

$$1a) y[n] + 0.2y[n-1] = x[n] - x[n-1]$$

$$h[n] + 0.2h[n-1] = \delta[n] - \delta[n-1]$$

$$h[0] = -0.2h[-1] + \delta[0] - \delta[-1] = 1$$

$$h[1] = -0.2h[0] + \delta[1] - \delta[0] = -0.2 - 1 = -1.2$$

$$h[2] = -0.2h[1] = 0.24$$

$$h[3] = -0.2h[2] = -0.048$$

$$\vdots$$
$$h[n] = (-0.2)^n (+1.2) \quad \text{for } n \geq 1$$

$$b) y[n] + 1.2y[n-1] = 2x[n-1]$$

$$h[n] + 1.2h[n-1] = 2\delta[n-1]$$

$$h[0] = -1.2h[-1] + 2\delta[-1] = 0$$

$$h[1] = -1.2h[0] + 2\delta[0] = 2$$

$$h[2] = -1.2h[1] + 2\delta[1] = -1.2(2)$$

$$h[3] = -1.2h[2] = (-1.2)^2(2)$$

$$\vdots$$

$$h[n] = (-1.2)^{n-1}(2) \quad \text{for } n \geq 1$$

$$c) y[n] = 0.24 (x[n] + x[n-1] + x[n-2] + x[n-3])$$

$$h[n] = 0.24 (\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3])$$

$$= \begin{cases} 0.24 & 0 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

$$d) y[n] = x[n] + 0.5x[n-1] + x[n-2]$$

$$h[n] = \delta[n] + 0.5\delta[n-1] + \delta[n-2]$$

$$h[n] = \begin{bmatrix} 1 & .5 & 1 \end{bmatrix}, \quad h[n] = 0 \quad \text{all other } n$$

$\uparrow$   
 $n=0$

$$2. a) x[n] = u[n] - u[n-4], v[n] = 0.5^n u[n]$$

$$\begin{aligned} x[n] * v[n] &= \sum_{k=-\infty}^{\infty} x[k] v[n-k] \\ &= \sum_{k=-\infty}^{\infty} (u[k] - u[k-4]) 0.5^{n-k} u[n-k] \end{aligned}$$

if  $0 \leq n \leq 4$

$$= \sum_{k=0}^n 0.5^{n-k} = 0.5^n \frac{1-2^{n+1}}{1-2} = -(0.5^n - 2)$$

if  $4 < n$

$$= \sum_{k=0}^4 0.5^{n-k} = 0.5^n \frac{1-2^5}{1-2} = -0.5^n + 0.5^{n-5}$$

$$b) x[n] = [1 \ 4 \ 8 \ 2], v[n] = [0 \ 1 \ 2 \ 3 \ 4]$$

$x[n]$	1	4	8	2	0	0	0	0	0
$v[n]$	0	1	2	3	4	0	0	0	0
	0	1	2	3	4	16	32	8	
		0	4	8	12	24	6		
			0	8	16	4			
				0	2				
$y[n] =$	[ 0	1	6	19	34	44	38	8	0 ]
	↑								
	$n=0$								

$$c) x[n] = u[n], v[n] = 2(.8)^n u[n]$$

$$\begin{aligned}
 x[n] * v[n] &= \sum_{k=-\infty}^{\infty} u[k] 2(.8)^{n-k} u[n-k] \\
 &= \sum_{k=0}^n 2(.8)^{n-k} \\
 &= 2(.8)^n \sum_{k=0}^n (.8)^{-k} = 2(.8)^n \frac{1 - 1.25^{n+1}}{1 - 1.25} \\
 &= -8[0.8^n - 1.25], \quad n \geq 0 \\
 &= -8(.8)^n + 10, \quad n \geq 0
 \end{aligned}$$

d)

$$y[n] = \sum_{k=-\infty}^{\infty} u[k-1] 2 (.5)^{n-k} u[n-k]$$

$$= \sum_{k=1}^n 2 (.5)^{n-k} \quad , \quad n \geq 1$$

$$= 2 (.5)^n \sum_{k=1}^n 2^k$$

$$= 2 (.5)^n \left( \sum_{k=0}^n 2^k - 1 \right)$$

$$= 2 (.5)^n \left[ \frac{1 - 2^{n+1}}{1 - 2} - 1 \right]$$

$$= 2 (.5)^n (-2 + 2^{n+1})$$

$$= -(.5)^{n-2} + 4 \quad , \quad n \geq 1$$