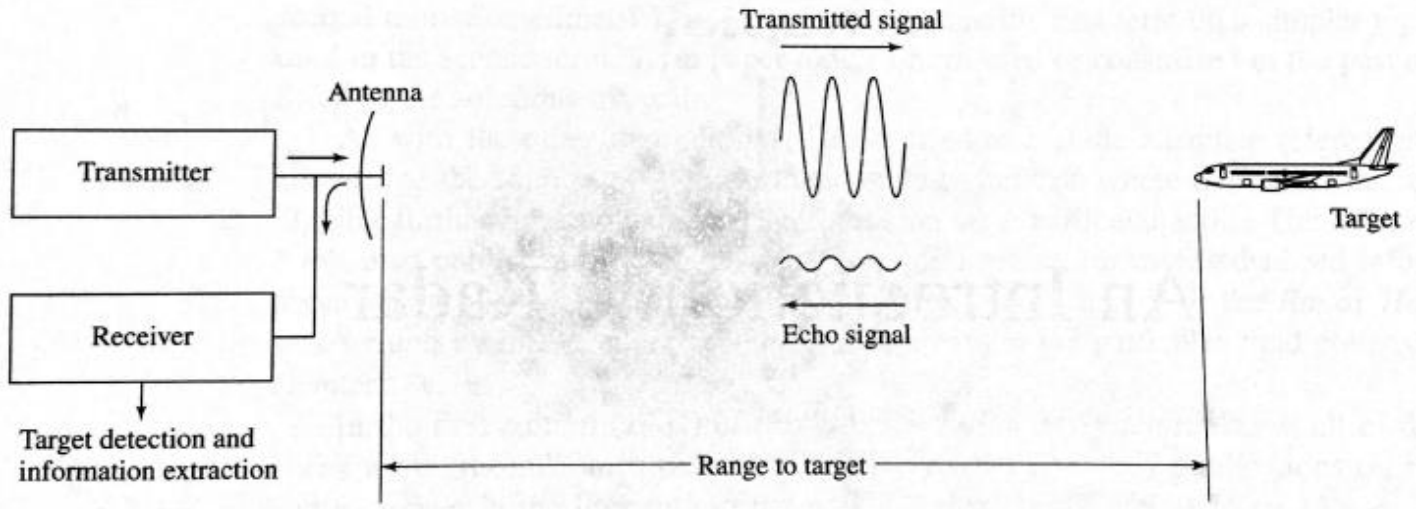
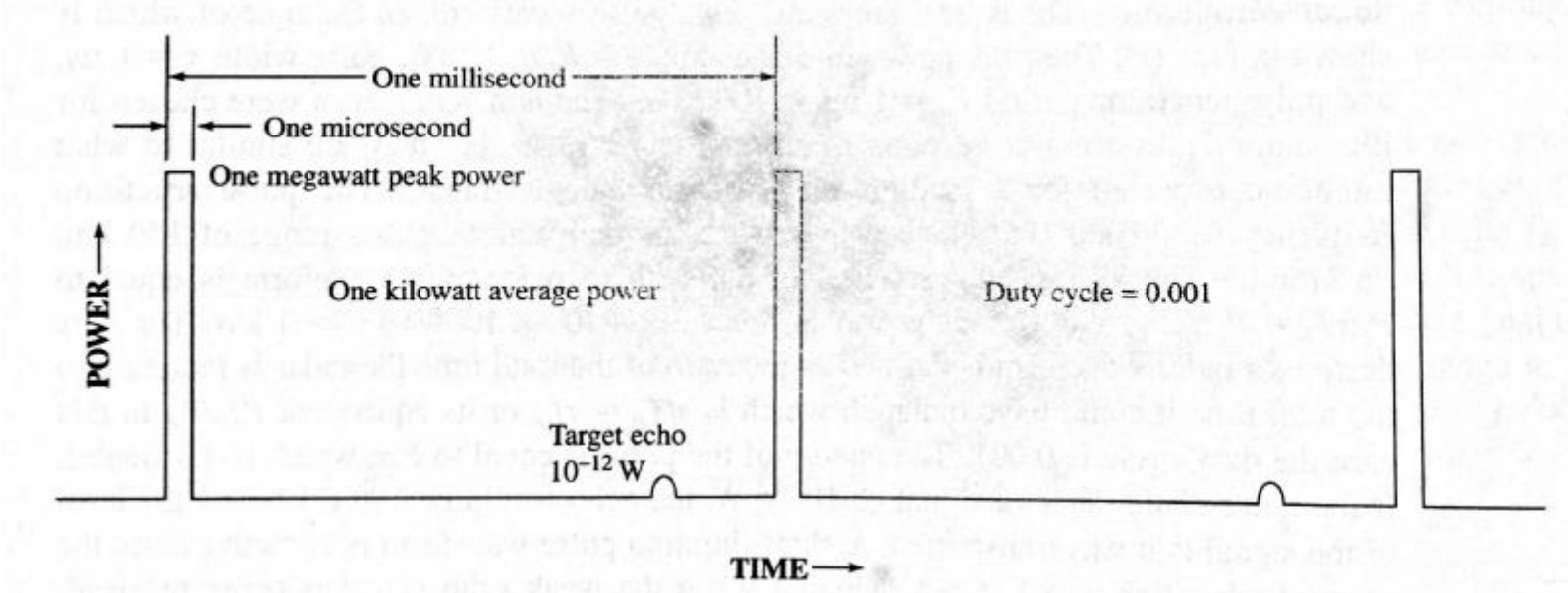
Radar (Radio Detection And Ranging) Principle



- Most common radar waveform: a series of short-duration "rectangular-shaped" pulses modulating a sinewave carrier [pulse train]
- Range to the target: determined by the time T_R for the radar signal to travel to the target and back [$R=c*T_R/2$]
- R(km) = 0.15 T_R (us) or R(nmi) = 0.081 T_R [us]
- E.g. 1us of round trip corrsponds to a range of 150m, 492 feet, 0.081 nautical mile or 0.093 statute mile

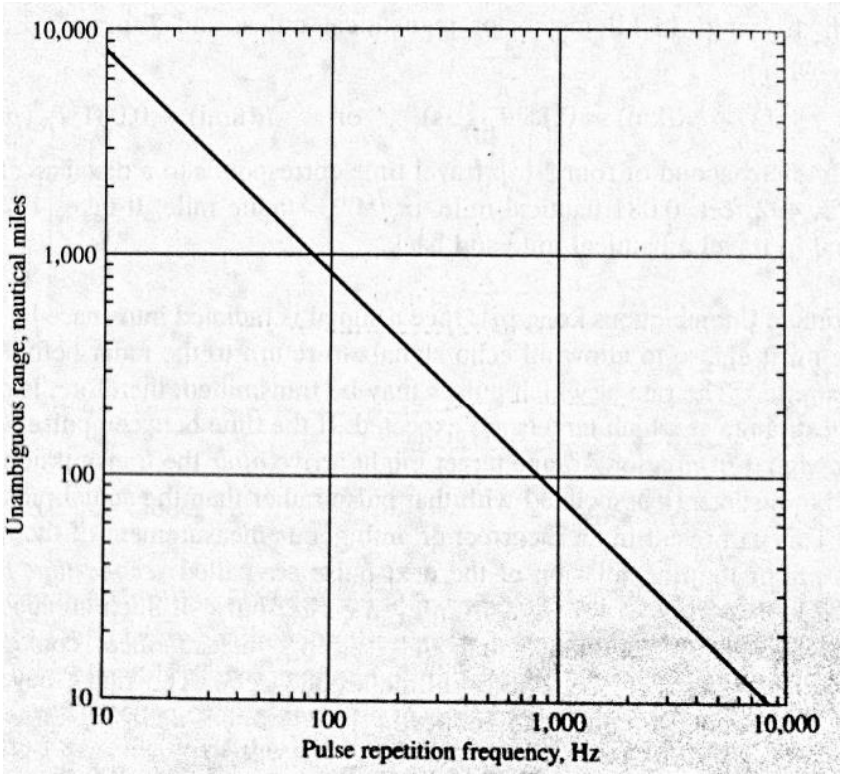
1 nautical mile = 1nmi = 1,852m and 1 statute mile = 1mi = 1,609m

Radar Waveforms



- Pulse width: τ
- Pulse repetition frequency (prf): f_p
- Time between consecutive pulses: T_p
- Average power P_{av}= Peak power $P_t * \tau / T_p$
- Duty cycle is $\tau / T_p = \tau * f_p = P_{av} / P_t$
- If T_p is too short, echo from a long range target might arrive after the transmission of the next pulse and be mistakenly associated with that pulse instead of the actual pulse transmitted earlier (second-time-around-echo or multiple-time-around-echo)

Maximum Unambiguous Range



- Maximum Unambiguous Range: the range beyond which targets appear as second-time-around echos $R_{un} = c\,T_p / 2 = c / 2f_p$

1 nautical mile = 1nmi = 1,852m and 1 statute mile = 1mi = 1,609m

Radar Range Equation

Antenna Gain G, A_e = antenna effective area, A=antenna physical area,
ρ_α = radiation efficiency, λ=c/f=wavelength

$$G = \frac{\text{maximum power density radiated by a directive antenna}}{\text{power density radiated by a lossless isotropic antenna with the same power input}}$$

$$G = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \rho_\alpha A}{\lambda^2}$$

$$\text{Power density at range } R \text{ from a directive antenna} = \frac{P_t G}{4\pi R^2}$$

$$\text{Reradiated power density back at the radar} = \frac{P_t G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2}$$

$$P_r = \frac{P_t G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2} \cdot A_e = \frac{P_t G A_e \sigma}{(4\pi)^2 R^4}$$

The target intercepts a portion of incident energy and reradiates it
(σ: radar cross section, minimum detected signal by the receiver $P_r = S_{min}$)

$$R_{\max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{\min}} \right]^{1/4}$$

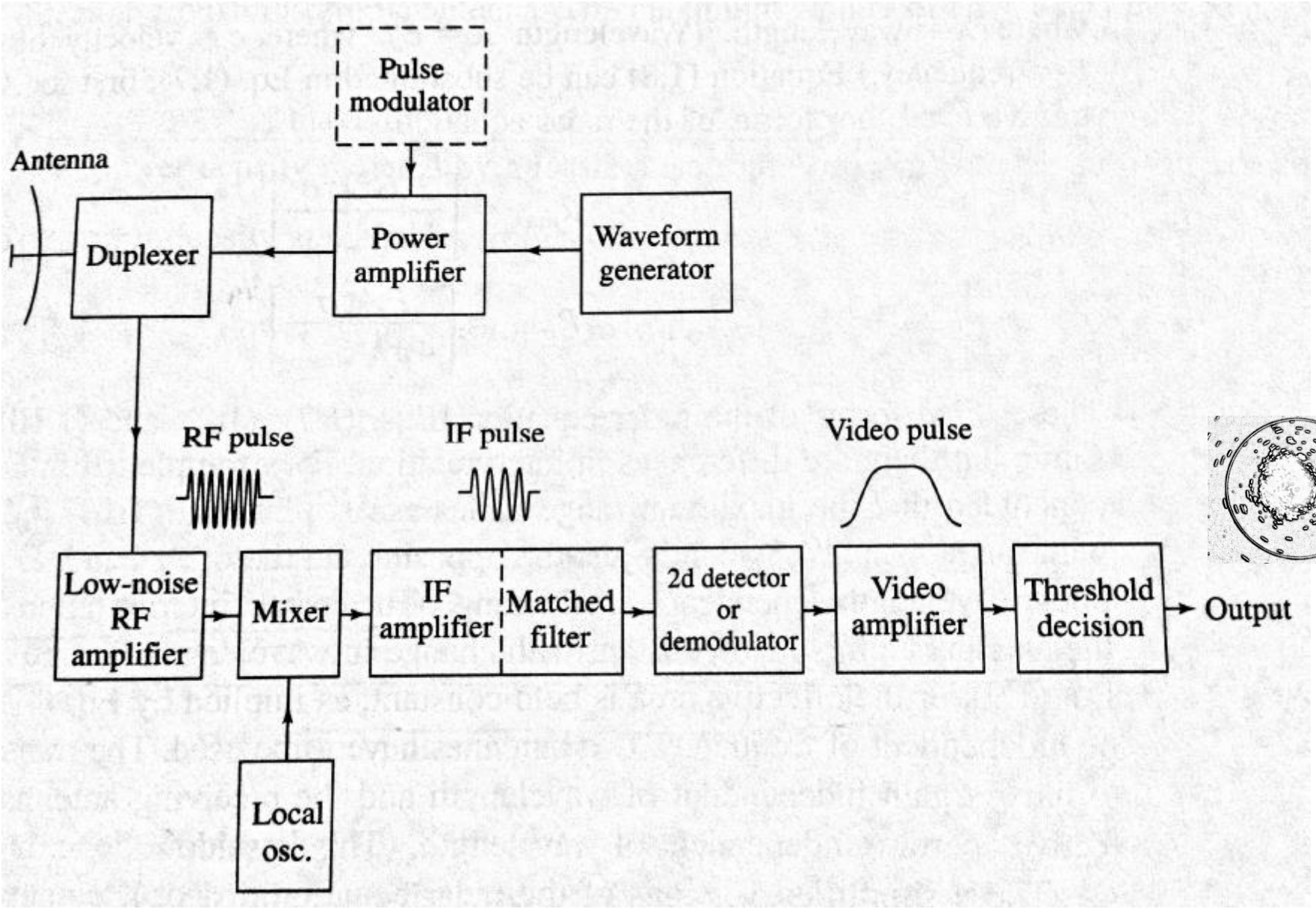
Radar Range Equation forms

$$R_{\max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{\min}} \right]^{1/4}$$

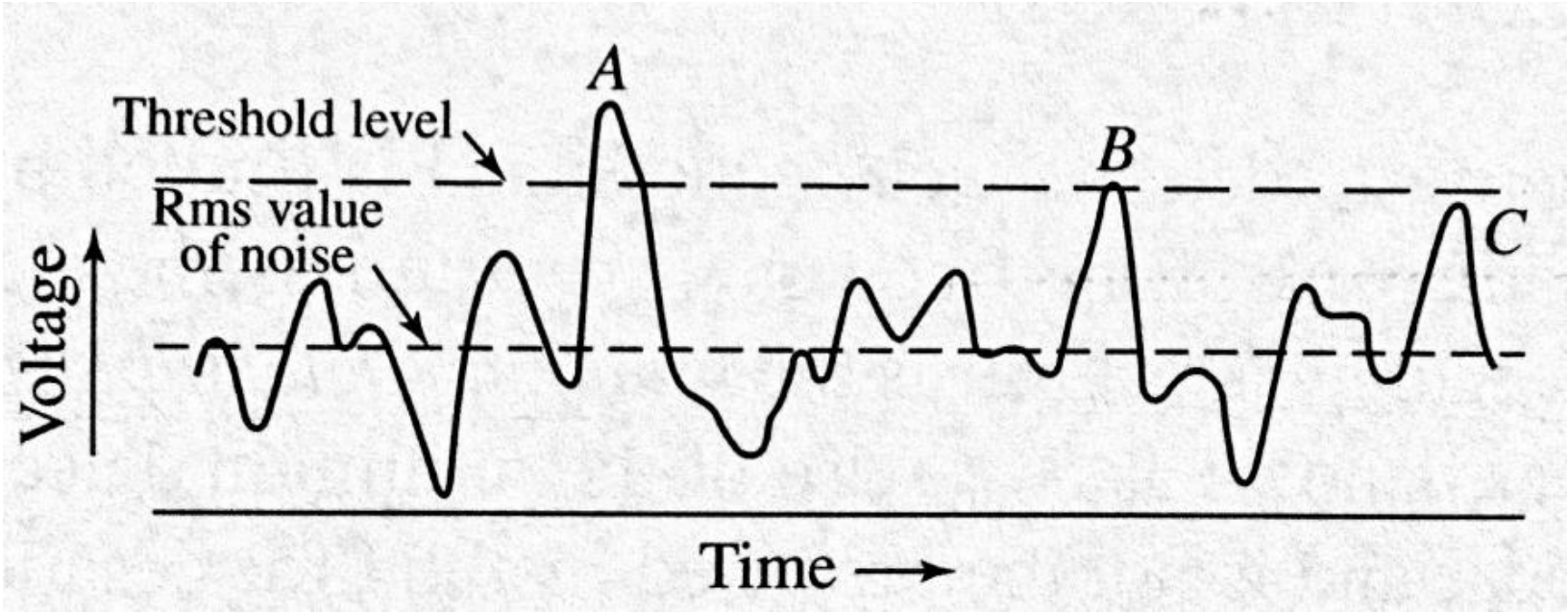
$$R_{\max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{\min}} \right]^{1/4}$$

$$R_{\max} = \left[\frac{P_t A_e^2 \sigma}{4\pi \lambda^2 S_{\min}} \right]^{1/4}$$

Block Diagram of a Conventional Pulse Radar with Superheterodyne Receiver



Detection of Signals in Noise



Detection of a radar signal is based on establishing a threshold at the output of the receiver

The output is assumed to be from a matched-filter receiver (maximizes the output signal-to-noise ratio; maximizes detectability and is not designed to preserve the signal shape)

Depending on threshold level, there are issues of

- False alarm
- Missed target detection

Receiver Noise

Internal receiver noise: Thermal noise (Johnson noise), that is directly proportional to the bandwidth and the absolute temperature (degrees Kelvin) of the ohmic portions

$$\text{available thermal-noise power} = kTB_n$$

k = Boltzmann's constant = 1.38×10^{-23} J/deg

B_n = noise bandwidth (not the same as half-power or 3db bandwidth

$$B_n = \frac{\int_0^\infty |H(f)|^2 \, df}{|H(f_o)|^2}$$

H(f) = frequency-response function of IF amplifier (filter)

f_o = frequency of the maximum response (usually at midband)

Noise bandwidth is the bandwidth of the equivalent rectangular filter with the same noise power output with the one with H(f)

Half-Power Bandwidth: separation between two points with $|H(f)|=0.707\ |H(f_o)|$

Commonly half-power bandwidth B is used as approximation for noise bandwidth B_n

Receiver Noise and Signal-to-Noise Ratio

Noise figure F_n = measure of the noise out of a real receiver to that of an ideal receiver with only thermal noise

[typically evaluated in a standard temperature To=290K (62F), close to room temperature]

$$F_n = \frac{\text{noise out of practical receiver}}{\text{noise out of ideal receiver at std temp } T_0} = \frac{N_{out}}{kT_0BG_a}$$

G_a = available gain (ratio of signal out S_{out} to signal in S_{in} with both input and output matched to deliver maximum output power), $N_{in} = k\,T_o\,B_n$ for an ideal receiver

$$F_n = \frac{S_{in}/N_{in}}{S_{out}/N_{out}} \qquad S_{in} = \frac{kT_0BF_nS_{out}}{N_{out}}$$

Minimum detectable signal S_{min} = value of S_{in} that corresponds to the minimum detectable signal-to-noise ratio at the output of the IF, $(S_{out}/N_{out})_{min}$

$$S_{min} = kT_0BF_n\left(\frac{S_{out}}{N_{out}}\right)_{min}$$

$$R_{max}^4 = \frac{P_tGA_e\sigma}{(4\pi)^2kT_0BF_n(S/N)_{min}}$$

Probability Density Functions

Probability density function (pdf): expresses probability as a density rather than discrete values; more appropriate for continuous functions of time (e.g. noise in radar receiver)

$$p(x) = \lim_{\substack{\Delta x \to 0 \\ N \to \infty}} \frac{(\text{number of values within } \Delta x \text{ at } x)/\Delta x}{\text{total number of values} = N}$$

$$\text{probability } (x_1 < x < x_2) = \int_{x_1}^{x_2} p(x)\, dx$$

$$\int_{-\infty}^{\infty} p(x)\, dx = 1$$

Average value of a variable function $\phi(x)$ (e.g. DC component of a current)

$$\langle \phi(x) \rangle_{\text{av}} = \int_{-\infty}^{\infty} \phi(x) p(x)\, dx$$

Moments of the Random Variable *x*

Mean (average) value of x:
First moment of x
DC component of electrical voltage/current

$$\langle x \rangle_{\text{av}} = m_1 = \int_{-\infty}^{\infty} x p(x)\, dx$$

Mean Square value of x:
Second moment of x
When multiplied by the resistance,
it gives the average power

$$\langle x^2 \rangle_{\text{av}} = m_2 = \int_{-\infty}^{\infty} x^2 p(x)\, dx$$

Variance:
The mean square deviation of x about its mean m_1 (Second central moment)
If x is a noise current, its product with resistance is the mean power of AC

$$\sigma^2 = \langle (x - m_1)^2 \rangle_{\text{av}} = \int_{-\infty}^{\infty} (x - m_1)^2 p(x)\, dx = m_2 - m_1^2$$

Standard deviation σ is the square root of variance:
Root mean square of the AC component

Commonly Used Probability Density Functions

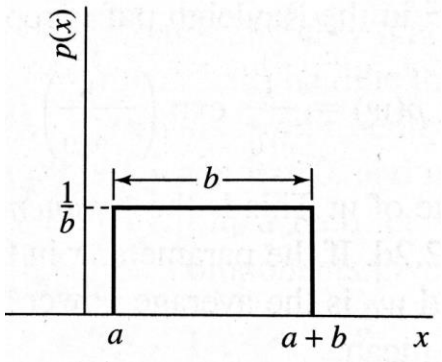
Uniform pdf

Describes the phase of a random sinewave relative to a particular origin of time [0,2π]

Describes the distribution of the round-off (Quantizing) error in numerical computations and ADC's

k=1/b, Average value = a+(b/2), Variance = $b^2/12$

$$p(x) = k \quad \text{for } a < x < a + b$$
$$= 0 \quad \text{for } x < a \text{ and } x > a + b$$



Gaussian pdf

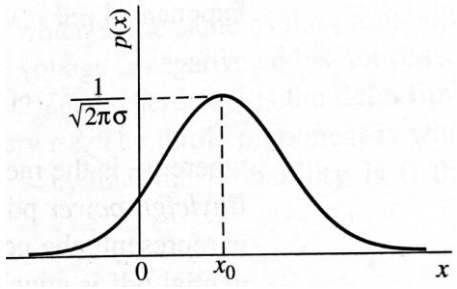
Describes many sources of noise, including receiver thermal noise

pdf has a "bell-shaped" appearance

Mean value = x_o , Variance = σ^2

Central limit theorem: the pdf of the sum of a large number of independently distributed quantities approaches the gaussian pdf no matter what the individual pdf's might be

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x - x_0)^2}{2\sigma^2}\right]$$



Commonly Used Probability Density Functions

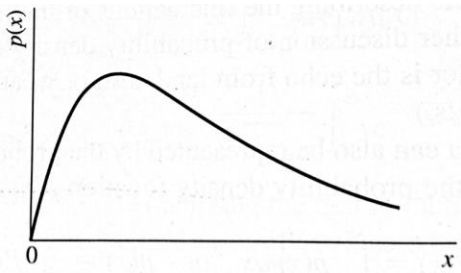
Rayleigh pdf

Describes the envelope of a narrowband filter (such as the IF filter of a radar receiver) when the input noise voltage is gaussian

Describes the statistical behavior of the radar cross section of some targets and clutter

Mean square value is m_2 Standard deviation is equal to sqrt((4/π)-1)=0.523* mean

$$p(x) = \frac{2x}{m_2} \exp\left(-\frac{x^2}{m_2}\right) \qquad x \geq 0$$



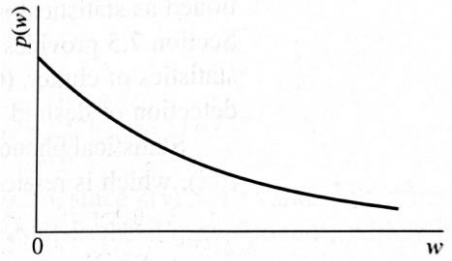
Exponential (Rayleigh power pdf)

When replacing x^2 in Rayleigh with w

Describes the power (w_o is the average power) when x is the Rayleigh voltage pdf

Standard deviation is equal to the mean

$$p(w) = \frac{1}{w_0} \exp\left(-\frac{w}{w_0}\right) \qquad w \geq 0$$



Probability distribution function

$$P(x) = \int_{-\infty}^{x} p(x)\,dx \quad \text{or} \quad p(x) = \frac{d}{dx}P(x)$$

Example

Calculation of the mean value (DC) of the voltage output of a half-wave linear rectifier when the input is thermal noise (gaussian thermal voltage of zero mean)

pdf of the zero mean gaussian noise voltage *x* at the input

$$p(x)\,dx = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \qquad -\infty < x < \infty$$

output *y* of a half-wave rectifier for input *x*

$$\begin{aligned} y &= ax \qquad x \geq 0, \\ y &= 0 \qquad x < 0 \end{aligned}$$

$$p(y)\,dy = \frac{1}{\sqrt{2\pi a\sigma}} e^{-\frac{y^2}{2a^2\sigma^2}} dy + \frac{1}{2}\delta(y)\,dy + 0 \qquad y \geq 0$$

Prob(y>0)=Prob(x>0) Prob(y=0)=Prob(x<0) Prob(y<0)=0

$$m_1 = \frac{1}{\sqrt{2\pi a\sigma}} \int_0^\infty y e^{-\frac{y^2}{2a^2\sigma^2}} dy + \frac{1}{2}\int_{-\infty}^{\infty} y\delta(y)\,dy$$

Second Integral=0

$m_1 = \alpha\sigma/\mathrm{sqrt}(2\pi)$

Probability of False Alarm

pdf of the receiver noise at the input of the IF filter: gaussian pdf with zero mean value and mean square value of Ψ_o (mean noise power)

$$p(v) = \frac{1}{\sqrt{2\pi\Psi_0}} \exp\left(-\frac{v^2}{2\Psi_0}\right)$$

pdf of the envelope R when gaussian noise goes through the IF filter is Rayleigh

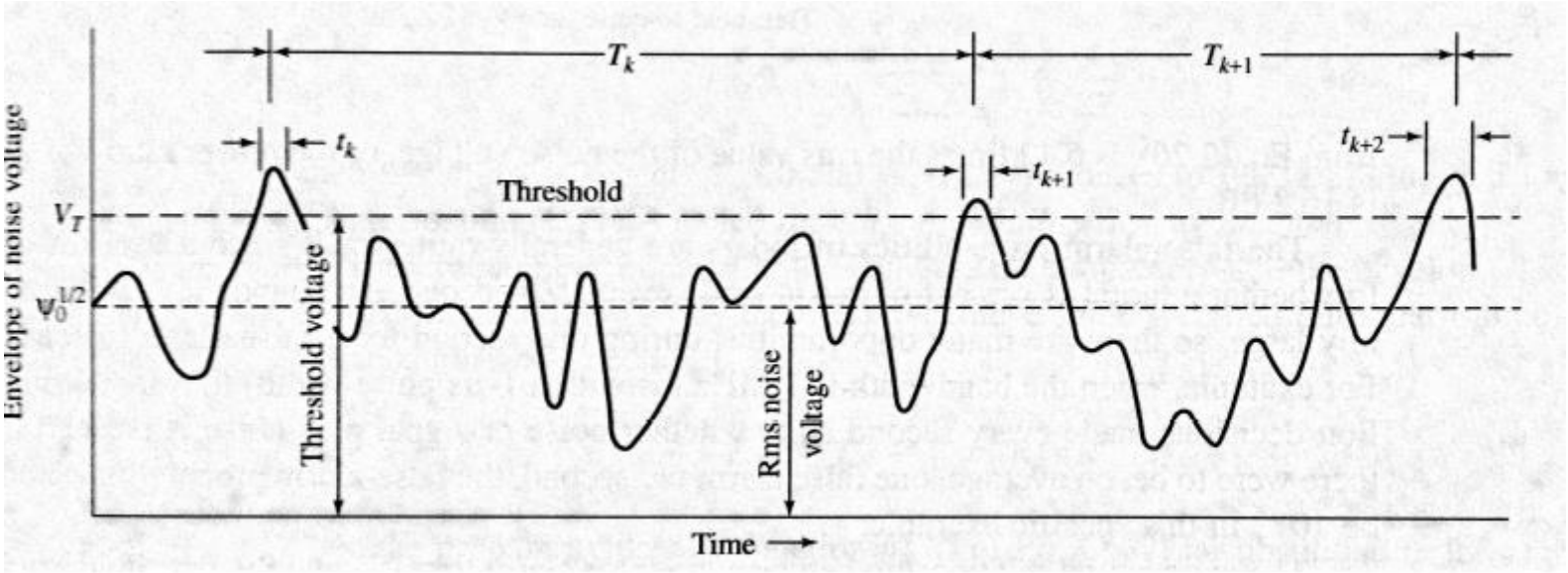
$$p(R) = \frac{R}{\Psi_0} \exp\left(-\frac{R^2}{2\Psi_0}\right)$$

Probability of false alarm P_{fa} = probability that the envelope of noise voltage will exceed V_T

$$\text{Probability } (V_T < R < \infty) = \int_{V_T}^{\infty} \frac{R}{\Psi_0} \exp\left(-\frac{R^2}{2\Psi_0}\right) dR = \exp\left(\frac{-V_T^2}{2\Psi_0}\right)$$

False-alarm Time

False alarm time: the average time between crossings of the decision threshold when noise alone is present *Much better measure of the effect of the noise on radar*



$$T_{\text{fa}} = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} T_k$$

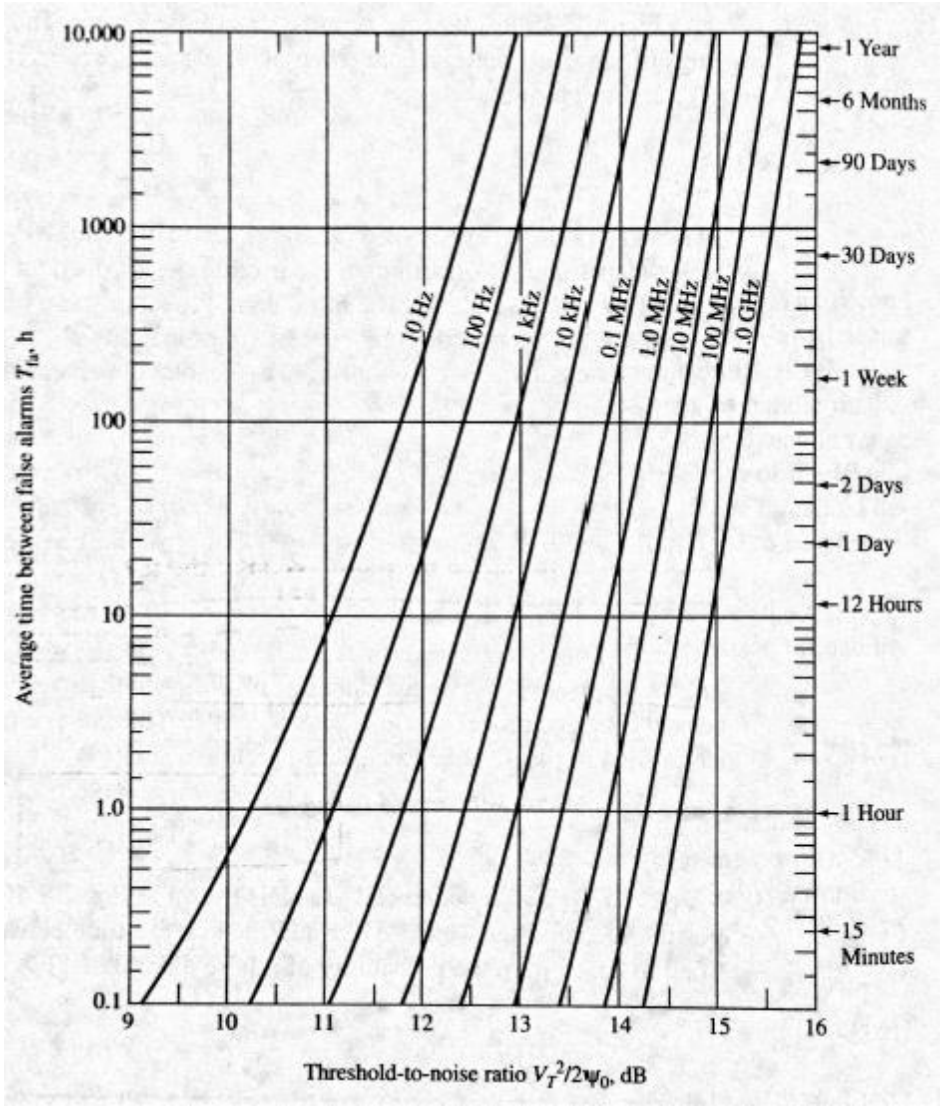
T_k is the time between crossings of threshold V_T by noise envelope

$$P_{\text{fa}} = \frac{\sum_{k=1}^{N} t_k}{\sum_{k=1}^{N} T_k} = \frac{\langle t_k \rangle_{\text{av}}}{\langle T_k \rangle_{\text{av}}} = \frac{1}{T_{\text{fa}} B}$$

P_{fa} = time envelope is above the threshold over the total time of operation

Average duration of threshold crossing ~ ~IF Bandwidth B

Probability of False Alarm / False Alarm Time



$$T_{fa} = \frac{1}{B} \exp\left(\frac{V_T^2}{2\psi_0}\right)$$

False alarm time is very sensitive to small variations of threshold
(e.g. B=1MHz, a value of $10\log(V_t^2/2\Psi_o)$=13.2db results in T_{fa}=20 min; a 0.5db decrease in threshold to 12.7db decreases the false-alarm time by an order of magnitude to about 2min.

False alarm probabilities are Generally quite small as a decision to whether a target is present or not is made every 1/B second

Probability of Detection

(Rice) pdf of the envelope R at the video output with an input of a sinewave of amplitude A along with gaussian noise mean square value of Ψo (mean noise power)

$$p_s(R) = \frac{R}{\Psi_0} \exp\left(-\frac{R^2 + A^2}{2\Psi_0}\right) I_0\left(\frac{RA}{\Psi_0}\right)$$

I_o (z) = modified Bessel function of zero-order and a large-argument Z approximation

$$I_0(Z) = \frac{e^Z}{\sqrt{2\pi Z}}\left(1 + \frac{1}{8Z} + \cdots\right)$$

Probability of detecting the signal P_d = probability that the envelope R will exceed V_T

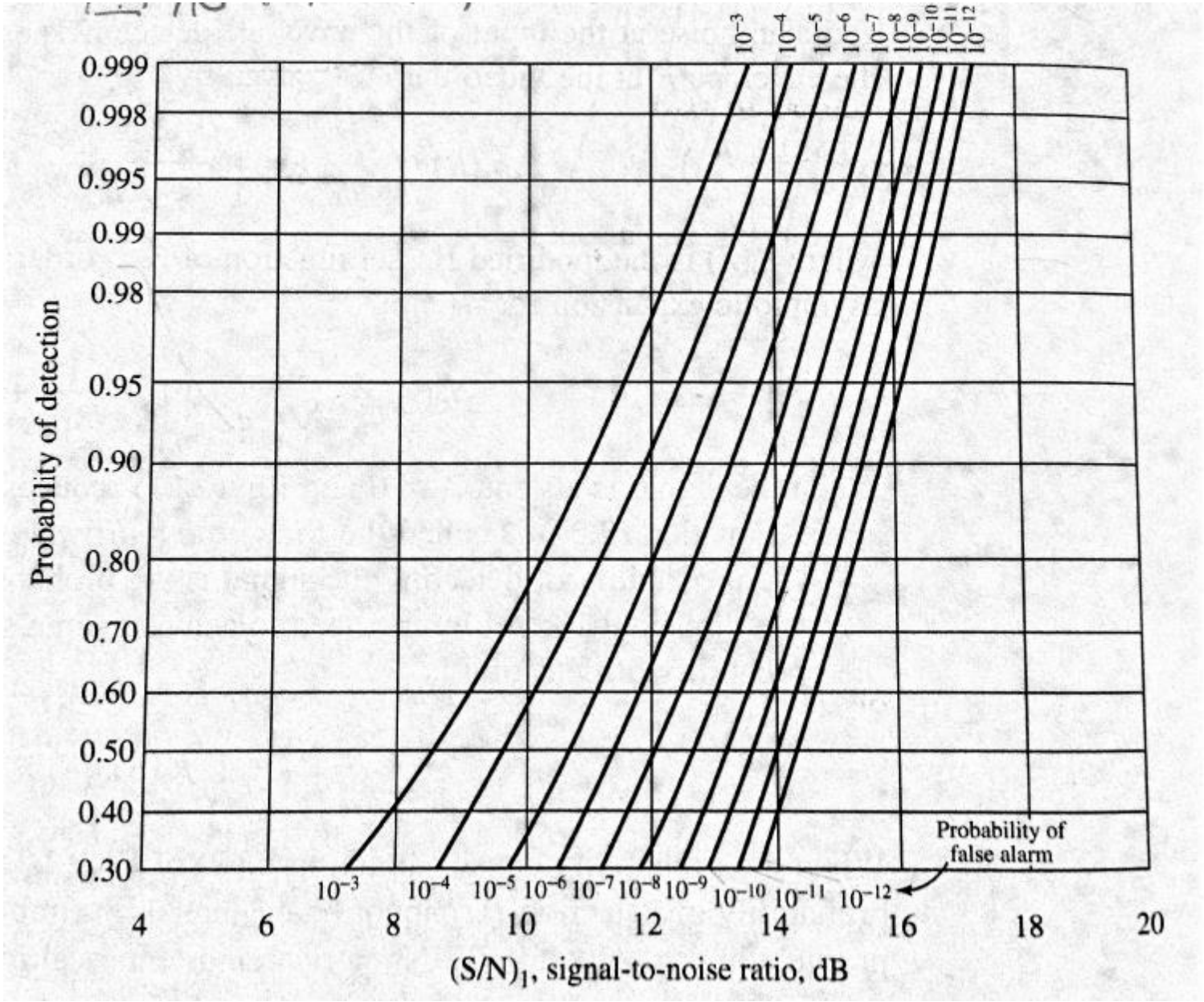
$$P_d = \int_{V_T}^{\infty} p_s(R)\, dR$$

Better to use signal-to-noise ratio S/N instead of $(A^2/2\Psi_o)$

$$\frac{A}{\Psi_0^{1/2}} = \frac{\text{signal amplitude}}{\text{rms noise voltage}} = \frac{\sqrt{2}\ (\text{rms signal voltage})}{\text{rms noise voltage}}$$

$$= \left(2\frac{\text{signal power}}{\text{noise power}}\right)^{1/2} = \left(\frac{2S}{N}\right)^{1/2}$$

Probability of Detection



Probability of Detection

From the specified detection and false-alarm probabilities, the minimum signal-to-noise ratio is found

Albersheim approximate expressions for a single pulse
(accurate to within 0.2dB for P_{fa} between 10^{-3} and 10^{-7} and P_d between 0.1 and 0.9)

$$S/N = A + 0.12AB + 1.7B$$

$$A = ln\,[0.62/P_{fa}] \quad \text{and} \quad B = ln\,[P_d/(1 - P_d)]$$

⚠ S/N : numerical, not db