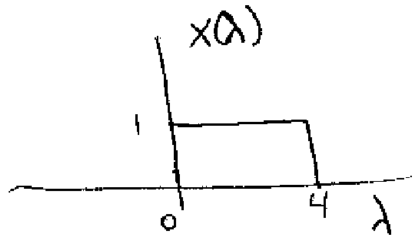
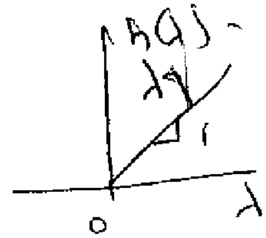
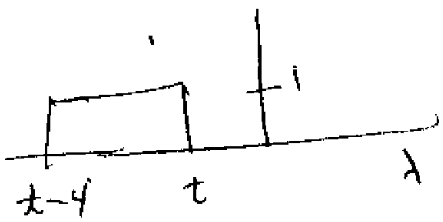
1 $x(t) = u(t) - u(t-4)$ $\quad h(t) = r(t)$





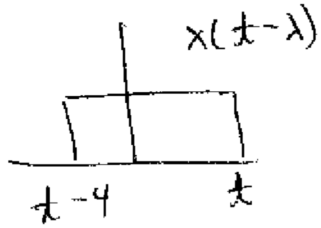
since $h(t)$ is more complicated, flip & switch x instead of h. That is, compute $h(t) * x(t)$

$$= \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$



i) $t < 0$, then $h(t) * x(t) = 0$

ii)

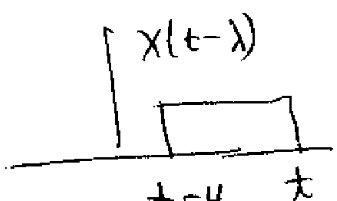


if $0 < t$ & $t-4 < 0$

or $0 < t < 4$

$$h(t) * x(t) = \int_0^t \lambda\, d\lambda = \frac{\lambda^2}{2}\Big|_0^t = \frac{t^2}{2}$$

iii)



if $4 < t$

$$h(t) * x(t) = \int_{t-4}^{t} \lambda\, d\lambda = \frac{\lambda^2}{2}\Big|_{t-4}^{t}$$

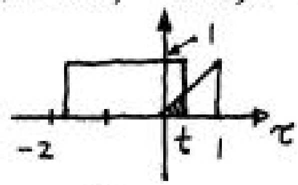
$$= \frac{t^2}{2} - \frac{(t-4)^2}{2}$$

$$h(t) * x(t) = \begin{cases} 0 & t < 0 \\ t^2/2 & 0 \le t < 4 \\ 4t - 8 & 4 \le t \end{cases}$$

2. (a) $x(t) = \delta(t)$, $h(t) = e^{-2t}u(t)$

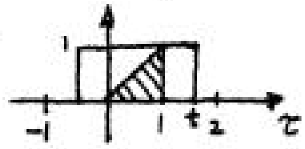
$y(t) = x(t) * h(t) = \delta(t) * h(t) = h(t) = e^{-2t}u(t)$

(b) $0 \le t < 1$,



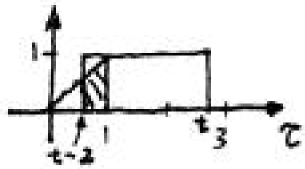
$y(t) = \int_0^t x(\tau)d\tau = \int_0^t \tau\, d\tau = \frac{\tau^2}{2}\Big]_0^t$
$= \frac{t^2}{2}$

$1 \le t < 2$,



$y(t) = \frac{1}{2}$

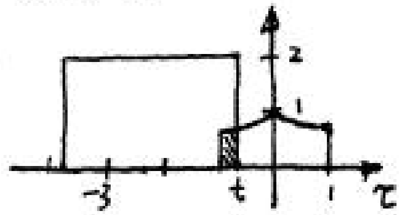
$2 \le t < 3$,



$y(t) = \int_{t-2}^1 x(\tau)d\tau = \frac{\tau^2}{2}\Big]_{t-2}^1$
$= \frac{1}{2} - \frac{(t-2)^2}{2}$

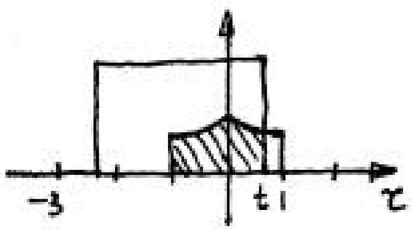
elsewhere, $y(t) = 0$

(c) $-1 \le t < 0$,



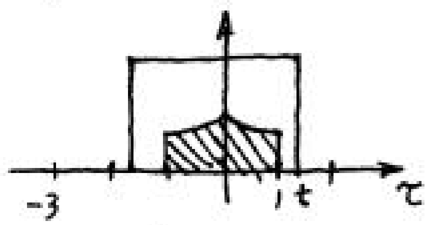
$y(t) = 2\int_{-1}^t e^{\tau}d\tau = 2e^{\tau}\Big]_{-1}^t$
$= 2e^t - 2e^{-1}$
$= 2(e^t - e^{-1})$

$0 \le t < 1$,



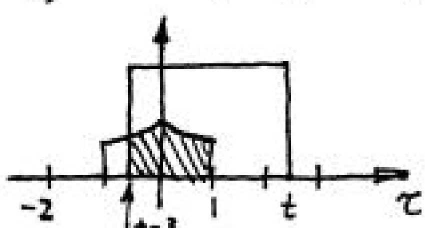
$y(t) = 2\int_{-1}^0 e^{\tau}d\tau + 2\int_0^t e^{-\tau}d\tau$
$= 2e^{\tau}\Big]_{-1}^0 + (-2)e^{-\tau}\Big]_0^t$
$= 2 - 2e^{-1} - 2e^{-t} + 2$
$= 4 - 2e^{-1} - 2e^{-t}$

$1 \le t < 2$,



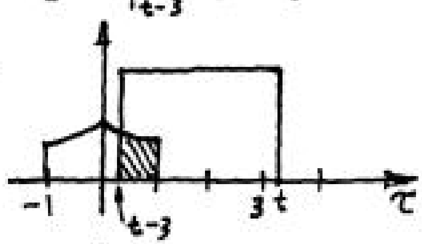
$y(t) = 2\int_{-1}^0 e^{\tau}d\tau + 2\int_0^1 e^{-\tau}d\tau$
$= 2(1 - e^{-1}) + 2(1 - e^{-1})$
$= 4(1 - e^{-1})$

$2 \le t < 3$,



$y(t) = 2\int_{t-3}^0 e^{\tau}d\tau + 2\int_0^1 e^{-\tau}d\tau$
$= 2(1 - e^{t-3}) + 2(1 - e^{-1})$
$= 4 - 2e^{-1} - 2e^{t-3}$

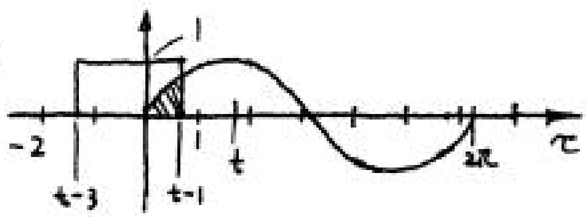
$3 \le t < 4$,



$y(t) = 2\int_{t-3}^1 e^{-\tau}d\tau = -2e^{-\tau}\Big]_{t-3}^1$
$= 2e^{-t+3} - 2e^{-1}$
$= 2(e^{3-t} - e^{-1})$

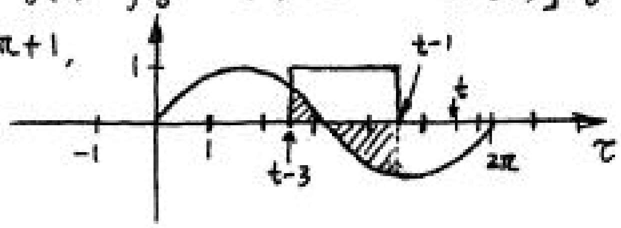
elsewhere, $y(t) = 0$

(d) $1 \le t < 3$,



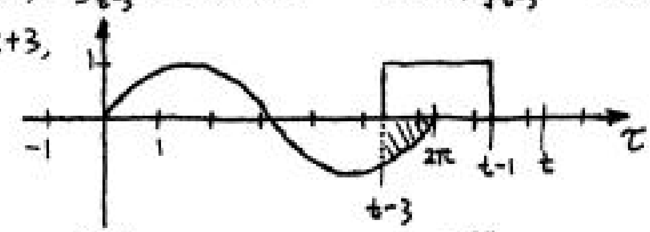
$$y(t) = \int_0^{t-1} \sin(\tau)\,d\tau = -\cos(\tau)\Big]_0^{t-1} = 1 - \cos(t-1)$$

$3 \le t < 2\pi + 1$,



$$y(t) = \int_{t-3}^{t-1} \sin(\tau)\,d\tau = -\cos(\tau)\Big]_{t-3}^{t-1} = \cos(t-3) - \cos(t-1)$$

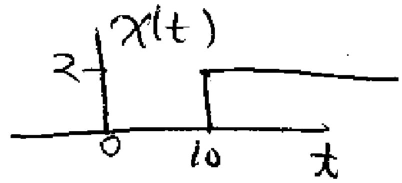
$2\pi + 1 \le t < 2\pi + 3$,

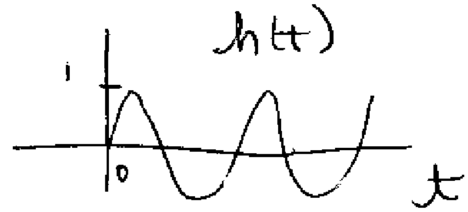


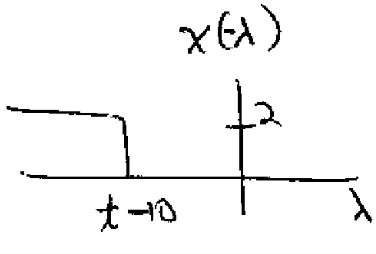
$$y(t) = \int_{t-3}^{2\pi} \sin(\tau)\,d\tau = -\cos(\tau)\Big]_{t-3}^{2\pi} = \cos(t-3) - 1$$

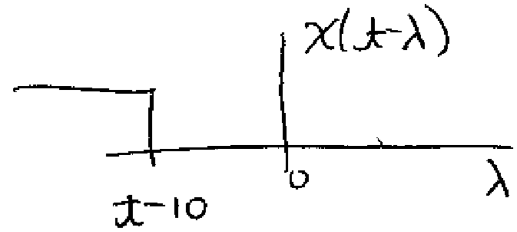
elsewhere. $y(t) = 0$

3





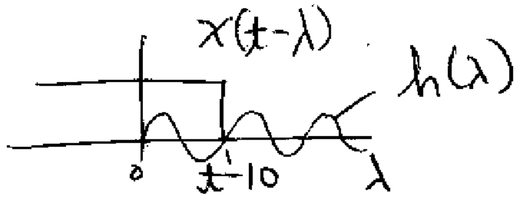




$$y(t) = x(t) * h(t)$$

$y(t) = 0$ if $t \le 10$

if $t \ge 10$



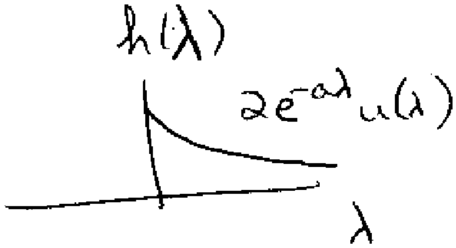
$$y(t) = \int_0^{t-10} 2\sin(2\lambda)\, d\lambda$$

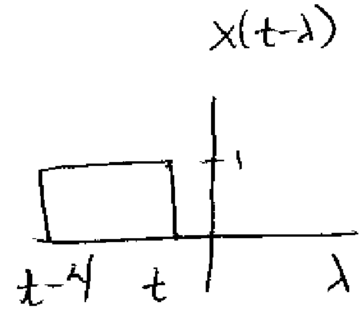
$$= -\cos(2\lambda)\Big|_0^{t-10}$$

$$= -\cos\left(2(t-10)\right) + 1$$

$$y(t) = \begin{cases} 0 & t \le 10 \\ 1-\cos(2(t-10)) & t > 10 \end{cases}$$

4





$t<0, \quad y(t)=0$

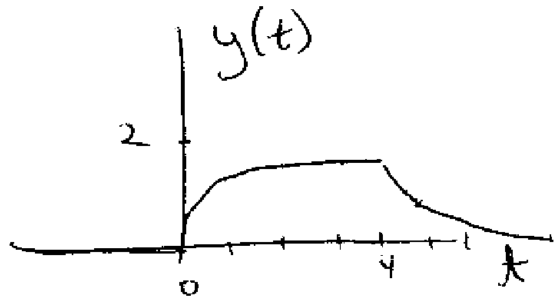
$0 \le t < 4, \quad y(t) = \int_0^t 2e^{-a\lambda}\,d\lambda = \frac{2}{-a} e^{-a\lambda}\Big|_0^t = \frac{2}{-a}\left(e^{-at}-1\right)$

$4 \le t, \quad y(t) = \int_{t-4}^{t} 2e^{-a\lambda}\,d\lambda = \frac{2}{-a} e^{-a\lambda}\Big|_{t-4}^{t} = \frac{2}{a}\left(e^{-a(t-4)} - e^{-at}\right)$

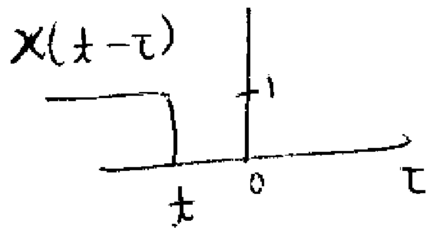
$$y(t) = \begin{cases} 0 & t<0 \\ \frac{2}{a}\left(1-e^{-at}\right) & 0 \le t < 4 \\ \frac{2}{a}\left(e^{-a(t-4)} - e^{-at}\right) & 4 \le t \end{cases}$$

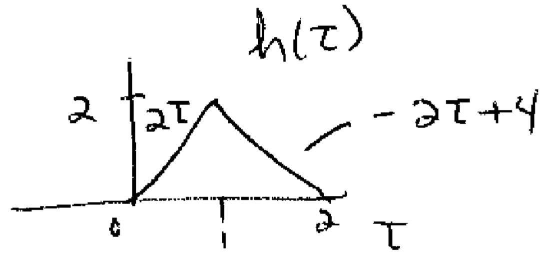
if $a=1$

$$y(t) = \begin{cases} 0 & t<0 \\ 2\left(1-e^{-t}\right) & 0 \le t < 4 \\ 2\left(e^{-(t-4)} - e^{-t}\right) & 4 \le t \end{cases}$$



5.





i) $t<0, \quad y(t)=0$

ii) $0 \le t < 1, \quad y(t) = \int_0^t 2\tau \, d\tau = t^2$

iii) $1 \le t < 2, \quad y(t) = \int_0^1 2\tau \, d\tau + \int_1^t -2\tau+4 \, d\tau$

$$= 1 + (-\tau^2 + 4\tau)\Big|_1^t = 1 - t^2 + 4t + 1 - 4)$$

$$= -2 - t^2 + 4t$$

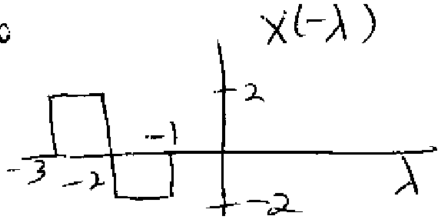
iv) $2 \le t, \quad y(t) = \int_0^1 2\tau \, d\tau + \int_1^2 -2\tau + 4 \, d\tau$

$$= 2$$

$$y(t) = \begin{cases} 0 & t<0 \\ t^2 & 0 \le t < 1 \\ -t^2+4t-2 & 1 \le t < 2 \\ 2 & 2 \le t \end{cases}$$

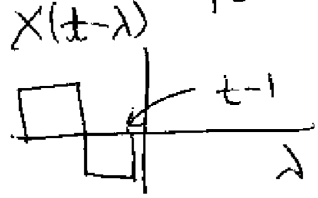
6

$v(t) * x(t)$ is easier to do

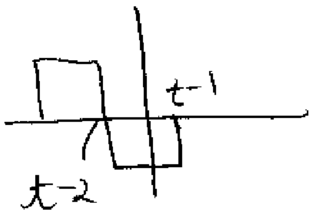


1) if $t-1 < 0$ (or $t < 1$)

$v(t) * x(t) = 0$



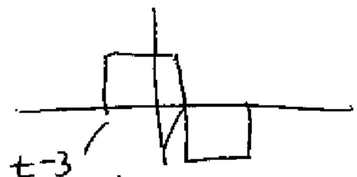
2)



if $t-1 > 0$ & $t-2 < 0$ (or $1 < t < 2$)

$$v(t) * x(t) = \int_0^{t-1} (-2) e^{-\lambda} d\lambda = 2e^{-\lambda}\Big|_0^{t-1} = 2\left(e^{-(t-1)} - 1\right)$$

3) if $t-2 > 0$ & $t-3 < 0$ (or $2 < t < 3$)

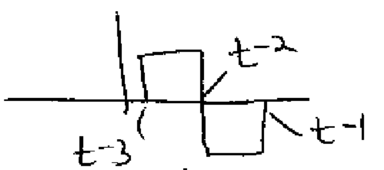


$$v(t) * x(t) = \int_0^{t-2} 2e^{-\lambda} d\lambda + \int_{t-2}^{t-1} (-2) e^{-\lambda} d\lambda$$

$$= -2\left(e^{-(t-2)} - 1\right) + 2\left(e^{-(t-1)} - e^{-(t-2)}\right)$$

$$= 2\left(1 + e^{-(t-1)} - 2e^{-(t-2)}\right)$$

4) if $t > 3$



$$v(t) * x(t) = \int_{t-3}^{t-2} 2e^{-\lambda} d\lambda + \int_{t-2}^{t-1} (-2) e^{-\lambda} d\lambda = 2\left(e^{-(t-1)} - 2e^{-(t-2)} + e^{-(t-3)}\right)$$

$$x(t) * v(t) = \begin{cases} 0 & t < 1 \\ 2e^{-(t-1)} - 2 & 1 \le t < 2 \\ 2 + 2e^{-(t-1)} - 4e^{-(t-2)} & 2 \le t < 3 \\ 2e^{-(t-1)} - 4e^{-(t-2)} + 2e^{-(t-3)} & t \ge 3 \end{cases}$$