Consider the Type-II second order loop show below. Let $k_{\phi} = 2\pi V/rad$, $k_v = 2\pi \cdot 10 \text{ MHz } rad/sec-V$, $\tau_1 = \tau_2 = 160 \mu\text{sec}$, $f_{\text{ref}} = 200 \text{ kHz}$, and $15000 \leq N \leq 16000$.

1. (20 pts) Calculate the damping factor $\zeta$, natural frequency $\omega_n$, and loop BW $\omega_c$ of the PLL while it is operating in the linear range.

2. (20 pts) If the phase detector phase noise is flat at $L(f_m) = -135 \text{ dBC/Hz}$, calculate the integrated phase noise over the loop BW.

3. (20 pts) Determine the lock-in frequency range $\omega_L$ for the PLL using analytic methods.
4. (40 pts) Using numerical simulation, estimate the pull-in time $t_p$ for the loop to settle for the worst case $\Delta \omega$, when $N$ switches from 15000 to 16000 ($\Delta \omega = 2\pi \cdot 1000 \cdot f_{ref}$). Note that outside the linear range, the PFD has an odd symmetry periodic function of $k_\phi(\Delta \theta) = k_\phi \mod(\Delta \theta, 2\pi)$. Include simulation results, and compare them to an appropriate analytic result. Explain any discrepancy between simulation and analytic results.
Problem 1.

\[ N := 15500 \quad k_\phi := 2\pi \quad k_v := 2\cdot10^7 \quad \tau_1 := 160\cdot10^{-6} \quad \tau_2 := 160\cdot10^{-6} \]

\[ \omega_{\text{ref}} := 2\pi \cdot 200\cdot10^3 \]

\[ K := \frac{k_\phi \cdot k_v}{N} \quad K \approx 2.547\cdot10^4 \quad 20\cdot\log(K) = 88.121 \]

\[ \omega_n := \frac{K}{\sqrt{\tau_1}} \quad f_n := \frac{\omega_n}{2\pi} \quad \zeta := \frac{\tau_2}{2\omega_n} \]

\[ \omega_n = 1.262\cdot10^4 \quad f_n = 2.008\cdot10^3 \quad \zeta = 1.009 \]

\[ H(s) := \frac{2\zeta \omega_n s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]

\[ \omega_c := \omega_n \left(1 + 2\zeta^2 + \sqrt{2 + 4\zeta^2 + 4\zeta^4}\right)^{1/2} \quad 20\cdot\log\left(\frac{H(j\omega_c)}{N}\right) = -3.01 \]

\[ \omega_c = 3.151\cdot10^4 \quad f_c := \frac{\omega_c}{2\pi} \quad f_c = 5.014\cdot10^3 \]
Problem 2

$L_{\phi} := -135$

Approximate solution:

$\Delta \phi := L_{\phi} + 20 \cdot \log(N) + 10 \cdot \log(f_c)$

$\Delta \phi = -14.191$ dBc

Exact solution:

$\Delta \phi := L_{\phi} + 20 \cdot \log(N) + 10 \cdot \log(B)$

$\Delta \phi = -12.302$ dBc

Problem 3

$\omega := 2 \cdot \zeta \cdot \omega_n$

$\omega = 2.547 \cdot 10^4$

$f_L := \frac{\omega}{2 \pi}$

$f_L = 4.054 \cdot 10^3$

Another way: Solve nonlinear equation manually:

$H(s) := \frac{1 + s \cdot \tau}{s \cdot \tau}$

$\omega := 2.61854 \cdot 10^4$

$f_L := \frac{\omega}{2 \pi}$

$f_L = 4.168 \cdot 10^3$

$\omega = 2.999$

For reference:

$t_L := \frac{2 \pi}{\omega_n}$

$t_L = 4.98 \cdot 10^{-4}$

Problem 4

$\Delta \omega := 2 \cdot \pi \cdot \left| \frac{3 \cdot 10^9}{16000} - \frac{3 \cdot 10^9}{15000} \right|$  

$\Delta \omega = -7.854 \cdot 10^4$

$\left| \frac{\Delta \omega}{\omega} \right| = 2.999$

$\Delta \omega$ is outside of lock-in range
For active loop filter, $\omega_p$ is theoretically infinite. The pull-in time may be estimated from one of several published formulas:

Rhode:

$$t_p := \frac{1}{\pi^2} \frac{\Delta\omega_o}{\zeta \omega_n^3}$$

$$t_p = 3.083 \cdot 10^{-4}$$

Best (most detailed derivation):

$$t_p := \frac{2}{16} \frac{\Delta\omega_o^2}{\zeta \omega_n^3}$$

$$t_p = 1.877 \cdot 10^{-3}$$

Gardner, Brennan:

$$t_p := \frac{1}{2} \frac{\Delta\omega_o^2}{\zeta \omega_n^3}$$

$$t_p = 1.521 \cdot 10^{-3}$$

**Numerical Simulation of Pull-in Process**

**PFD Model**

$$v_\phi(\Delta\theta) := k_\phi \cdot \text{mod}(\Delta\theta, 2\pi)$$

$$\Delta\theta_i := -6\pi + \frac{i}{Npts} \cdot (12\pi)$$
Simulation of Closed Loop PLL Acquisition using Finite Difference Integration Method

\[
\frac{\Delta \omega}{2\pi} = -1.25 \cdot 10^4
\]

\[
\Delta t := 10^{-1} \cdot \sqrt{\frac{t_1}{k_v k_{\phi}}}
\]

\[
\Delta t = 6.366 \cdot 10^{-8}
\]

\[
t_{\text{max}} := \text{Npts} \Delta t
t_{\text{max}} = 3.183 \cdot 10^{-4}
\]

\[
K_1 := \frac{k_v}{t_1}
\]

\[
K_1 = 2.534 \cdot 10^7
\]

\[
K_2 := k_v k_{\phi} \frac{\tau_2}{t_1}
\]

\[
K_2 = 2.547 \cdot 10^4
\]

\[
K_2 := 2.534 \cdot 10^7
\]

\[
K_2 := 2.547 \cdot 10^4
\]

\[
gle 1 \text{.. Npts} \quad t_0 := 0 \quad t_j := j \cdot \Delta t
\]

\[
\omega_j := \omega_{j-1} + \Delta t \cdot K_1 \left[ \phi \left( \omega_{\text{ref}} - \omega_{j-1} \right)^t \right] + \Delta t \cdot K_2 \left[ \phi \left( \omega_{\text{ref}} - \omega_{j-1} \right)^t \right]
\]

Final Frequency Error:

\[
\frac{\omega_{\text{Npts}} - \omega_{\text{ref}}}{\omega_{\text{ref}}} = -1.94299 \cdot 10^{-5}
\]

\[
t_p := t_{\text{max}}
t_p = 3.183 \cdot 10^{-4}
\]
Reference Oscillator
Amp = 1 v
Freq = 200e+3 Hz
Phase = 0 deg

Charge Pump modeled as moving window average

Loop Filter
1 Sections
Quant Bits = None
Num1 = (160.e-6)s+1
Den1 = (160.e-6)s

VCO
Amp = 707.1e-3 v
Freq = 187.5e+3 Hz
Phase = 0 deg
Mod Gain = 643 Hz/v

Max Rate (Port 0) = 40e+6 Hz
(Token 2)

Charge Pump Model
Operator: Average
Time Window = 20.e-6 sec

PFD model:
D flip-flop with NAND reset
Power Spectrum of w4 (dBm 50 ohms)

Frequency in Hz (dF = 333.3 Hz)