1 a) $4\sin(100t)u(t) \longleftrightarrow \frac{400}{s^2+100^2}$

b) $4\sin(100t-10)u(t-0.1) = 4\sin(100(t-0.1))$

$4\sin(100t) \longleftrightarrow \frac{400}{s^2+100^2}$

$4\sin(100(t-.1))u(t-.1) \longleftrightarrow \frac{400}{s^2+100^2}e^{-0.1s}$

c) $2u(t)+\delta(t-4)-\cos(5t)u(t) \longleftrightarrow \frac{2}{s}+e^{-4s}-\frac{s}{s^2+25}$

d) $tu(t)-2(t-2)u(t-2)+(t-3)u(t-3) \longleftrightarrow \frac{1}{s^2}-\frac{2}{s^2}e^{-2s}+\frac{1}{s^2}e^{-3s}$

e) $u(t)-e^{-2t}\cos(10t) \longleftrightarrow \frac{1}{s}-\frac{s+2}{(s+2)^2+10^2}$

2 a) $X(s)=\frac{10(s+1)}{s^2+4s+3}=\frac{10\cancel{(s+1)}}{\cancel{(s+1)}(s+3)}=\frac{10}{s+3}$

$x(t)=10e^{-3t}u(t)$

b) $X(s)=\frac{10(s+1)}{s^2+4s+8}=\frac{10(s+1)}{(s+2)^2+4}=\frac{10(s+2)}{(s+2)^2+4}-\frac{10}{(s+2)^2+4}$

$x(t)=\left(10e^{-2t}\cos(4t)-5e^{-2t}\sin(4t)\right)u(t)$

c) $X(s)=\frac{2s+100}{(s+1)(s+8)(s+10)}=\frac{c_1}{s+1}+\frac{c_2}{s+8}+\frac{c_3}{s+10}$

$c_1=X(s)(s+1)\Big|_{s=-1}=1.555 \qquad c_2=-4.667 \qquad c_3=4.44$

$x(t)=\left(1.555e^{-t}-4.667e^{-8t}+4.44e^{-10t}\right)u(t)$

d) $X(s) = \frac{10(s+1)}{s^2+4s+3} e^{-2s} = X_1(s) e^{-2s}$

from a) $x_1(t) = 10e^{-3t} u(t)$

so $x(t) = 10e^{-3(t-2)} u(t-2)$

e) $X(s) = \frac{20}{s(s^2+10s+16)} = \frac{C_1}{s} + \frac{C_2}{s+2} + \frac{C_3}{s+8}$

$C_1 = \frac{5}{4}$, $C_2 = \left.\frac{20}{s(s+8)}\right|_{s=-2} = \frac{-20}{12} = \frac{-5}{3}$, $C_3 = \left.\frac{20}{s(s+2)}\right|_{s=-8} = \frac{5}{12}$

$x(t) = \left(\frac{5}{4} - \frac{5}{3} e^{-2t} + \frac{5}{12} e^{-8t}\right) u(t)$

f) $X(s) = \frac{10(s+1)}{s(s^2+4s+8)} = \frac{10(s+1)}{s((s+2)^2+4)} = \frac{C_1}{s} + \frac{C_2 s + C_3}{(s+2)^2+4}$

$C_1 = 5/4$

$10(s+1) = \frac{5}{4}(s^2+4s+8) + C_2 s^2 + C_3 s \Rightarrow C_2 = -\frac{5}{4}, C_3 = 5$

$X(s) = \frac{5/4}{s} + \frac{-5/4\, s + 5}{(s+2)^2+4} = \frac{5/4}{s} - \frac{5/4(s+2)}{(s+2)^2+4} + \frac{15/2}{(s+2)^2+4}$

$x(t) = \frac{5}{4} u(t) - \frac{5}{4} e^{-2t} \cos(2t) u(t) + \frac{15}{4} e^{-2t} \sin(2t)\, u(t)$

3a) $X(s) = \dfrac{10(s+1)}{s(s^2+4s+3)}$

poles of $sX(s)$ are $-3, -1$ $\Rightarrow$ limit exists

$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) = \dfrac{10}{3}$

b) $X(s) = \dfrac{10(s+1)}{s(s^2+4s+8)}$

poles of $sX(s)$ are $-2 \pm 2j$

$<0 \Rightarrow$ limit exists

$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) = \dfrac{5}{4}$

c) $X(s) = \dfrac{10(s+1)}{s(s^2+2s-3)}$

poles of $sX(s)$ are $3, -1$; $3>0$ so no limit exists

4 a) $X(s) = \dfrac{2s+100}{(s+2)(s+6)(s+10)}$

$x(t) = c_1 e^{-2t} + c_2 e^{-6t} + c_3 e^{-10t}, \quad t \geq 0$

b) $X(s) = \dfrac{2s+100}{s(s+1)(s+8)(s-4)}$

$x(t) = c_1 + c_2 e^{-t} + c_3 e^{-8t} + c_4 e^{4t}, \quad t \geq 0$

c) $X(s) = \dfrac{s-40}{(s+1)(s+8)(s+10)}$, $x(t) = c_1 e^{-t} + c_2 e^{-8t} + c_3 e^{-10t}, \quad t \geq 0$

d) $X(s) = \dfrac{10(s+1)}{s(s^2+4s+3)} = \dfrac{10(s+1)}{s(s+1)(s+3)}$

$x(t) = c_1 + c_2 e^{-t} + c_3 e^{-3t}, \quad t \geq 0$

e) $X(s) = \dfrac{10(s+1)}{s(s^2+4s+8)} = \dfrac{10(s+1)}{s((s+2)^2+4)}$

$x(t) = c_1 + c_2 e^{-2t} \cos(2t+\theta), \quad t \geq 0$

f) $X(s) = \dfrac{s+1}{s(s^2+4)(s+8)}$, $x(t) = c_1 + c_2 \cos(2t+\theta) + c_3 e^{-8t}, \quad t \geq 0$

g) $X(s) = \dfrac{20(s+1)}{(s^2+16)((s+4)^2+25)(s+1)}$, $x(t) = c_1 \cos(4t+\theta_1) + c_2 e^{-4t} \cos(5t+\theta_2) + c_3 e^{-t}, \quad t \geq 0$