MRTD APPLIED TO COMPLEX GEOMETRY AIR–DIELECTRIC INTERFACES IN 3-D MICROWAVE STRUCTURES

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Abstract- The MRTD scheme is applied to the analysis of microwave structures with complex air-dielectric interfaces. A new formulation for modeling air-dielectric interfaces is introduced and applied to different microwave structures. MRTD results are compared to those generated by FDTD, and it is shown that MRTD retains substantial savings in memory.

I Introduction

MRTD is a new time-domain technique with a wide variety of applications [1–4]. These applications include solving linear and non-linear problems. The MRTD technique is derived by the use of Multiresolution analysis in the discretization of time-domain Maxwell's equations. In the general case, Battle-Lemarie functions and pulse functions are applied in a moment method solution of the space and time derivatives of Maxwell's equations. The most important advantage of the MRTD technique is the demonstrated improvement over conventional FDTD in terms of memory requirements. Overall, the MRTD technique exhibits as much as two orders of magnitude in memory improvement over FDTD.

In previous work [4] MRTD modeling of air-dielectric interfaces proved to be memory consuming. The advantage in memory savings over conventional FDTD could not be preserved for structures with complex dielectric geometries, such as micromachined circuits [5]. In this paper, a new formulation for the modeling of air-dielectric interfaces is proposed and applied to the simulation of 3-D microwave structures. Specifically, three different structures are studied using the new dielectric formulation in MRTD. First, a cavity resonator with a dielectric block perturbation is investigated. Second, a coplanar waveguide to microstrip (CPW/MS) transition [6] is studied. The paper concludes with the modeling of a membrane microstrip line transition [5]. In each case, results for MRTD are compared with FDTD. Overall MRTD maintains an advantage in memory requirements over FDTD.

II 3-D MRTD Modeling of Air-Dielectric Interfaces

In general, to model non-homogeneous dielectric material we express the electric field (**E**) as:

$$\mathbf{D} = \varepsilon(\vec{r},t)\,\mathbf{E} \quad , \tag{1}$$

where **D** represents the electric flux vector and $\varepsilon(\vec{r},t)$ the space– and time–dependent permittivity tensor. Without loss of generality, equation (1) is discretized using scaling and pulse functions in space and time domain as expansion factors in the method of moments. The general procedure for this discretization is described in detail in [4]. In this case, the procedure yields the following equation describing the relation between **D** and **E** in the MRTD formulation:

$$ {}_kD^{\phi x}_{l+1/2,m,n} = \sum_{k',l',m',n'=-\infty}^{+\infty} \varepsilon^{\phi x}_{(x)l+1/2,l+1/2'}\,\varepsilon^{\phi x}_{(y)m,m'}\,\varepsilon^{\phi x}_{(z)n,n'}\,\varepsilon^{x}_{(t)k,k'}\;{}_{k'}E^{\phi x}_{l'+1/2,m',n'} \quad , \tag{2}$$

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where the epsilon coefficients $\varepsilon^{\phi x}_{(\kappa)m,m'}$ and $\varepsilon^{x}_{(t)k,k'}$ are integrals given by

$$\varepsilon^{\phi x}_{(\kappa)m,m'} = \frac{1}{\Delta\kappa} \int \phi_m(\kappa)\, \varepsilon_x(\kappa)\, \phi_{m'}(\kappa)\, d\kappa \qquad (3)$$

and

$$\varepsilon^{x}_{(t)k,k'} = \frac{1}{\Delta t} \int h_k(t)\, \varepsilon_x(t)\, h_{k'}(t)\, dt \quad , \qquad (4)$$

with $\kappa = (x, y, z)$. The functions $\phi_m(\kappa)$ are Battle-Lemarie scaling functions and $h_k(t)$ are pulse functions. The indices l, m, n and k are the discrete space and time indices related to the space and time coordinates via $x = l\Delta x$, $y = m\Delta y$, $z = n\Delta z$ and $t = k\Delta t$, where Δx, Δy, Δz and Δt represent the space and time discretization intervals in x–, y–, z– and t–direction. The integral functions describing $\varepsilon^{\phi x}_{(\kappa)m,m'}$ are necessary due to the non-localized properties of the Battle-Lemarie functions. Unlike FDTD, which has a single value of ε for each cell, MRTD has distributed values of ε across several cells.

Now, consider the equation that describes a dielectric that varies along three coordinate directions:

$$\varepsilon^{\phi x}_{(x,y,z)} = \frac{1}{\Delta v} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \varepsilon_x(x,y,z) \cdot \phi_l(x)\, \phi_{l'}(x)\, \phi_m(y)\, \phi_{m'}(y) \cdot \phi_n(z)\, \phi_{n'}(z)\, dx\, dy\, dz \qquad (5)$$

where $\Delta v = \Delta x \Delta y \Delta z$. This equation results in a matrix with $[nx \times nx \times ny \times ny \times nz \times nz]$ elements, with nx, ny, nz equal to the discretization in the x–, y– and z–directions respectively. For a problem with a large number of air–diectric discontinuities, this would mean a prohibitively high number of dielectric terms.

However, it is possible to separate equation (5) into the following:

$$\varepsilon^{\phi x}_{(x,y,z)} = \frac{1}{\Delta x} \int_{-\infty}^{+\infty} \varepsilon_x(x)\, \phi_l(x)\, \phi_{l'}(x)\, dx \cdot \frac{1}{\Delta y} \int_{-\infty}^{+\infty} \varepsilon_x(y)\, \phi_m(y)\, \phi_{m'}(y)\, dy \cdot \frac{1}{\Delta z} \int_{-\infty}^{+\infty} \varepsilon_x(z)\, \phi_n(z)\, \phi_{n'}(z)\, dz \qquad (6)$$

which results in the product of three matrices of elements $[nx \times nx]$, $[ny \times ny]$ and $[nz \times nz]$. This procedure is generalized to any canonical cartesian air–dielectric interface and exhibits a tremendous savings in memory. Additionally, it is equivalent to the method shown in [4]. It should be noted that the dielectric matrix terms are evaluated *a priori* and not recomputed over each time step.

III Analysis of 3D Microwave Structures

The first structure under consideration is a resonant cavity 6 cm on a side with a dielectric cube located in one corner of the structure [4]. This cube has a relative permittivity equal to 6 and is 2 cm on a side as shown in Figure 1. Simulations on this structure are carried out in MRTD with a discretization of $8 \times 8 \times 8$ and FDTD with discretizations of $8 \times 8 \times 8$ and $40 \times 40 \times 40$. It should be noted that in the previous dielectric formulation for MRTD [4], we would need 8^6 dielectric matrix terms for the discretization. However, in the new formulation we require three matrices, each with 8^2 elements. It should be noted that the old MRTD formulation used almost 25 Mbytes to store the dielectric matrices, while the new method uses less than 10 Kbytes.

In this case, 60,000 time steps were used for both FDTD and MRTD simulations. MRTD used a time step of $\Delta t = 3.0 \cdot 10^{-12} s$ while FDTD used a time step of $\Delta t = 2.0 \cdot 10^{-12} s$. Results between the different simulations are compared in Table 1. Note that the new MRTD achieved a high degree of correlation with the old MRTD and with the FDTD result with a discretization of $40 \times 40 \times 40$. FDTD results for the lower discretization ($8 \times 8 \times 8$) produced unacceptable values. The execution times in Table 1 are from a 400 MHz Pentium Processor based PC.

	Frequency (GHz)	Exec. Time (s)
MRTD (8x8x8)[new]	3.497	1205.6
MRTD (8x8x8)[old]	3.497	4580.39
FDTD (8x8x8)	3.45	670.23
FDTD (40x40x40)	3.50	4306.5

Table 1: Frequency and time comparison for a resonant cavity cavity w/ a corner dielectric

The second structure under consideration is a CPW-MS Transition [6], shown in Figure 2. This is a proximity coupled overlay transition, where electromagnetic coupling occurs in the overlap region between the coplanar waveguide and the microstrip. Both MRTD and FDTD are applied to the modeling of this structure. In this case, a dielectric truncation of 24.5 mils is assumed around the CPW lines as described in [6]. In the FDTD codes, a mesh of $92 \times 153 \times 54$ is used. This mesh includes the 6 cells of PML added in the x– and z– directions and the 10 cells of PML added in the y– directions, with $\sigma_{max}^{Ex} = 167.0$, $\sigma_{max}^{Ey} = 276.0$, and $\sigma_{max}^{Ez} = 216.0$. These values allow a numerical coefficient of reflection below -50 dB. A Gaussian pulse is applied to excite the microstrip with $f_{max} = 410$ GHz and is placed 40 mils from the discontinuity created by the overlap. In the MRTD case a mesh of $46 \times 63 \times 22$ is applied which includes 3 cells of PML along x, 6 PML in y and 5 PML cells in z. The σ_{max}^{E} and Gaussian excitation values are similar to the FDTD case above. Additionally the air-dielectric interface in this case is non-homogeneous only along the z-direction. Here two separate dielectric matrices are applied to describe the air-dielectric interface, with dimensions of $[22 \times 22]$ and $[11 \times 11]$. The first matrix is used to describe the area around the CPW lines and the second is used for the region below the CPW conductor. Figure 3 shows the S-parameter comparisons between FDTD and MRTD. Note that we have excellent correlation between the two methods.

The final structure simulated is a micromachined membrane supported microstrip line transition [5], shown in Figure 4. This is a transition from a 50Ω to 73Ω to 106Ω microstrip line, with a complex–geometry air-dielectric interface under the 106Ω line. In the FDTD codes, a mesh of $84 \times 224 \times 32$ is used, which does not include PML values. It is found that lower FDTD discretizations give unacceptable results. In the MRTD codes a discretization of $48 \times 62 \times 10$ discretization is applied. PML values are chosen to give reflection below -60 dB. Additionally, there is a dielectric discontinuity along all three coordinate directions in this case. The problem is solved in MRTD by dividing the problem into regions and taking the appropriate products of dielectric matrices in each region. Overall, three dielectric matrices of dimensions $[18 \times 18]$ are applied to this problem. Figure 5 shows S-parameter comparisons between FDTD and MRTD. Once again the correlation is excellent between the two methods.

IV Conclusion

A new method for modeling complex geometry air–dielectric interfaces in MRTD is presented. This new method allows MRTD to model structures with air–dielectric interfaces in a memory efficient manner. Results between FDTD and MRTD for three microwave structures are shown, with excellent correlation between the two methods.

V Acknowledgments

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References

[1] M.Krumpholz, L.P.B.Katehi, "MRTD: New Time Domain Schemes Based on Multiresolution Analysis", IEEE Trans. Microwave Theory Tech., pp. 555-572, 1996.

[2] E. Tentzeris, R. Robertson and L. Katehi, "Space– and Time–Adaptive Gridding Using MRTD Technique", Proc. MTT-S 1997, pp. 337–340.

[3] L. Roselli, E. Tentzeris and L. Katehi, "Non–Linear Circuit Characterization Using MRTD", Proc. MTT–S 1998, pp. 1397–1400.

[4] R. Robertson, E. Tentzeris, M. Krumpholz and L. Katehi, "Modeling of Dielectric Cavity Structures using MRTD", Int. Journal of Numerical Modeling, Jan-Feb. 1998.

[5] T. Weller, PhD Thesis, pp. 135-142, University of Michigan, 1993.

[6] R. Robertson, E. Tentzeris, T. Ellis and L. Katehi, "Characterization of a CPW–MS Transition for Antenna Applications", Proc. AP-S 1998, pp. 1380–1383.

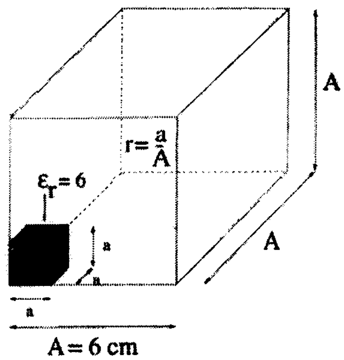

Figure 1: Resonant cavity with a dielectric cube perturbation (ε_r=6)

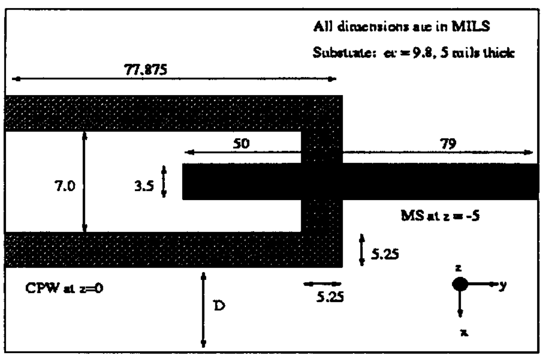

Figure 2: Coplanar Waveguide to Microstrip (CPW/MS) Transition [6].

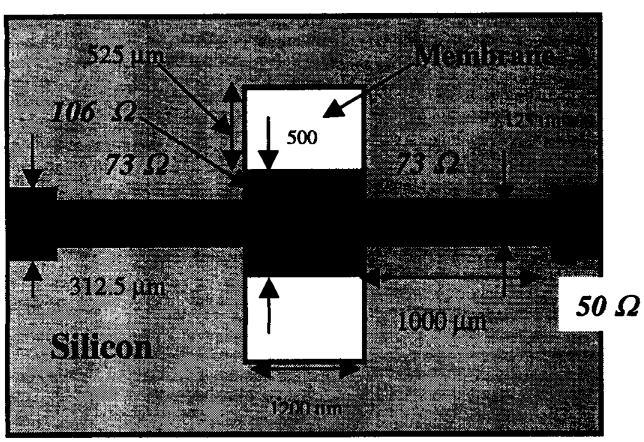

Figure 4: Membrane Microstrip Line 50-73-106 Ω Transition (Top View) [5].

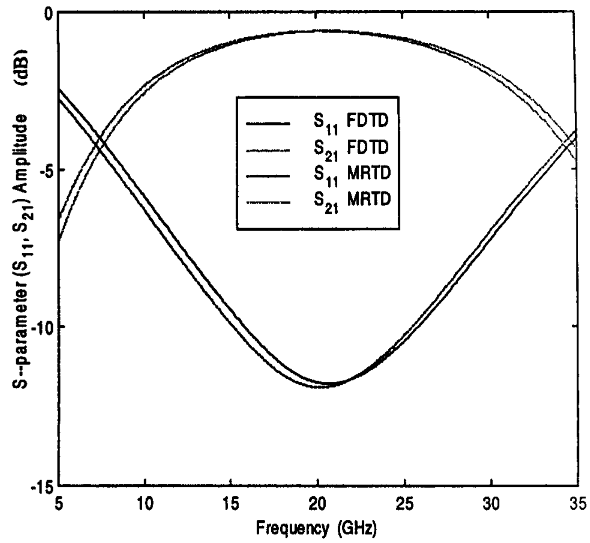

Figure 3: *S-parameter* results for the CPW/MS Transition: FDTD vs. MRTD Comparison.

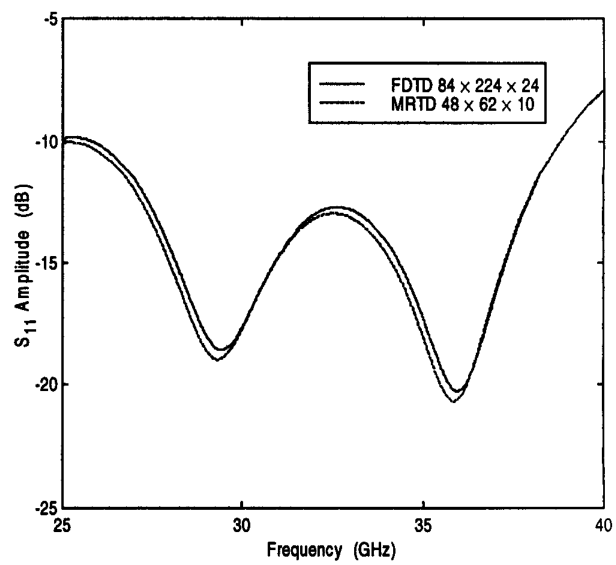

Figure 5: *S-parameter* results for the Membrane Microstrip Line: FDTD vs. MRTD Comparison.