Tracking and recognition of airborne targets via commercial television and FM radio signals

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Introduction

- Multistatic system using commercial transmissions
  - System remains covert
  - No cost of building transmitters
  - Coverage of low altitude targets
- Television signals
  - Low practical bandwidth
  - On *all the time*
  - Good doppler resolution
  - Poor range resolution
  - Need high SNR receivers
- FM radio
  - Better range resolution, but still not very good
- Bayesian approach (Srivastava, Grenander, Miller)
  - Prior $p(\mathbf{x})$ based on Newtonian dynamics
  - Data likelihood $p(\mathbf{y}|\mathbf{x})$
  - Inference via posterior $p(\mathbf{x}|\mathbf{y})$
- Denote target class

$$\alpha \in \mathcal{A} = \{\text{F-15, F-18, Mig-29, Mirage, 747, Cessna}, \ldots\}$$

Track Parameterization

- Path in *inertial reference frame*

$$\mathbf{p}(t) = [p_1(t)\ \ p_2(t)\ \ p_3(t)]^T \in \Re(3)$$

- Orientation relative to inertial reference frame

$$\mathbf{O}(t) \in SO(3)$$

— Avoids sin's and cos's associated with Euler angles

- Angular velocity

$$\mathbf{q}(t) = [q_1(t)\ \ q_2(t)\ \ q_3(t)]^T$$

- Translational velocity in *body reference frame*

$$\mathbf{v}(t) = [v_1(t)\ \ v_2(t)\ \ v_3(t)]^T$$

- Positions, orientations, velocities related by

$$\begin{aligned} \dot{\mathbf{p}}(t) &= \mathbf{O}(t)\mathbf{v}(t) \\ \dot{\mathbf{O}}(t) &= \mathbf{O}(t)\mathbf{Q}(t) \end{aligned}$$

where

$$\mathbf{Q}(t) = \begin{bmatrix} 0 & -q_3(t) & q_2(t) \\ q_3(t) & 0 & -q_1(t) \\ -q_2(t) & q_1(t) & 0 \end{bmatrix}$$

Target Dynamics

- Velocities governed by SDEs

$$\begin{aligned} \dot{\mathbf{v}}(t) + \mathbf{Q}(t)\mathbf{v}(t) &= \mathbf{f}(t) \\ \mathbf{I_m}\dot{\mathbf{q}}(t) + \mathbf{Q}(t)\mathbf{I_m}\mathbf{q}(t) &= \boldsymbol{\tau}(t) \end{aligned}$$

—— Translational force $\mathbf{f}(t)$
—— Rotational torque $\boldsymbol{\tau}(t)$
—— Rotational inertials $\mathbf{I_m} = diag(I_1, I_2, I_3)$

- Discretization yields

$$\begin{aligned} \frac{\mathbf{v}[k+1] - \mathbf{v}[k]}{\delta_t} + \mathbf{Q}[k]\mathbf{v}[k] &= \mathbf{f}[k] \\ \mathbf{I_m}\frac{\mathbf{q}[k+1] - \mathbf{q}[k]}{\delta_t} + \mathbf{Q}[k]\mathbf{I_m}\mathbf{q}[k] &= \boldsymbol{\tau}[k] \end{aligned}$$

$$\begin{aligned} \mathbf{p}[k+1] &= \mathbf{p}[k] + \delta_t\mathbf{O}[k]\mathbf{v}[k] \\ \mathbf{O}[k+1] &= \mathbf{O}[k] + \delta_t\mathbf{O}[k]\mathbf{Q}[k] \end{aligned} \tag{1}$$

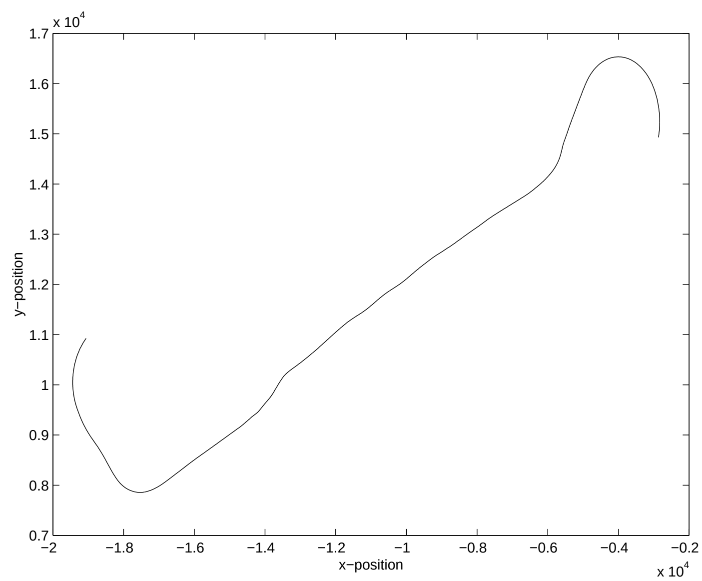
- To avoid accumulation of errors, use

$$\mathbf{O}[k+1] = \mathbf{O}[k]\exp(\delta_t\mathbf{Q}[k])$$

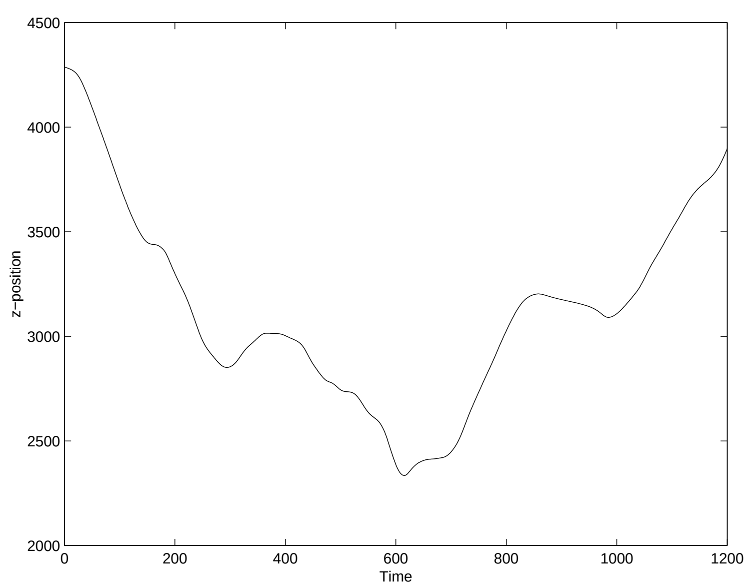
instead of (1)

Sample Flight Path

- Flown in Silicon Graphics flight simulator
- Path in X-Y plane:

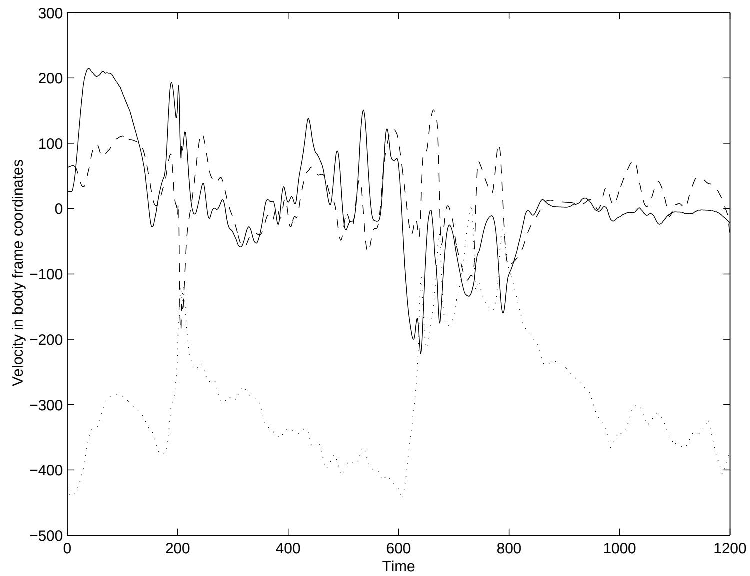


- Altitude (Z-coordinate):

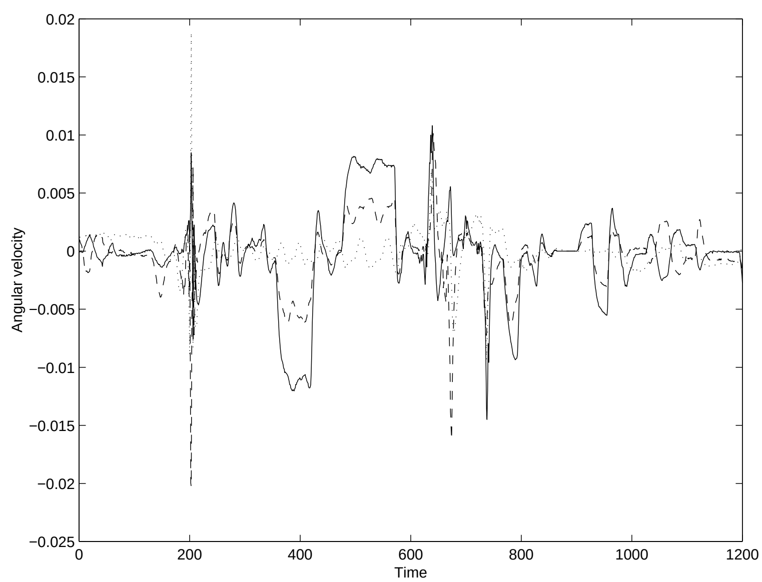


Extracted Velocities

- Body-frame velocities **v**

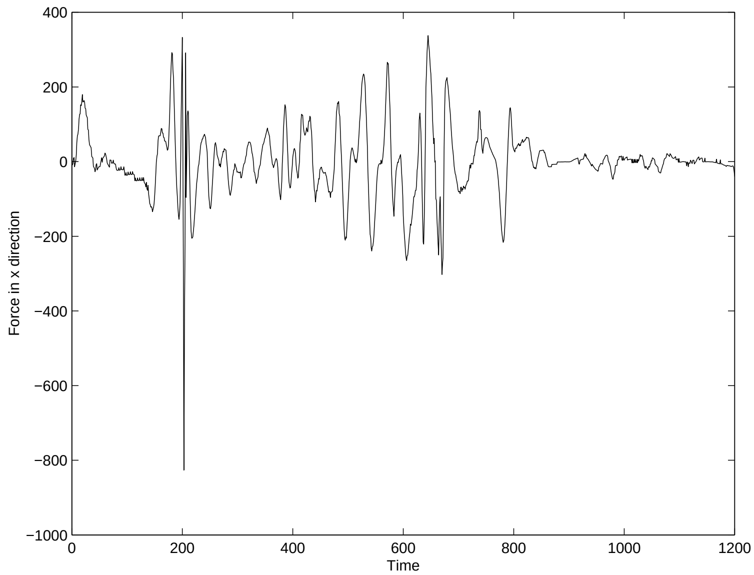


- Angular velocities **q**

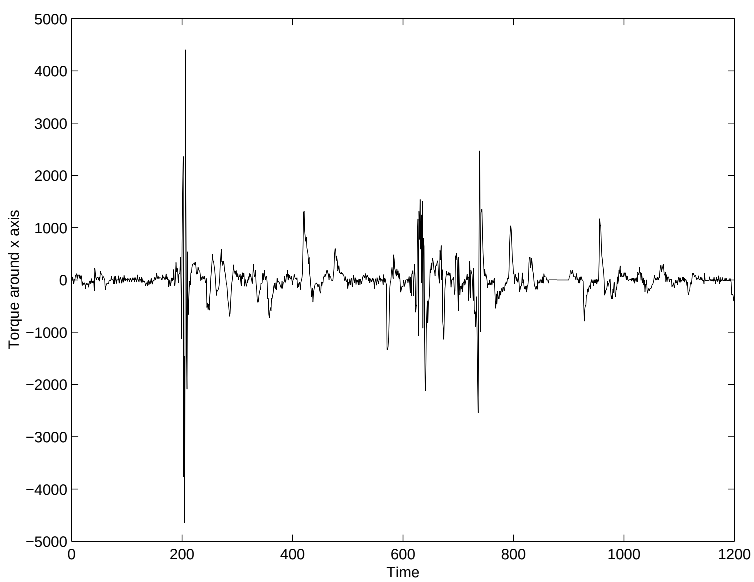


Extracted Forces

- Translational force $\mathbf{f}_1$

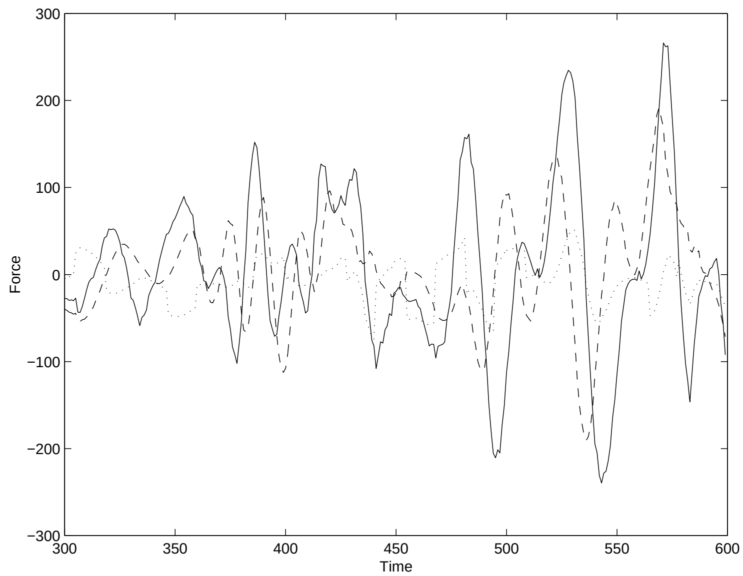


- Torque $\boldsymbol{\tau}_1$

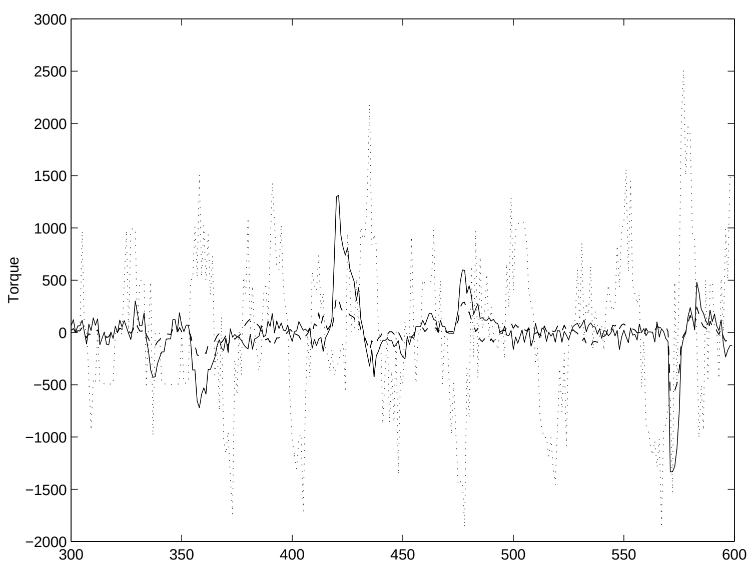


Extracted Forces (zooming in)

- Translational forces $\mathbf{f}$



- Torques $\boldsymbol{\tau}$



Fitting a Gaussian Model

- Estimated translational force covariance

$$\mathbf{\Sigma}_f = \begin{bmatrix} 79917 & 36421 & -18819 \\ 36421 & 96630 & -17301 \\ -18819 & -17301 & 51533 \end{bmatrix} \times 10^2$$

$$\mathbf{\Sigma}_f = \underbrace{\begin{bmatrix} 88535 & 16860 & 8201 \\ 16860 & 39152 & 462 \\ 8201 & 462 & 6479 \end{bmatrix}}_{\text{using smaller chunk}} \times 10$$

- Estimated torque covariance

$$\mathbf{\Sigma}_\tau = \begin{bmatrix} 1576 & -521 & 9524 \\ -521 & 1416 & -9385 \\ 9524 & -9385 & 140464 \end{bmatrix} \times 10^2$$

$$\mathbf{\Sigma}_\tau = \underbrace{\begin{bmatrix} 7585 & 2485 & -1723 \\ 2485 & 1141 & -553 \\ -1723 & -553 & 6182 \end{bmatrix}}_{\text{using smaller chunk}} \times 10$$

Fitting an α-Stable Model

- α-stable densities not generally available in closed form
- Linear combinations of α-stable r.v.'s are α-stable
- Characteristic exponent $\alpha \in (0, 2]$
  — Lower α's yield heavier tails
  — Gaussian: $\alpha = 2$
  — Cauchy: $\alpha = 1$
- Dispersion $\gamma > 0$
  — Analogous to variance
- α estimated by the asymptotic extreme value method of Tsihrintzis and Chysostomos (IEEE Sig. Proc., June 1996)
- γ estimated via fractional lower order moments

	α	γ
f_1	1.0501	27.3
f_2	0.7197	6.7
f_3	1.0482	21.1
τ_1	1.1996	172.7
τ_2	1.0469	42.4
τ_3	1.0008	207.6

Television Signals

- Three groups, 6 MHz spacing in each group:
  — Lower VHF, channels 2-6, 55.25-79.25 MHz
  — Upper VHF, channels 7-13, 175.25-221.25 MHz
  — UHF, channels 14-83, 471.25-885.25 MHz

- Around 85% of the power is in the carrier

- Model reflected signals as doppler shifted sinusoids

- Assume one receiving station
  — N-element array of isotropic sensors

- S = number of transmitters

- M = number of targets

Received Data

- After quadrature demodulation:

$$\mathbf{r}^{(s)}(t) = \sum_{m=1}^{M} \mathbf{D}[\mathbf{p}^{(m)}(t)] b(\mathbf{p}^{(m)}(t), \mathbf{O}^{(m)}(t), \alpha^{(m)}) \times \exp\{j\omega_D[\mathbf{p}(t)^{(m)}, \dot{\mathbf{p}}^{(m)}(t)]t\} + \mathbf{w}(t)$$

— b complex reflectance
— ω_D doppler shift
— $\mathbf{w}$ noise
— $\omega_D << \omega_C$, so use narrowband approximation

- Direction vector $\mathbf{D}$ given by

$$\mathbf{D}(\mathbf{p}, \omega_D) \approx \begin{bmatrix} \exp(j\frac{\omega_C}{c}\frac{\mathbf{p}^T\mathbf{a}_1}{\|\mathbf{p}\|}) \\ \vdots \\ \exp(j\frac{\omega_C}{c}\frac{\mathbf{p}^T\mathbf{a}_N}{\|\mathbf{p}\|}) \end{bmatrix}$$

— $\mathbf{a}_1, \cdots, \mathbf{a}_N$ = positions of sensors

Target Reflectance

- Each target type, frequency, incident angle, and observation angle gives different reflectance
- Prediction via XPATCH
  —— Shooting and bouncing array approximation
  —— Not accurate at low frequencies
- Prediction via FISC
  —— Full method-of-moments solution
  —— Uses "fast multilevel multipole algorithm"
  —— Accurate at low frequencies
  —— Memory requirements impractical at high frequencies
- FISC and XPATCH output ξ normalized so that $|\xi|^2$ is the radar cross section. Adjust magnitude based on path loss according to the radar equation

$$b = \text{phase}(\xi)\sqrt{P_R} = \xi \frac{\lambda\sqrt{P_T G_T G_R}}{(4\pi)^{3/2} R_T R_R}$$

  —— G_T and G_R antenna gains
  —— R_T and R_R ranges to transmitter and receiver
  —— P_T transmitted power
- Large databases of ξ's needed

Doppler Shift

- Doppler velocity $v_D =$

$$
\begin{aligned}
&\tfrac{d}{dt}(R_T + R_R) = \tfrac{d}{dt}(\|\mathbf{p}-\mathbf{s}\| + \|\mathbf{p}-\mathbf{r}\|) \\
= \; & \tfrac{d}{dt}\Bigg(\sqrt{[(p_x(t)-s_x]^2 + [p_y(t)-s_y]^2 + [p_z(t)-s_z]^2} \\
+ \; & \sqrt{[(p_x(t)-r_x]^2 + [p_y(t)-r_y]^2 + [p_z(t)-r_z]^2}\Bigg) \\
= \; & \Bigg(\frac{[p_x(t)-s_x]\dot{p}_x(t)+[p_y(t)-s_y]\dot{p}_y(t)+[p_z(t)-s_y]\dot{p}_z(t)}{\sqrt{[p_x(t)-s_x]^2+[p_y(t)-s_y]^2+[p_z(t)-s_z]^2}} \\
+ \; & \frac{[p_x(t)-r_x]\dot{p}_x(t)+[p_y(t)-r_y]\dot{p}_y(t)+[p_z(t)-r_y]\dot{p}_z(t)}{\sqrt{[p_x(t)-r_x]^2+[p_y(t)-r_y]^2+[p_z(t)-r_z]^2}}\Bigg) \qquad (2)
\end{aligned}
$$

In practice, we compute v_D using the last expression via a discrete approximation to $\dot{\mathbf{p}}$

- Associated doppler shift is

$$f_D = -\frac{v_D}{\lambda} \text{ or } \omega_D = -\frac{2\pi v_D}{\lambda}$$

Inference via the Posterior

- Recursive estimation
  —— Extended Kalman filtering
  —— Explicit nonlinear filtering (Daum)

- Jump-diffusion processes (Grenander & Miller)
  —— Stochastic gradient ascent for continuous parameters
  —— Jumps for discrete parameters
  —— Gibbs and Metropolis style jumps

- Representation via a population of samples
  —— "Conditional Density Propagation" (Blake & Isard)
  —— "Bayesian bootstrap filter" (Gordon)
  —— Inspired by *factor sampling*
  —— Employ population of N samples
  —— Propagate samples forward under dynamical model
  —— Resample population based on likelihood

Recursive Estimation

- $\mathbf{x}[k]$ = parameters, $\mathbf{y}[k]$ = data
- $\mathbf{y}[k]$'s indep. conditioned on $\mathbf{y}[k]$'s:

$$p(\mathbf{y}[k], \ldots, \mathbf{y}[0] | \mathbf{x}[k], \ldots, \mathbf{x}[0]) = \prod_{i=0}^{k} p(\mathbf{y}[i] | \mathbf{x}[i]).$$

- Recursive update:

$$\begin{aligned}
& p(\mathbf{x}[k+1] | \mathbf{y}[k+1], \ldots, \mathbf{y}[0]) \\
= \; & \frac{1}{Z} p(\mathbf{y}[k+1] | \mathbf{x}[k+1]) p(\mathbf{x}[k+1] | \mathbf{y}[k], \ldots, \mathbf{y}[0]) \\
= \; & \frac{1}{Z} p(\mathbf{y}[k+1] | \mathbf{x}[k+1]) \\
\times \; & \int p(\mathbf{x}[k+1] | \mathbf{x}[k]) p(\mathbf{x}[k] | \mathbf{y}[k], \ldots, \mathbf{y}[0]) d\mathbf{x}[k]
\end{aligned}$$

- In traditional Kalman filtering:
  —— Dynamic and observation eqns. are linear
  —— System inputs and noise are Gaussian
  —— Densities are Gaussian
  —— Just need means and covariances
- Extended Kalman filter for nonlinear systems
  —— Linearize around current estimate
- Traditional nonlinear filtering
  —— Needs explicit form for the density
  —— Extremely complicated

The CONDENSATION Algorithm

- Independenly developed by two groups
  — Blake & Isard: "Conditional Density Propagation"
  — Gordon: "Bayesian bootstrap filter"

- Inspired by *factored sampling* for static scenes
  — Want to sample from $p(\mathbf{x}|\mathbf{y}) \propto p(\mathbf{y}|\mathbf{x})p(\mathbf{x})$
  — Suppose $p(\mathbf{x})$ easy to sample
  — Suppose $p(\mathbf{y}|\mathbf{x})$ hard to sample but easy to evaluate
  — Draw N samples $\mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(N)}$ from $p(\mathbf{x})$
  — Choose from among them with probability

$$Pr\{\text{Choose } \mathbf{x}^{(n)}\} = \frac{p(\mathbf{y}|\mathbf{x}^{(n)})}{\sum_{m=1}^{N} p(\mathbf{y}|\mathbf{x}^{(m)})}$$

CONDENSATION Con't

- Use a pool of N samples $\mathbf{x}^{(1)}[k], \ldots, \mathbf{x}^{(N)}[k]$

- Attach a probability $\pi^{(n)}[k]$ to each sample

- At each step k:

  1. For $n = 1...N$,
     (a) $\mathbf{x}^{chosen} = \mathbf{x}^{(n)}[k]$ with prob. $\pi^{(n)}[k]$
     (b) Draw $\mathbf{x}^{(n)}[k+1]$ from $p(\mathbf{x}[k+1]|\mathbf{x}^{chosen})$.
     (c) Assign $\tilde{\pi}^{(n)} = p(\mathbf{y}[k+1]|\mathbf{x}^{(n)}[k+1])$
  2. Let

$$\pi^{(n)}[k+1] = \frac{\tilde{\pi}^{(n)}}{\sum_{m=1}^{N} \tilde{\pi}^{(m)}}$$

- Compute cond. mean estimates of functions of $\mathbf{x}[k]$ by

$$E[f(\mathbf{x}[k])|\mathbf{y}[k], \ldots, \mathbf{y}[0]] \approx \sum_{n=1}^{N} \pi^{(n)}[k] f(\mathbf{x}^{(n)}[k])$$