ECE 3065

Plane - Wave Propagation in Lossy Media

Conducting medium $\Rightarrow \sigma \neq 0$

$$\nabla^2 \tilde{E} - \gamma^2 \tilde{E} = 0$$

with: $\gamma^2 = -\omega^2 \mu \varepsilon_c = -\omega^2 \mu (\varepsilon' - j\varepsilon'')$

$$\varepsilon' = \varepsilon, \quad \varepsilon'' = \sigma/\omega$$

$$\gamma = \underbrace{\alpha}_{\text{attenuation constant}} + j \underbrace{\beta}_{\text{phase constant}}$$

$$(\alpha + j\beta)^2 = (\alpha^2 - \beta^2) + j2\alpha\beta = -\omega^2\mu\varepsilon' + j\omega^2\mu\varepsilon''$$

$$\Rightarrow \begin{cases} \alpha = \omega \left\{ \dfrac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\dfrac{\varepsilon''}{\varepsilon'}\right)^2} - 1 \right] \right\}^{1/2}, & (\text{Np/m}) \\ \beta = \omega \left\{ \dfrac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\dfrac{\varepsilon''}{\varepsilon'}\right)^2} + 1 \right] \right\}^{1/2} & (\text{rad/m}) \end{cases}$$

valid for any medium

Assume plane wave: $\tilde{E} = \hat{x}\, \tilde{E}_x(z)$ $\xrightarrow{+\hat{z}}$

$$\frac{d^2 \tilde{E}_x(z)}{dz^2} - \gamma^2 \tilde{E}_x(z) = 0 \Rightarrow \tilde{E}(z) = \hat{x}\, \tilde{E}_x(z) = \hat{x}\, E_{x0}\, e^{-\gamma z} = \hat{x}\, E_{x0}\, e^{-\alpha z} e^{-j\beta z}$$

$\tilde{H} = \dfrac{1}{\eta_c} (\hat{k} \times \tilde{E})$, η_c = intrinsic impedance of lossy medium

$$\tilde{H}(z) = \hat{y}\, \tilde{H}_y(z) = \hat{y}\, \frac{\tilde{E}_x(z)}{\eta_c} = \hat{y}\, \frac{E_{x0}}{\eta_c} e^{-\alpha z} e^{-j\beta z}$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon'}} \left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2} \quad (\Omega)$$

η_c : complex number $\Rightarrow$ $\tilde{E}(z)$, $\tilde{H}(z)$ NOT IN PHASE for lossy materials