Chapter 12 Problems

1. a) $\ddot{y} + 2\dot{y} + 4y = 2v$

let $x_1 = y$, $x_2 = \dot{y}$,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} v$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

b) $\ddot{y} - 4y = v$

let $x_1 = y$, $x_2 = \dot{y}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

c) $y[n+2] + 2y[n+1] + 4y[n] = 2v[n]$

let $x_1[n] = y[n]$
$x_2[n] = y[n+1]$

$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} x_1[n] \\ x_2[n] \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} v[n]$$

$$y[n] = \begin{bmatrix} 1 & 0 \end{bmatrix} x[n]$$

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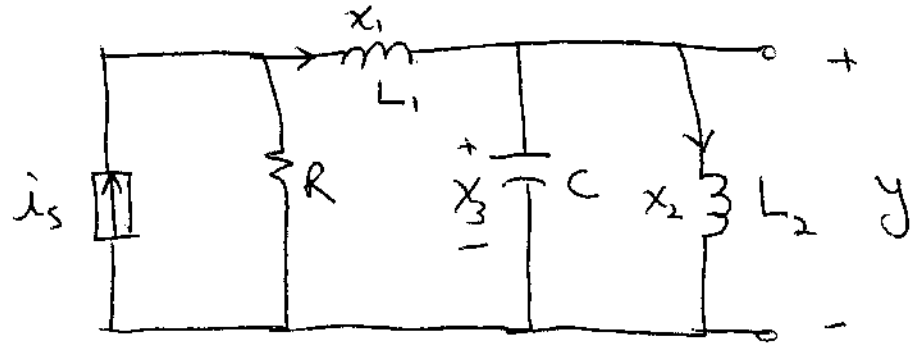

, let $v = i_s$

choose the state variables as inductor currents and capacitor voltages (as shown in the diagram).

Perform Kirchoff's voltage law to get loop equations.

① $L_1 \dot{x}_1 + x_3 + R(-i_s + x_1) = 0$

② $L_2 \dot{x}_2 = x_3$

Perform Kirchoff's current law to get current relationships

③ $x_1 = C\dot{x}_3 + x_2$

Rewrite as

$$\dot{x}_1 = \frac{1}{L_1}(-Rx_1 - x_3 + Ri_s)$$

$$\dot{x}_2 = \frac{1}{L_2}x_3$$

$$\dot{x}_3 = \frac{1}{C}(x_1 - x_2)$$

In state space form,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -R/L_1 & 0 & -1/L_1 \\ 0 & 0 & 1/L_2 \\ 1/C & -1/C & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} R/L_1 \\ 0 \\ 0 \end{bmatrix} v$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

3. a) $\dot{x} = \begin{bmatrix} -3 & -1 \\ -4 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$

$y = \begin{bmatrix} 4 & -1 \end{bmatrix} x$

$$(sI-A)^{-1} = \begin{bmatrix} s+3 & 1 \\ 4 & s+2 \end{bmatrix}^{-1} = \begin{bmatrix} s+2 & -1 \\ -4 & s+3 \end{bmatrix} \frac{1}{s^2+5s+2}$$

$$H(s) = C(sI-A)^{-1}B$$

$$= \begin{bmatrix} 4 & -1 \end{bmatrix} \begin{bmatrix} s+2 & -1 \\ -4 & s+3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1}{s^2+5s+2}$$

$$= \begin{bmatrix} 4 & -1 \end{bmatrix} \begin{bmatrix} s+2 \\ -4 \end{bmatrix} \frac{1}{s^2+5s+2}$$

$$= \frac{4s+12}{s^2+5s+2}$$

b) $x[n+1] = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix} x[n] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v[n]$

$y[n] = \begin{bmatrix} 1 & -1 \end{bmatrix} x[n] + v[n]$

$$(zI - A)^{-1} = \begin{bmatrix} z & -1 \\ 1 & z+3 \end{bmatrix}^{-1} = \begin{bmatrix} z+3 & 1 \\ -1 & z \end{bmatrix} \frac{1}{z^2+3z+1}$$

$$H(z) = C(zI - A)^{-1}B + D$$

$$= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} z+3 & 1 \\ -1 & z \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{z^2+3z+1} + 1$$

$$= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ z \end{bmatrix} \frac{1}{z^2+3z+1} + 1$$

$$= \frac{1-z}{z^2+3z+1} + 1$$

$$= \frac{z^2+2z+2}{z^2+3z+1}$$

4 a) $H(s) = \dfrac{4s+12}{s^2+5s+2}$

$$Y(s) = H(s)\,\frac{1}{s}$$

$$= \frac{6}{s} + \frac{(-.33)}{s+4.56} + \frac{(-5.67)}{s+0.438}$$

$$y(t) = 6 - 0.33e^{-4.56t} - 5.67e^{-0.438t}, \quad t \geq 0$$

if solving for x(t), use the following:

$$X(s) = (sI-A)^{-1}BV(s)$$

$$= \begin{bmatrix} s+2 \\ -4 \end{bmatrix} \frac{1}{s^3+5s^2+2s}$$

$$= \begin{bmatrix} \dfrac{s+2}{(s^2+5s+2)s} \\[2ex] \dfrac{-4}{(s^2+5s+2)s} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{s} - \dfrac{0.136}{s+4.56} - \dfrac{0.864}{s+0.438} \\[2ex] \dfrac{-2}{s} - \dfrac{0.213}{s+4.56} + \dfrac{2.21}{s+0.438} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} 1 - 0.136e^{-4.56t} - 0.864e^{-0.438t} \\ -2 - 0.213e^{-4.56t} + 2.21e^{-0.438t} \end{bmatrix}, \quad t \geq 0$$

b) if solving for $y[n]$, use

$$Y(z) = C(zI - A)^{-1}B + D$$

if solving for both $x[n]$ and $y[n]$, solve for $x[n]$ using

$X(z) = (zI - A)^{-1} B V(z)$ first, then use $y[n] = Cx[n] + v[n]$

$$X(z) = \begin{bmatrix} z+3 & 1 \\ -1 & z \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{z^2+3z+1} \frac{z}{z-1}$$

$$= \begin{bmatrix} \dfrac{z}{(z-1)(z^2+3z+1)} \\[2ex] \dfrac{z^2}{(z-1)(z^2+3z+1)} \end{bmatrix} = \begin{bmatrix} \dfrac{0.2}{z-1} - \dfrac{0.32}{z+2.6} + \dfrac{0.12}{z+0.38} \\[2ex] \dfrac{0.2}{z-1} + \dfrac{0.847}{z+2.6} - \dfrac{0.047}{z+0.38} \end{bmatrix}$$

let $W(z) = X(z)z \Rightarrow x[n] = w[n-1]$

$$w[n] = \begin{bmatrix} 0.2u[n] - 0.32(-2.6)^n u[n] + 0.12(-0.38)^n u[n] \\ 0.2u[n] + 0.847(-2.6)^n u[n] - 0.047(-0.38)^n u[n] \end{bmatrix}$$

$$x[n] = \begin{bmatrix} 0.2 - .32(-2.6)^{n-1} + 0.12(-0.38)^{n-1} \\ 0.2 + 0.847(-2.6)^{n-1} - 0.047(-0.38)^{n-1} \end{bmatrix} u[n-1]$$

$$y[n] = [1 \ \ -1]\, x[n] + v[n]$$

$$y[n] = -1.167(-2.6)^{n-1}u[n-1] + 0.167(-0.38)^{n-1}u[n] + u[n]$$