ECE2030A Introduction to Computer Engineering
Fall 2008
Homework Assignment #1

Assigned 09/08/08 **Due in the first 5 min in class 09/15/08**
No late turn-in accepted

1. (10%) Determine the numbers for all the question marks (?) shown in the following equations. Note that all these numbers are "**signed**" numbers and are represented by two's complement using 2 bytes. Please show how you derive your answers step-by-step. You will not receive any credit if you only write down your answers with no derivation.

1.1. $(-183)_{10} = (?)_{16}$

Note that the number is negative. First convert $(183)_{10}$ to base 2.

$183 = 128 + 32 + 16 + 4 + 2 + 1$

$= 2^7 + 2^5 + 2^4 + 2^2 + 2^1 + 2^0$

$(183)_{10} = (0000\ 0000\ 1011\ 0111)_2$ [Note: leading zeros to ensure 2 bytes]

To get the negative complement :

→$(1111\ 1111\ 0100\ 1000)_2$

And then add 1.

$(-183)_{10}$→$(1111\ 1111\ 0100\ 1001)_2$

Now convert to base 16

$(-183)_{10} = (FF49)_{16}$

1.2. $(0125)_7 = (?)_9$

Convert to base 10

$(0125)_7 = 0(7^3) + 1(7^2) + 2(7^1) + 5(7^0)$

$= 0 + 49 + 14 + 5$

$(0125)_7 = (68)_{10}$

Now, convert to base 9

$(68)_{10} = 7(9^1) + 5(9^0)$

$(0125)_7 = (75)_9$

1.3. $(FFED)_{16} = (?)_{10}$

Convert to base 2

$(FFED)_{16} = (1111\ 1111\ 1110\ 1101)_2$

Notice that the leading (sign) bit is a one; therefore the answer will be negative. Apply two's complement and add one.

$= -(0000\ 0000\ 0001\ 0011)_2$

Now convert from base 2 to base 10.

$= -(2^4 + 2^1 + 2^0)_{10}$

$= -(16 + 2 + 1)_{10}$

$(FFED)_{16} = (-19)_{10}$

1.4. $(0111\ 0010\ 0000\ 1110\ 1111\ 1010\ 0110\ 1100)_2 = (?)_{16}$

Simple binary to hex conversion

$(0111\ 0010\ 0000\ 1110\ 1111\ 1010\ 0110\ 1100)_2 = (720EFA6C)_{16}$

1.5. $(79)_{10} = (142)_?$

Convert the digits of the right hand side one place at a time to base 10.

$(142)_? = (1(?^2) + 4(?^1) + 2(?^0))_{10}$

Now solve for ?

$0 = (?^2) + 4(?) - 77$

Using quadratic equation:

$$\frac{-4 \pm \sqrt{4^2 - 4(1)(-77)}}{2} = \frac{-4 \pm \sqrt{16 + 308}}{2} = \frac{-4 \pm \sqrt{324}}{2} = \frac{-4 \pm 18}{2} = -2 \pm 9$$

Therefore ? = -11, or 7. However, base must be positive.

$(79)_{10} = (142)_7$

2. (20%) Gary tried to solve the equation $5x^2 - 50x + 125 = 0$ for his algebra homework assignment. He came up with a solution x=5. All of a sudden, a flying saucer from Vega landed in his backyard and a little green man coming out from the craft told him their solution of this equation in fact is x=8 based on their number system. We (or Gray) use base-10 on earth, what are the possible base(s) these outer space visitors use in their number system?

$$5X^2 - 50X + 125 = 0$$

Let s = the unknown base.

$(5)_s(8^2)_{10} - (50)_s(8)_{10} + (125)_s = 0;$

Convert the coefficients one term at a time to base 10.

$(5)_s = 5(s^0) = (5)_{10}$

$(50)_s = 5(s^1) + 0(s^0) = (5s)_{10}$

$(125)_s = 1(s^2) + 2(s^1) + 5(s^0) = (s^2 + 2s + 5)_{10}$

$(5)_s(8^2)_{10} - (50)_s(8)_{10} + (125)_s = ((5)64 - 8(5s) + (s^2 + 2s + 5))_{10} = 0$
$= (320 - 40s + s^2 + 2s + 5)_{10}$
$= (325 - 38s + s^2)_{10}$

Now solve using the quadratic equation:

$$\frac{38 \pm \sqrt{38^2 - 4(1)(325)}}{2} = \frac{38 \pm \sqrt{1444 - 1300}}{2} = \frac{38 \pm \sqrt{144}}{2} = \frac{38 \pm 12}{2} = 19 \pm 6$$

Therefore, the possible bases are 13 and 25.

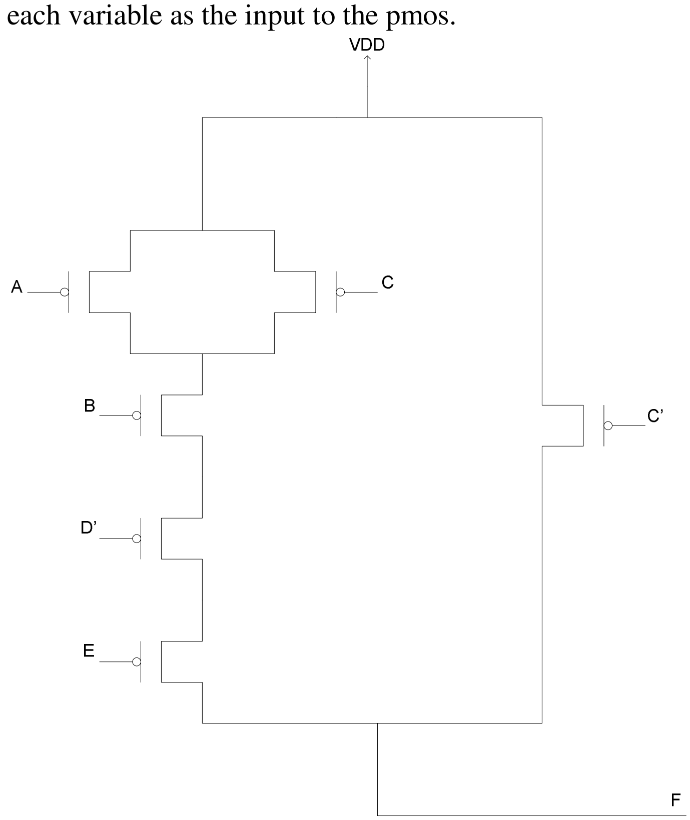
3. (20%) Draw CMOS circuits for the following two Boolean expressions. (You don't need to minimize the expression prior to your CMOS implementation. You only need to apply DeMorgan's Theorem and expand the XNOR function.)

$$F = \overline{(A \cdot C + \overline{\overline{B}} \cdot D)} \cdot \overline{(E + \overline{D})} + C$$

Method 1. Start with PUN.

$$F = \overline{(A \cdot C + \overline{\overline{B}} \cdot D)} \cdot \overline{(E + \overline{D})} + C$$
$$= \left(\overline{A \cdot C} \cdot \overline{\overline{\overline{B}} \cdot D}\right) \cdot (\overline{E} \cdot D) + C$$
$$= (\overline{A} + \overline{C}) \cdot \overline{B} \cdot D \cdot \overline{E} + C$$

Now invert each variable as the input to the pmos.



Then draw the nmos in complementary form.

Method 2. Start with PDN.

$$\overline{F} = \overline{\overline{\left(A \cdot C + \overline{\overline{B} \cdot D}\right)} \cdot \overline{\left(E + \overline{D}\right)} + C}$$

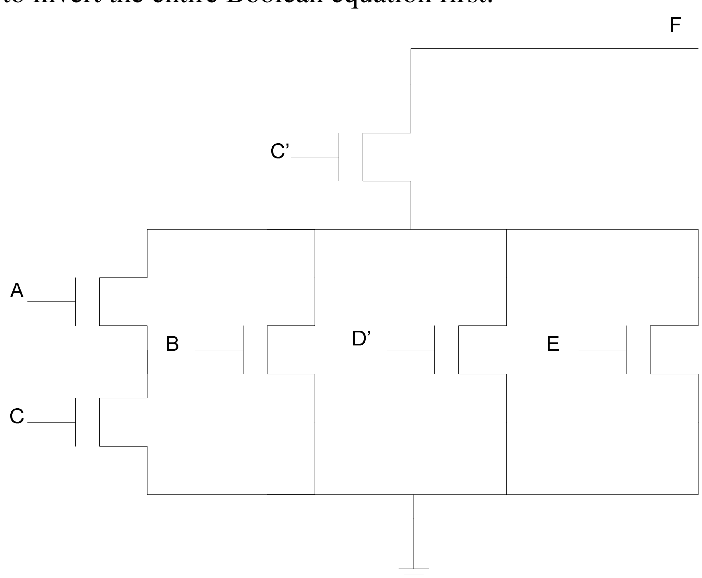
$$= \overline{\overline{\left(A \cdot C + \overline{\overline{B} \cdot D}\right)} \cdot \overline{\left(E + \overline{D}\right)}} \cdot \overline{C}$$

$$= \left[\overline{\overline{\left(A \cdot C + \overline{\overline{B} \cdot D}\right)}} + \overline{\overline{\left(E + \overline{D}\right)}}\right] \cdot \overline{C}$$

$$= \left(A \cdot C + \overline{\overline{B} \cdot D} + E + \overline{D}\right) \cdot \overline{C}$$

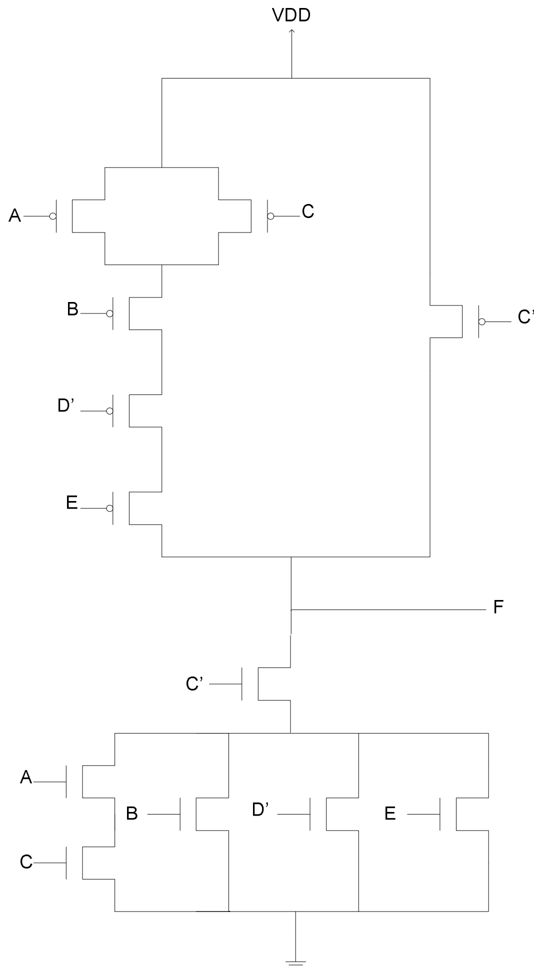
$$= (A \cdot C + B + \overline{D} + E) \cdot \overline{C}$$

Remember to invert the entire Boolean equation first.



Then draw the pmos in complementary form.

The final result regardless of the method used:



$$F = \overline{\overline{B \cdot D} \cdot \overline{E \oplus C} + C \cdot A}$$

[Note: with XOR there exist multiple representations for the logic]

Method 1. Start with PUN.

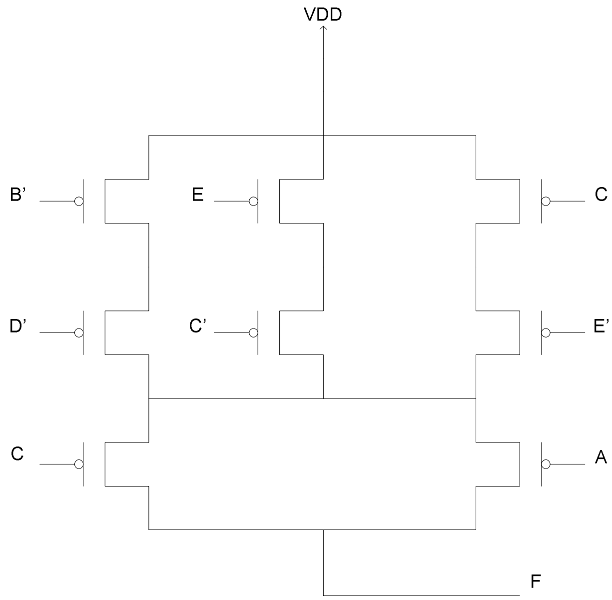
$$F = \overline{\overline{B \cdot D} \cdot \overline{E \oplus C} + C \cdot A} = \overline{\overline{B \cdot D} \cdot \overline{\left(\overline{E} \cdot C + \overline{C} \cdot E\right)} + C \cdot A}$$

$$= \overline{\overline{B \cdot D} \cdot \overline{\left(\overline{E} \cdot C + \overline{C} \cdot E\right)}} \cdot \overline{C \cdot A}$$

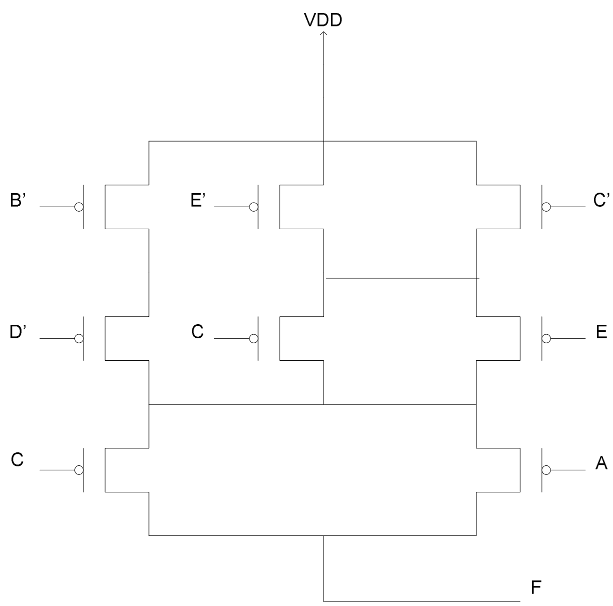
$$= \left[\overline{\overline{B \cdot D}} + \overline{\overline{\left(\overline{E} \cdot C + \overline{C} \cdot E\right)}}\right] \cdot \left(\overline{C} + \overline{A}\right)$$

$$= (B \cdot D + \overline{E} \cdot C + \overline{C} \cdot E) \cdot \left(\overline{C} + \overline{A}\right) \qquad \text{[Version 1]}$$

$$= (B \cdot D + (E + C) \cdot (\overline{C} + \overline{E})) \cdot \left(\overline{C} + \overline{A}\right) \qquad \text{[Version 2]}$$



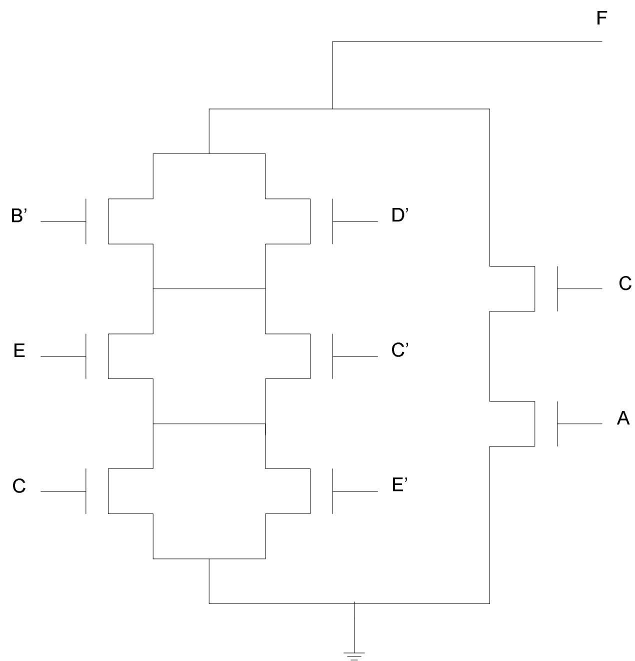
Version 1



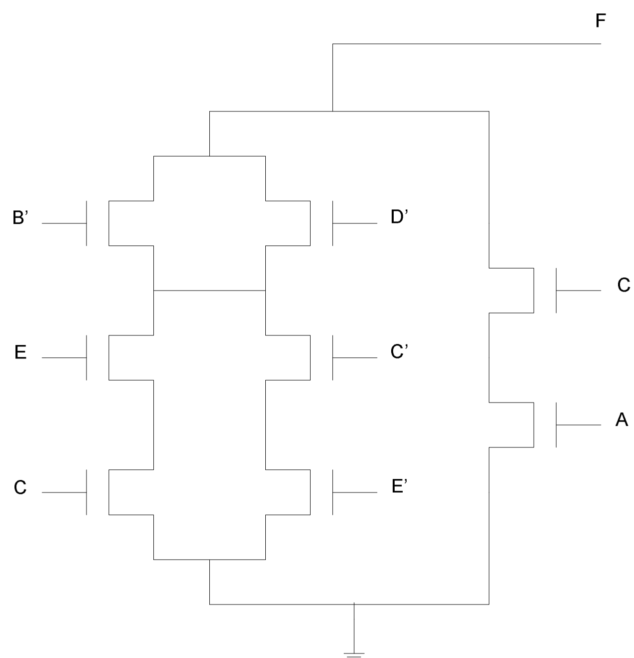
Version 2

Method 2. Start with PDN.

$$\overline{F} = \overline{\overline{\overline{B \cdot D} \cdot \overline{E \oplus C} + C \cdot A}}$$
$$= \overline{B \cdot D} \cdot \overline{E \oplus C} + C \cdot A$$
$$= (\overline{B} + \overline{D}) \cdot \overline{(\overline{E} \cdot C + \overline{C} \cdot E)} + C \cdot A$$
$$= (\overline{B} + \overline{D}) \cdot \overline{\overline{E} \cdot C} \cdot \overline{\overline{C} \cdot E} + C \cdot A$$
$$= (\overline{B} + \overline{D}) \cdot (E + \overline{C}) \cdot (C + \overline{E}) + C \cdot A \qquad \text{[Version 1]}$$
$$= (\overline{B} + \overline{D}) \cdot (E \cdot C + \overline{E} \cdot \overline{C}) + C \cdot A \qquad \text{[Version 2]}$$

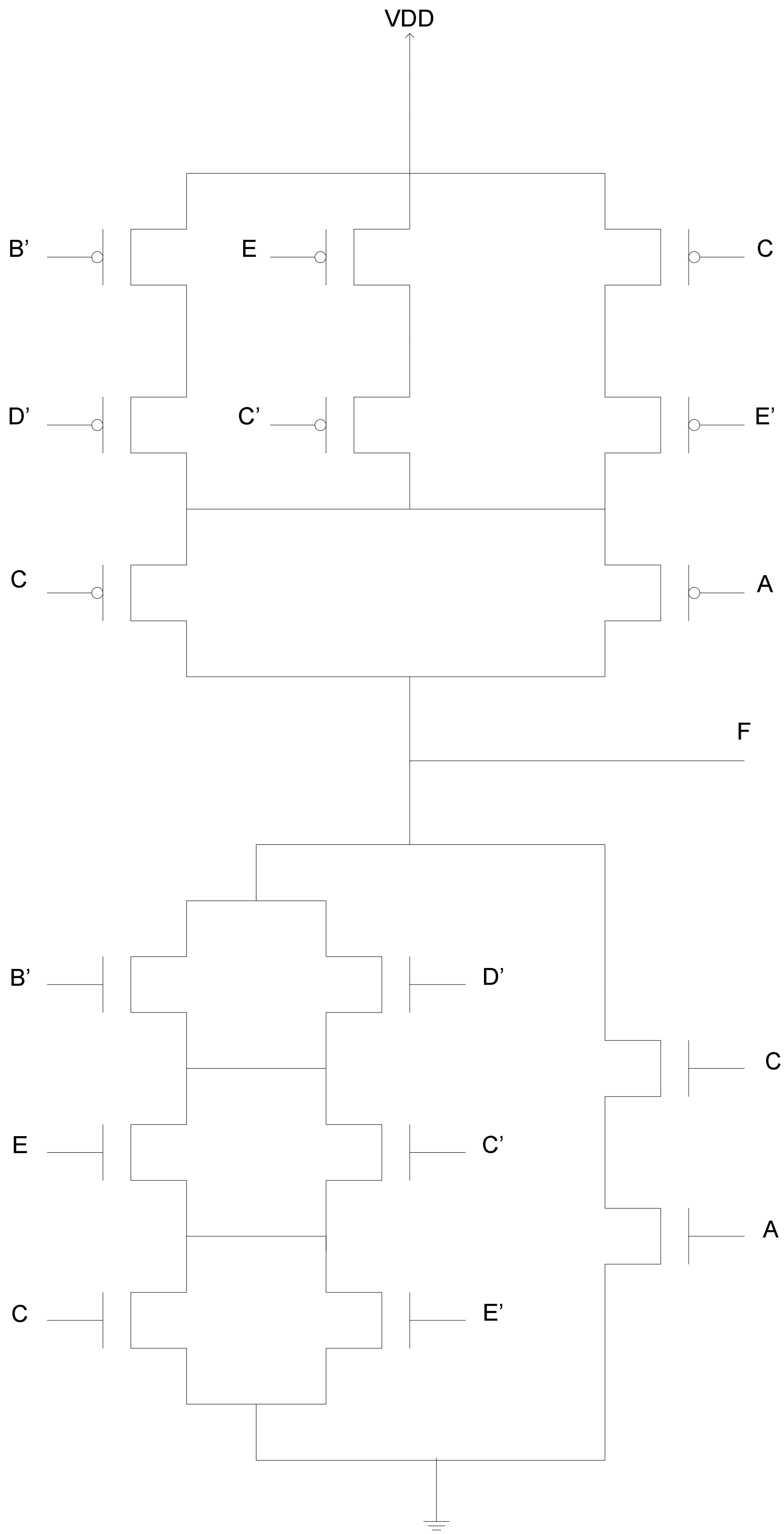


Version 1



Version 2

Any combination of the above pmos and nmos designs will work. One possible solution:


VDD
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E'

4. (10%) (1) Minimize the following Boolean equations as much as possible. (Your minimized equations should contain only "**NOT**", "**AND**" and "**OR**" operators.) Please derive your new expression using Boolean algebra only. (2) Then use Truth tables to verify the results of your minimized equations.

4.1. $\mathbf{A \oplus (A + B)}$

$$= A \cdot \overline{A + B} + \overline{A} \cdot (A + B)$$
$$= A \cdot \overline{A} \cdot \overline{B} + \overline{A} \cdot A + \overline{A} \cdot B = 0 \cdot B + 0 + \overline{A} \cdot B$$
$$= \mathbf{\overline{A} \cdot B}$$

AB	$A \oplus (A + B)$
00	$0 \oplus (0 + 0) = 0 \oplus 0 = 0$
01	$0 \oplus (0 + 1) = 0 \oplus 1 = 1$
10	$1 \oplus (1 + 0) = 1 \oplus 1 = 0$
11	$1 \oplus (1 + 1) = 1 \oplus 1 = 0$

AB	$\overline{A} \cdot B$
00	$1{\cdot}0 = 0$
01	$1{\cdot}1 = 1$
10	$0{\cdot}0 = 0$
11	$0{\cdot}1 = 0$

4.2. $\mathbf{A \oplus B \oplus (A \cdot B)}$

$$= A \oplus \left(\overline{B} \cdot (A \cdot B) + B \cdot \left(\overline{A \cdot B}\right)\right)$$
$$= A \oplus \left(\overline{B} \cdot A \cdot B + B \cdot \left(\overline{A} + \overline{B}\right)\right)$$
$$= A \oplus \left(0 \cdot A + B \cdot \overline{A} + B \cdot \overline{B}\right)$$
$$= A \oplus \left(B \cdot \overline{A} + 0\right)$$
$$= A \cdot \overline{B \cdot \overline{A}} + \overline{A} \cdot B \cdot \overline{A}$$
$$= A \cdot \left(\overline{B} + A\right) + \overline{A} \cdot B$$
$$= \mathbf{A + B}$$

AB	$A \oplus B \oplus (A \cdot B)$
00	$0 \oplus 0 \oplus (0 \cdot 0) = 0 \oplus 0 = 0$
01	$0 \oplus 1 \oplus (0 \cdot 1) = 0 \oplus 1 = 1$
10	$1 \oplus 0 \oplus (1 \cdot 0) = 1 \oplus 0 = 1$
11	$1 \oplus 1 \oplus (1 \cdot 1) = 1 \oplus 1 = 1$

AB	$A + B$
00	$0 + 0 = 0$
01	$0 + 1 = 1$
10	$1 + 0 = 1$
11	$1 + 1 = 1$

5. (10%) Prove the Consensus Theorem as shown below. Derive your proof using Boolean algebra from the Left-Hand Side (LHS) to the Right-Hand Side (RHS). Please DO NOT use truth table method nor apply duality principle directly to the LHS.

$$\mathbf{(X + Y) \cdot (\overline{X} + Z) \cdot (Y + Z) = (X + Y) \cdot (\overline{X} + Z)}$$

$$\text{LHS} = (X + Y) \cdot \left(\overline{X} + Z\right) \cdot (Y + Z)$$
$$= \left(X \cdot \overline{X} + X \cdot Z + Y \cdot \overline{X} + Y \cdot Z\right) \cdot (Y + Z)$$
$$= \left(0 + X \cdot Z + Y \cdot \overline{X} + Y \cdot Z\right) \cdot (Y + Z)$$
$$= X \cdot Y \cdot Z + Y \cdot Y \cdot \overline{X} + Y \cdot Y \cdot Z + X \cdot Z \cdot Z + Y \cdot \overline{X} \cdot Z + Y \cdot Z \cdot Z$$
$$= X \cdot Y \cdot Z + Y \cdot \overline{X} + Y \cdot Z + X \cdot Z + Y \cdot \overline{X} \cdot Z$$

$= Y \cdot Z \cdot (X + 1 + \overline{X}) + Y \cdot \overline{X} + X \cdot Z$
$= Y \cdot Z + Y \cdot \overline{X} + X \cdot Z$

$\text{RHS} = (X + Y) \cdot (\overline{X} + Z)$
$= X \cdot \overline{X} + X \cdot Z + Y \cdot \overline{X} + Y \cdot Z$
$= 0 + X \cdot Z + Y \cdot \overline{X} + Y \cdot Z$
$= X \cdot Z + Y \cdot \overline{X} + Y \cdot Z$

Therefore, LHS = RHS

6. (10%) Convert the following equation to its canonical SOP form and canonical POS form. Represent your canonical form in the format of **Σ** and **∏**. (Derive your answer use Boolean algebra. Do not use truth table.)

$$F(A, B, C) = \overline{A \cdot C} + \overline{A} \cdot B + \overline{B + \overline{C}}$$

$= (\overline{A} + \overline{C}) + \overline{A} \cdot B + \overline{B} \cdot C$
$= \overline{A} \cdot (1 + B) + \overline{C} + \overline{B} \cdot C$
$= \overline{A} + \overline{C} + \overline{B} \cdot C$

SOP: Expand each term to encompass all literals:
$\overline{A} = \overline{A} \cdot (B + \overline{B}) \cdot (C + \overline{C}) = \overline{A} \cdot B \cdot C + \overline{A} \cdot \overline{B} \cdot C + \overline{A} \cdot B \cdot \overline{C} + \overline{A} \cdot \overline{B} \cdot \overline{C}$
$= m(0) + m(1) + m(2) + m(3)$
$\overline{C} = \overline{C} \cdot (B + \overline{B}) \cdot (A + \overline{A}) = A \cdot B \cdot \overline{C} + A \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot B \cdot \overline{C} + \overline{A} \cdot \overline{B} \cdot \overline{C}$
$= m(0) + m(2) + m(4) + m(6)$
$\overline{B} \cdot C = \overline{B} \cdot C \cdot (A + \overline{A}) = A \cdot \overline{B} \cdot C + \overline{A} \cdot \overline{B} \cdot C = m(5) + m(1)$

Therefore
$F(A, B, C) = \sum m(0, 1,2,3,4,5,6)$

POS: Use the property [X+Y·Z = (X+Y) ·(X+Z)]
$F(A, B, C) = \overline{A} + \overline{C} + \overline{B} \cdot C$
$= \overline{A} + (\overline{C} + \overline{B}) \cdot (C + \overline{C})$
$= \overline{A} + (\overline{C} + \overline{B}) \cdot (1)$
$= \overline{A} + \overline{B} + \overline{C} = M(7)$

Therefore
$F(A, B, C) = \prod M(7)$

[Note: the terms of the POS plus the terms of the SOP accounts for all the 8 arrangement of literals]

7. (20%) Given the following canonical POS function in $\mathcal{B}^4$, please (1) Use Karnaugh map method to find a minimal SOP expression. (2) Draw your derived SOP function using CMOS.

$$F(A,B,C,D) = \prod M(3,6,7,11,12,13,15) + d(2,8)$$

Note the cells are indexed in the following manner. This is arranged such that only one bit is changed from one adjacent cell to another.

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Step 1. Fill in Maxterms and don't cares

AB \ CD	00	01	11	10
00			0	X
01			0	0
11	0	0	0	
10	X		0	

Step 2. Fill in remainder with 1s

AB \ CD	00	01	11	10
00	1	1	0	X
01	1	1	0	0
11	0	0	0	1
10	X	1	0	1

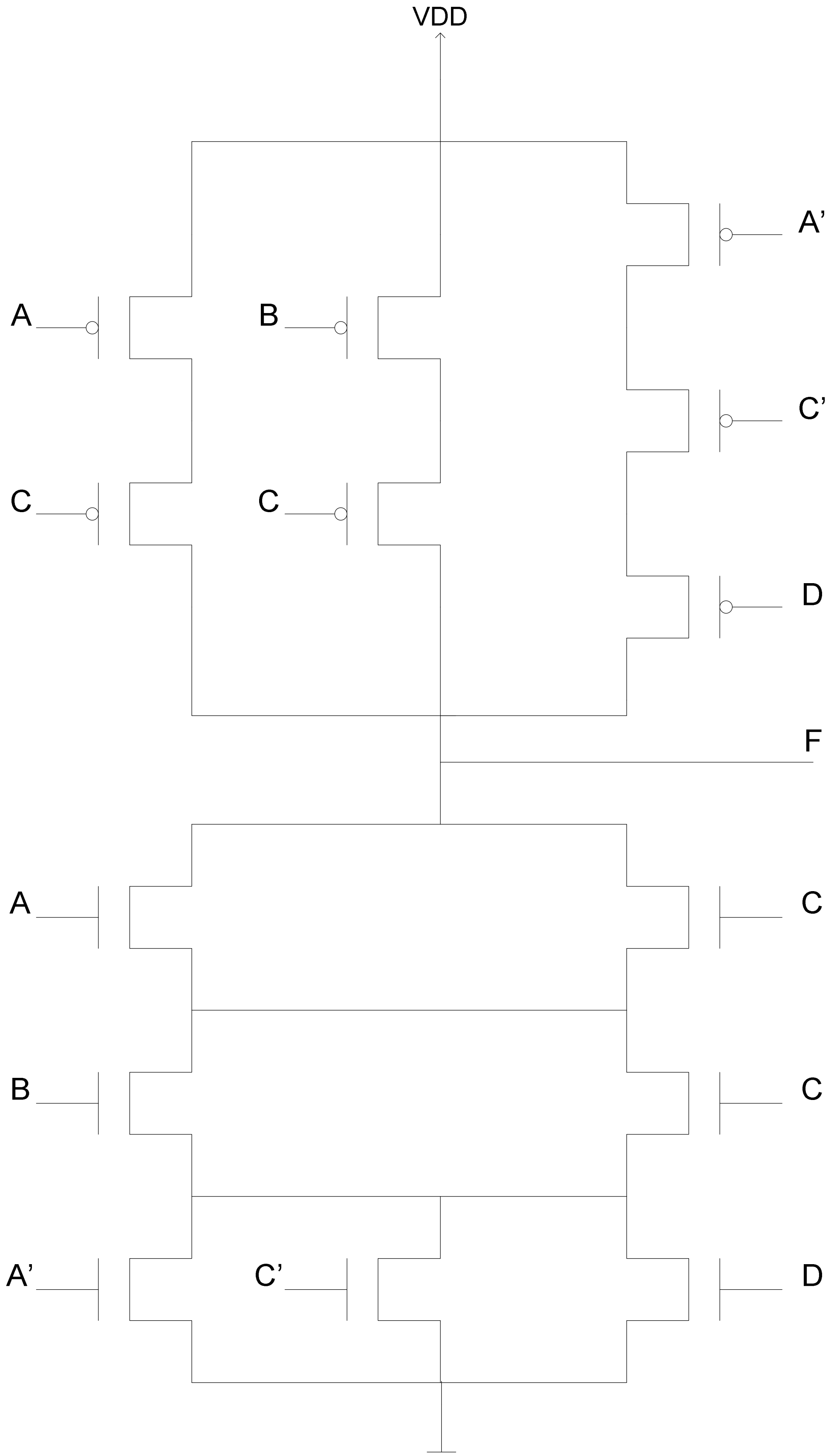
Step 3. Find Largest Prime Implicants

AB \ CD	00	01	11	10
00	1	1	0	X
01	1	1	0	0
11	0	0	0	1
10	X	1	0	1

$= \overline{A}\cdot\overline{C} + \overline{B}\cdot\overline{C} + A\cdot C\cdot\overline{D}$

The minimal SOP is:

$SOP = \overline{A}\cdot\overline{C} + \overline{B}\cdot\overline{C} + A\cdot C\cdot\overline{D}$


VDD
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B
C
C
A'
C'
D
F
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