

Statistical radar imaging of diffuse and specular targets using an expectation-maximization algorithm

Aaron D. Lanterman
(lanterma@ifp.wustl.edu)

Coordinated Science Laboratory
Univ. of Illinois
1308 W. Main
Urbana, IL 61801

This work was supported by DARPA Contract
F49620-98-1-0498, administered by AFOSR.
<http://www.ifp.uiuc.edu/~lanterma/darpa>

Models for Received Radar Data

- Received data $\mathbf{r} = \mathbf{\Gamma}^H \mathbf{c} + \mathbf{w}$, where
 - \mathbf{c} = complex target reflectance
 - $\mathbf{\Gamma}$ = linear observation mechanism
 - $\mathbf{w} \sim CN(0, N_0 I)$
- Examples of $\mathbf{\Gamma}$:
 - Simple range profiling: Samples of transmitted waveform (representing convolution)
 - Delay-doppler imaging: Transmitted waveform multiplied by sinusoids
 - Tomographic imaging: Transmitted waveform with partial Radon transform
- In target detection, \mathbf{c} is often treated as random vector. In target imaging, \mathbf{c} is more often treated as an unknown deterministic parameter. Here we explore random models for \mathbf{c} for imaging purposes.

Models for Target Reflectance

From Shapiro:

- Diffuse/speckle: $\mathbf{c}_d \sim CN(0, \mathbf{\Sigma})$
 - $\mathbf{\Sigma}$ is a diagonal covariance
 - $\mathbf{s} = \text{diag}(\mathbf{\Sigma})$ called the *scattering function*
 - Goal: estimate \mathbf{s}
- Specular/glint: $\mathbf{c}_s = \mathbf{b} \times \exp[j\theta]$
 - $\theta \sim$ i.i.d. uniform over $[0, 2\pi)$
 - \mathbf{b} is a deterministic *glint reflection coefficient*
 - \times is elementwise multiplication
 - Goal: estimate \mathbf{b}
- Mixed model: $\mathbf{c} = \mathbf{c}_d + \mathbf{c}_s$
 - Goal: estimate \mathbf{s} and \mathbf{b}
 - Tends to be overparameterized

EM Algorithm for Diffuse Imaging

- For diffuse imaging, $\mathbf{r} \sim CN(0, \mathbf{\Gamma}^H \mathbf{\Sigma} \mathbf{\Gamma} + N_0 \mathbf{I})$, yielding a structured covariance estimation problem
- No obvious closed-form formula for ML estimate
- Iterative EM algorithm by Snyder-O'Sullivan-Miller:

$$\sigma_i^{new} = \sigma_i^{old} - (\sigma_i^{old})^2 [\mathbf{\Gamma} \mathbf{K}^{-1} \mathbf{\Gamma}^H - \mathbf{\Gamma} \mathbf{K}^{-1} \mathbf{r} \mathbf{r}^H \mathbf{K}^{-1} \mathbf{\Gamma}^H]_{ii},$$

where

$$\mathbf{K} = \mathbf{\Gamma}^H \mathbf{\Sigma}^{old} \mathbf{\Gamma} + N_0 \mathbf{I}$$

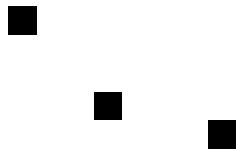
- Enjoys usual properties of EM algorithms
 - Likelihood increases at each iteration
 - Iterates guaranteed to be nonnegative

What About Specular Imaging?

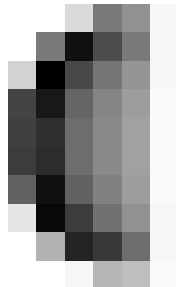
- Specular \mathbf{r} is vastly more complicated
 - Not aware of a closed form for the density on \mathbf{r}
- If the columns of $\mathbf{\Gamma}$ have a sufficient non-zero entries:
 - \mathbf{r} consists of sums of indep. 0-mean random variables
 - By CLT, *marginals* on \mathbf{r} approx. 0-mean Gaussian
 - \mathbf{r} “almost Gaussian” in the spirit of Mallows
- Motivates trying the diffuse EM algorithm on the specular case

Phantoms for Simulations

- Three point scatterers:

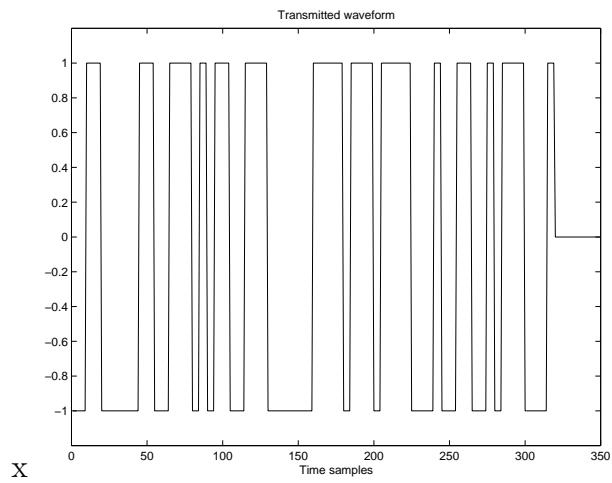


- Rotating sphere:

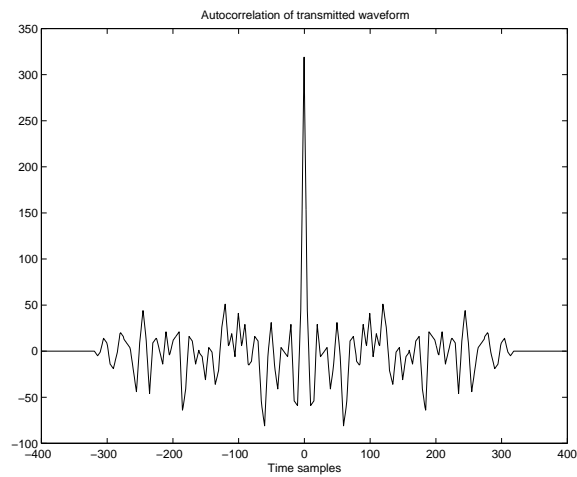


Transmitted Waveform

- Specular realization:

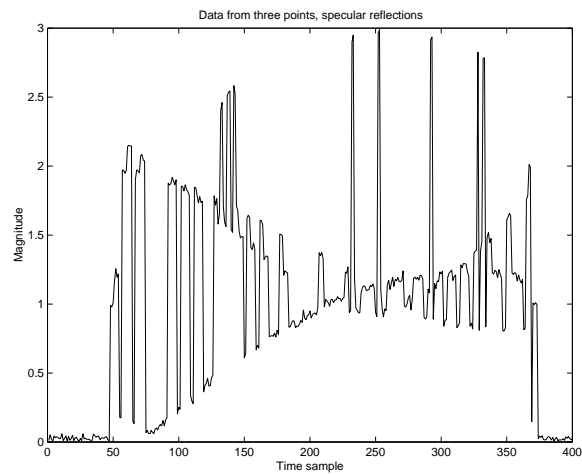


- Autocorrelation:



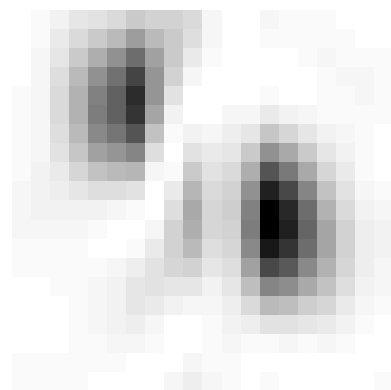
Data from Three Point Scatterer

- Data from three point scatterer:

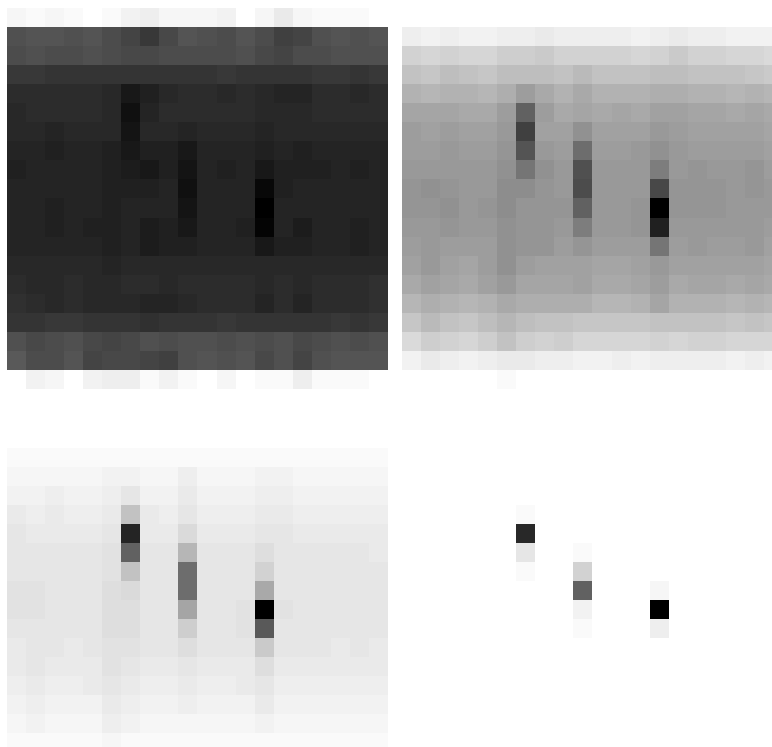


Results for Three Point Scatterer

- Matched filter output:

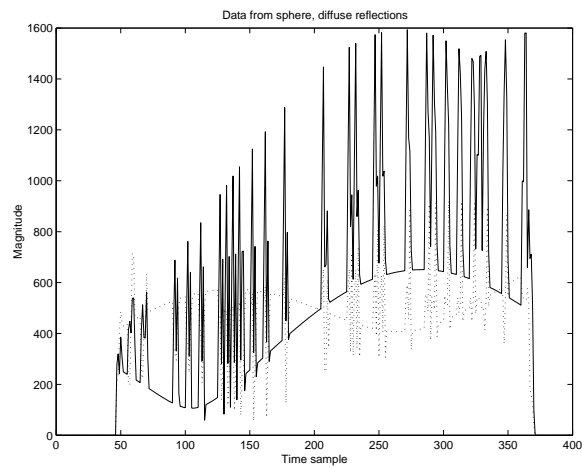


- At 1, 5, 10 and 20 EM iterations:

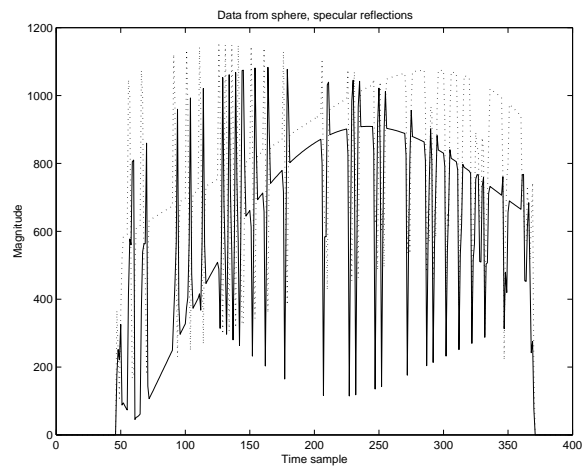


Data for the Sphere

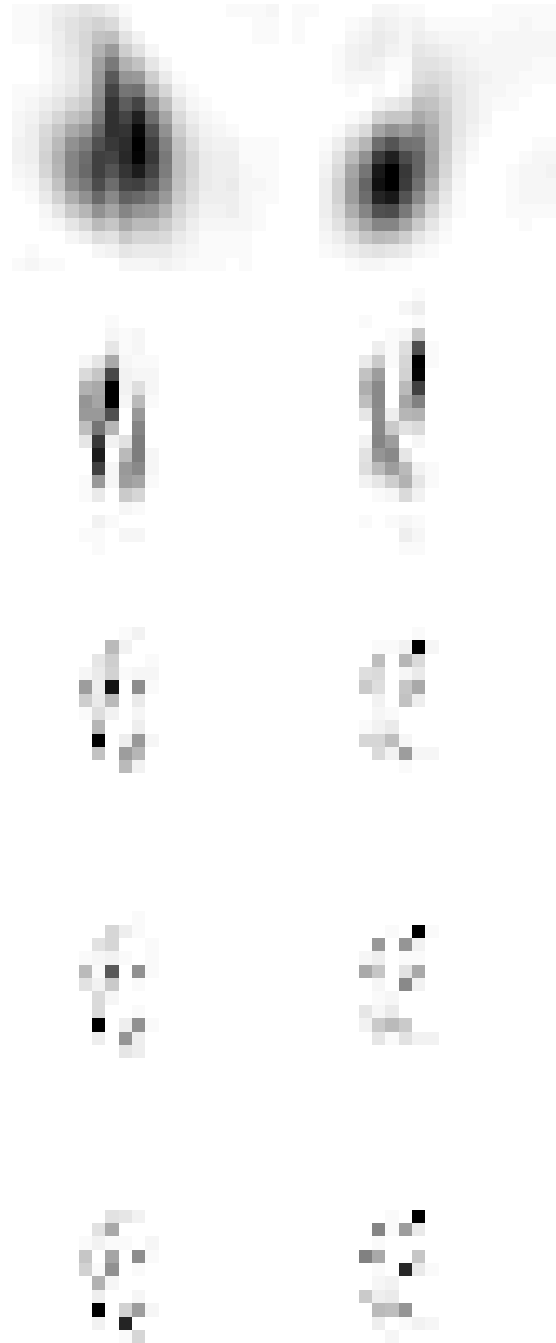
- Two diffuse realizations:



- Two specular realizations:



Results for Diffuse Sphere



Results for Specular Sphere



Regularization Techniques

- Grenander's Method of Sieves
 - B-spline basis for f (Moulin 92)
 - Wavelet basis for $\log(f)$ (Moulin 93)

- Penalized likelihood methods
 - Subtract penalty from likelihood

$$P(r|f) = L(r|f) - \alpha\Phi(f)$$

- Good's roughness penalty:

$$\Phi_G(f) = \int \left[\frac{d}{dx} \sqrt{f(x)} \right]^2 dx$$

- Good's is equivalent to O'Sullivan's I-divergence penalty
- Silverman's roughness penalty:

$$\Phi_S(f) = \int \left[\frac{d}{dx} \log f(x) \right]^2 dx$$

- Simple modification of EM algorithm produces penalized likelihood estimates; amounts to nonlinearly smoothing the result of the maximization step at each iteration
- Admits a Bayesian interpretation

Expectation-Maximization-Smoothing Algorithms

- Suggested by Silverman for emission tomography
- Try different kinds of *ad hoc* smoothing steps
- A particular choice of smoothing may not correspond to any particular penalized likelihood method
- Good performance shown in emission tomography
- However, it's hard to prove whether such algorithms converge, and even harder to show what they converge to

Directions for Future Work

- Implementation and comparison of various regularization techniques
- Current execution time of MATLAB implementation on Sun Enterprise 3500:

Image size	Total time	Time for inverse
20 x 20	15 seconds	6 seconds
32 x 32	8 minutes	4 minutes
40 x 40	32 minutes	13 minutes

- Improve computation time
 - Must find fast way of doing matrix inverse (or avoiding an explicit inverse altogether)
 - Speed up multiplies by $\mathbf{\Gamma}$
 - Fast EM Variants (SAGE, etc.)
- Statistical formulation provides criteria for radar waveform design (via Cramer-Rao bounds, etc.)
- Other applications
 - Radar astronomy
 - Direction finding?

Bibliography

- J. Shapiro, B.A. Capron, and R.C. Harney, “Imaging and target detection with a heterodyne-reception optical radar,” *Applied Optics*, 20(19):3292-3313, 1981.
- D.L. Snyder, J.A. O’Sullivan, and M.I. Miller, “The use of maximum-likelihood estimation for forming images of diffuse radar-targets from delay-doppler data,” *IEEE Trans. IT*, 35(3):536-548, 1989.
- C.L. Mallows, “Linear processes are nearly Gaussian”, *J. of Applied Prob.*, 4:313-329, 1968.
- P. Moulin, J.A. O’Sullivan, and D.L. Snyder, “A method of sieves for multiresolution spectrum estimation and radar imaging,” *IEEE Trans. IT*, 38(2):801-813, 1992.
- P. Moulin, “A wavelet regularization method for diffuse radar-target and speckle-noise reduction,” *J. of Mathematical Imaging and Vision*, 3(1):123-134, 1993.
- J.A. O’Sullivan, “Roughness penalties on finite domains,” *IEEE Trans. Image Proc.*, 4(9):1258-1268, 1995.
- B.W. Silverman, “On the estimation of a probability density function by the maximum penalized likelihood method,” *Annals of Stat.*, 10(3):795-810, 1982.
- B.W. Silverman, M.C. Jones, J.D. Wilson, and D.W. Nychka. “A smoothed EM approach to indirect estimation problems, with particular reference to stereology and emission tomography (with discussion),” *J. of the Royal Stat. Soc. B*, 52(2):271-324, 1990.