1. a) $\dot{y} + 4y = 3x$

$$(s+4)\,Y(s) = 3\,X(s)$$

$$H(s) = \frac{3}{s+4}$$

b) $\ddot{y} + 4\dot{y} + 20y = 2\dot{x} - x$

$$(s^2+4s+20)\,Y(s) = (2s-1)\,X(s)$$

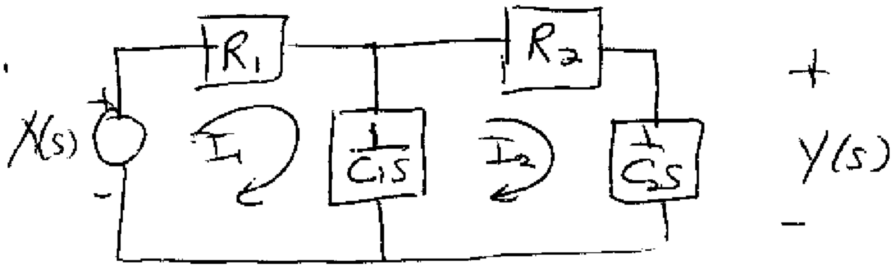
$$H(s) = \frac{2s-1}{s^2+4s+20}$$

c) $\dddot{y} - 3\ddot{y} + 4\dot{y} + 8y = 4\ddot{x} - 2\dot{x} + x$

$$(s^3 - 3s^2 + 4s + 8)\,Y(s) = (4s^2 - 2s + 1)\,X(s)$$

$$H(s) = \frac{4s^2 - 2s + 1}{s^3 - 3s^2 + 4s + 8}$$

2.



1) $X(s) = I_1 R_1 + (I_1 - I_2)\frac{1}{C_1 s}$ using Mesh analysis

2) $0 = R_2 I_2 + \frac{1}{C_2 s} I_2 + (I_2 - I_1)\frac{1}{C_1 s}$

$$0 = \left(R_2 + \frac{1}{C_2 s} + \frac{1}{C_1 s}\right) I_2 - I_1 \frac{1}{C_1 s}$$

$$\Rightarrow I_1 = C_1 s \left(R_2 + \frac{1}{C_2 s} + \frac{1}{C_1 s}\right) I_2$$

into 1): $X(s) = \left[\left(R_1 + \frac{1}{C_1 s}\right) C_1 s \left(R_2 + \frac{1}{C_2 s} + \frac{1}{C_1 s}\right) - \frac{1}{C_1 s}\right] I_2$

$$\Rightarrow Y(s) = \frac{1}{C_2 s} I_2 = \frac{1}{C_2 s} \frac{X(s)}{(R_1 C_1 s + 1)\left(\frac{R_2 C_1 C_2 s + C_1 + C_2}{C_1 C_2 s}\right) - \frac{1}{C_1 s}}$$

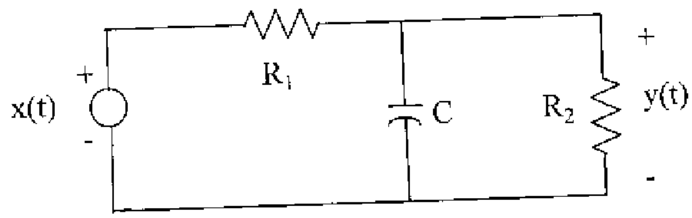
$$H(s) = \frac{1}{(R_1 C_1 s + 1)\frac{(R_2 C_1 C_2 s + C_1 + C_2)}{C_1} - \frac{C_2}{C_1}}$$

$$= \frac{C_1}{(R_1 C_1 s + 1)(R_2 C_1 C_2 s + C_1 + C_2) - C_2}$$

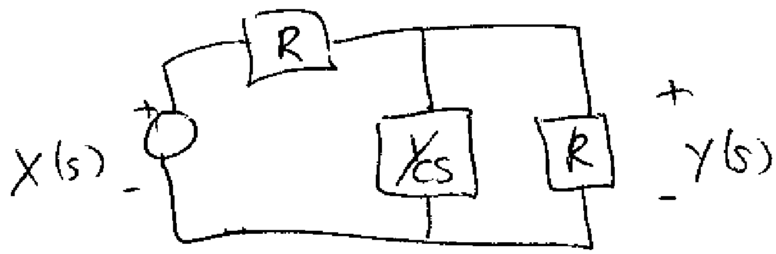
If $C_1 = C_2 = 100\,\mu f$ & $R_1 = R_2 = 2000\,\Omega$

$$H(s) = \frac{0.0001}{(.2s + 1)(.000025 + .0002) - .0001}$$

$$H(s) = \frac{25}{s^2 + 15s + 50}$$



3



$$R \parallel \frac{1}{CS} = \frac{1}{1/R + CS} = \frac{R}{1+RCS}$$

$$Y(s) = X(s)\,\frac{\frac{R}{1+RCS}}{R + \frac{R}{1+RCS}}$$ by voltage divider law

$$H(s) = \frac{R}{R + R^2CS + R}$$

$$= \frac{1000}{2000 + 100s} = \frac{10}{s+20}$$

4 $\ddot{y} + 8\dot{y} + 116y = 116x$

a) $(s^2 + 8s + 116)Y(s) = 116X(s)$

$$H(s) = \frac{116}{s^2+8s+116}$$

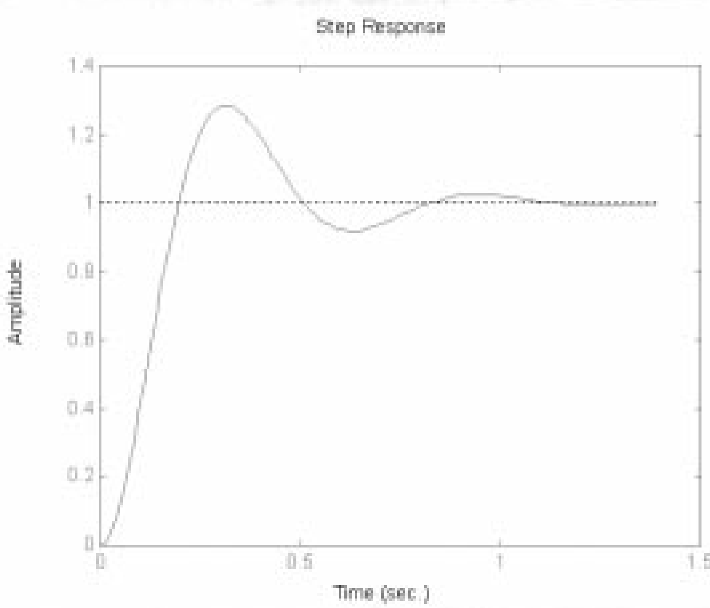
b) zeros: none

poles: $-4 \pm 10j$

c) $Y(s) = H(s)X(s) = \dfrac{116}{\left((s+4)^2+10^2\right)s}$

$y(t) = c_1 + c_2 e^{-4t}\cos(10t+\theta)$, $t \geq 0$

```
num= 116;
den = [1  8  116];
step (num, den)
title ('your name')
```



5. $\ddot{y} + 8\dot{y} + 12y = 12x$

a) $(s^2 + 8s + 12)\, Y(s) = 12\, X(s)$

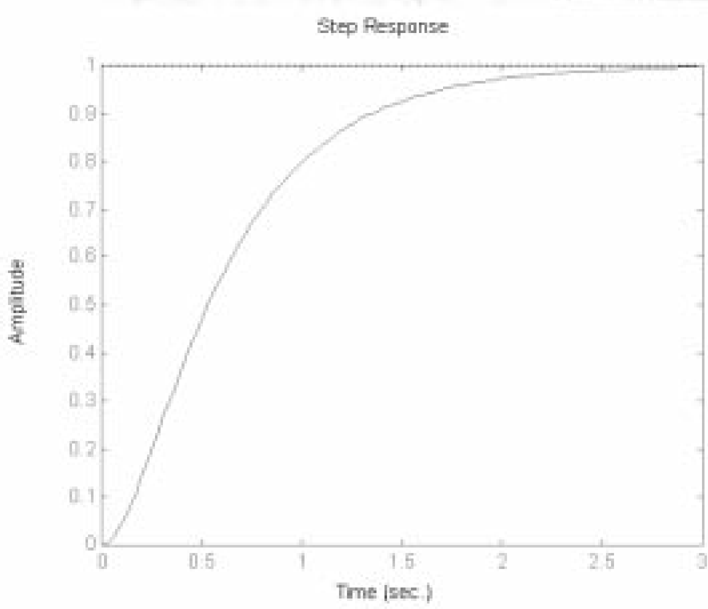
$$H(s) = \frac{12}{s^2 + 8s + 12}$$

b) zeros: none
poles: $-6, -2$

c) $Y(s) = H(s)\, X(s) = \dfrac{12}{s(s^2 + 8s + 12)}$

$$y(t) = c_1 + c_2 e^{-6t} + c_3 e^{-2t}, \quad t \geq 0$$

d) the system in Problem 4) has complex poles, which gives rise to oscillations in the response, while real poles do not



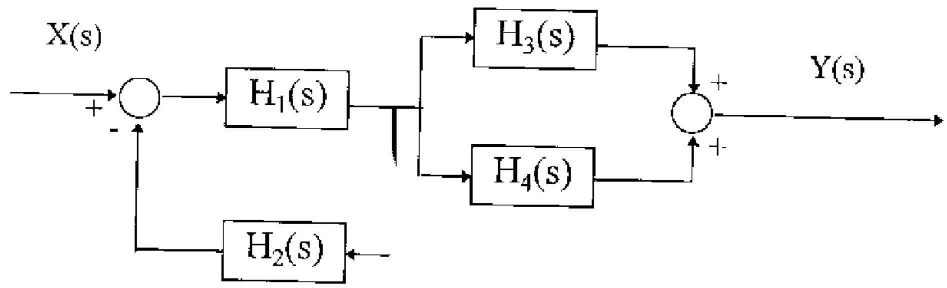
6.

$$H(s) = H_3(s)\,\frac{H_1(s) + H_2(s)}{1 + \big(H_1(s) + H_2(s)\big) H_4(s)}$$

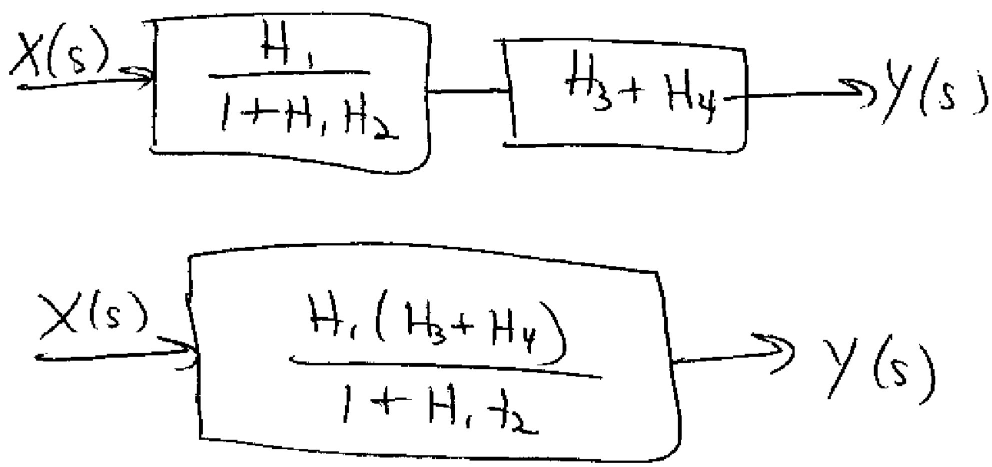
$$= \frac{0.1}{s+20}\left[\frac{2 + 10/s}{1 + (2 + 10/s)\,2/s+4}\right]$$

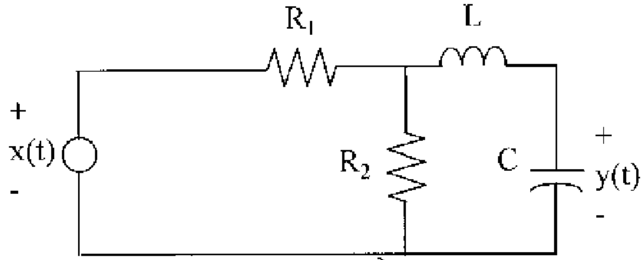
$$= \frac{.1}{s+20}\;\frac{2s(s+4) + 10(s+4)}{s(s+4) + 4s + 20}$$

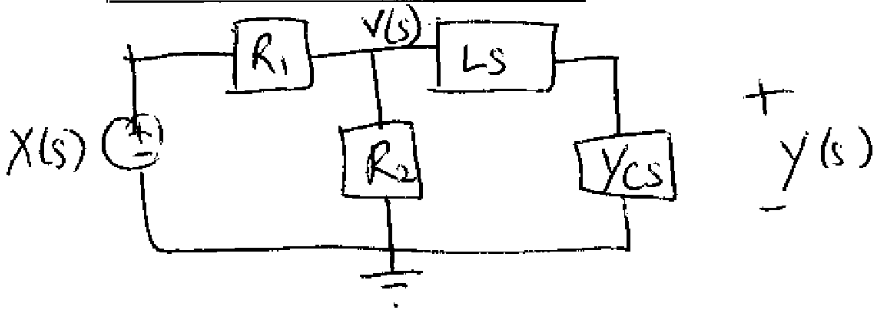
$$= \frac{0.2\,(s+5)(s+4)}{(s+20)(s^2 + 8s + 20)}$$


X(s)
H1(s)
H2(s)
H3(s)
H4(s)
Y(s)
+
-
+
+

7.


X(s)
$\frac{H_1}{1+H_1H_2}$
$H_3 + H_4$
Y(s)
X(s)
$\frac{H_1(H_3+H_4)}{1+H_1H_2}$
Y(s)





node analysis:

① $$\frac{V(s)-X(s)}{R_1} + \frac{V(s)}{R_2} + \frac{V(s)}{LS + 1/CS} = 0$$

$$V(s)\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{CS}{CLS^2+1}\right) = \frac{1}{R_1} X(s)$$

$$V(s) = \frac{1}{\cancel{R_1}}\left(\frac{\cancel{R_1}R_2(CLS^2+1)}{CR_1LS + R_1 + CR_2LS^2 + R_2 + CR_1R_2S}\right) X(s)$$

② $$\frac{Y(s)-V(s)}{LS} + \frac{Y(s)}{1/CS} = 0$$

$$Y(s) = \frac{1}{LS}\left(\frac{LS}{LCS^2+1}\right) V(s) = \frac{R_2}{CR_1L\,s^2 + CR_2LS^2 + CR_1R_2S + R_1 + R_2} X(s)$$

$$H(s) = \frac{R_2}{s^2 CL(R_1+R_2) + s\,CR_1R_2 + R_1 + R_2}$$

poles: $s^2 .004 + s\,400 + 4000 = 0$

poles at $-99989, -10$