookup tables.

Fig. 4 shows the results of Monte-Carlo simulations comparing the performance of the NPRML receiver, the DFTS receiver, and the DFE receiver discussed above. An eleven-tap Lorentzian channel was used to model $H(D)$, assuming a channel density of $pw_{50}/T = 2$, so that the discrete-time channel coefficients are given by:

$$h_k = \frac{1}{1 + k^2}, \quad \text{for } |k| \le 5. \qquad (6)$$

At $pw_{50}/T = 2$, $\gamma = 0.7$ accurately models the partial erasure phenomenon [2].

The solid curves of Fig. 4 correspond to a PR4 target response, whereas the dashed curves of Fig. 4 correspond to a minimum-phase target response. In both cases, an eleven-tap minimum-MSE approximation to the IIR equalizer $F(D)$ was used, and 10,000,000 data points were simulated for the detectors at each signal-to-noise ratio $2/N_0$.

TABLE I
THRESHOLD VALUES FOR THE SIMPLIFIED DFTS

| $\hat{b}_{k-1}$ | $\hat{b}_{k-2}$ | Threshold $\tau$ |
|---|---|---|
| 0 | – | $\sigma\gamma$ |
| ±2 | 0 | $\sigma(\gamma^2 - c_1\gamma + c_1)$ |
| ±2 | ±2 | $\sigma(\gamma^2(1 - c_1) + c_1\gamma)$ |

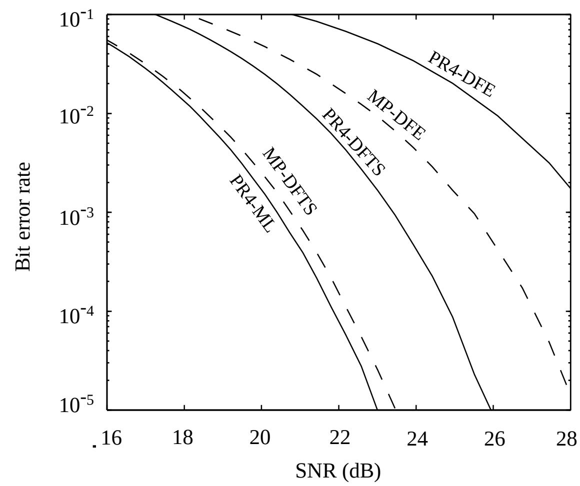

Fig. 4. Performance comparison of NPRML, DFE, and DFTS, with $\gamma = 0.7$.

The results of Fig. 4 show that the minimum-phase targeted receivers significantly outperform the PR4-targeted receivers. This is because the all-pass equalizer incurs no noise enhancement, and also because the energy of the minimum-phase impulse response is maximally concentrated in the first two taps. Thus, the slight increase in complexity of the DFTS and DFE receivers using a minimum-phase target as compared to a PR4 target is well-justified by a large performance gain. (Results for the MP-ML receiver are not shown in Fig. 4 because of its prohibitive complexity.)

## VI. CONCLUSION

We have proposed a reduced-complexity implementation of the DFTS receiver that requires no multiplications. For both minimum-phase and PR4 target responses, the simplified DFTS receiver achieves a significant performance improvement over the DFE, with only a small increase in complexity. In addition, the simplified MP-DFTS approaches the performance of the PR4-ML receiver but with a much lower complexity.

## REFERENCES


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