EE-4078: Analog Filtering via Discrete-Time Filtering: 1997–

When the input to a discrete-time system is a sequence derived by sampling a continuous-time signal, we can use our knowledge of sampling and reconstruction to interpret the effect of the filter on the original continuous-time signal. Consider the system depicted in Fig. 1.

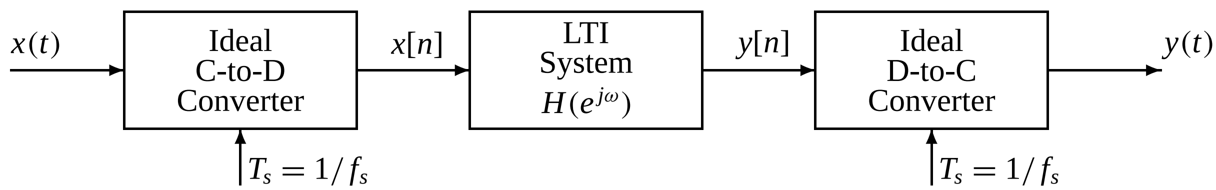

Figure 1: System for doing discrete-time filtering of continuous-time signals.

Now suppose that the input is

$$x(t) = Ae^{j\phi}e^{j\Omega t}$$

so that the input sequence to the discrete-time filter is

$$x[n] = x(nT_s) = Ae^{j\phi}e^{j\Omega nT_s} = Ae^{j\phi}e^{j\omega n}$$

where $\omega = \Omega T_s$. If the frequency of the continuous-time signal is such that $|\Omega| < \pi/T_s$ (no aliasing), then the normalized discrete-time frequency is such that $|\omega| < \pi$.

Now we know that the output of the discrete-time system for a complex exponential input is

$$y[n] = H(e^{j\omega})Ae^{j\phi}e^{j\omega n}$$

where if we make the substitution $\omega = \Omega T_s$, then

$$y[n] = H(e^{j\omega T_s})Ae^{j\phi}e^{j\omega T_s n}$$

Recall that the ideal D/C converter reconstructs all frequency components in the band $|\omega| < \pi$ as continuous-time frequencies $|\omega| < \pi/T_s$. Since we assume that no aliasing occurs, it follows that the output of the ideal C/D converter is

$$y(t) = H(e^{j\omega T_s})Ae^{j\phi}e^{j\omega t}$$

for all frequencies such that $-\pi/T_s < \omega < \pi/T_s$. Thus, the overall system of Fig. 1 behaves as if it is a linear time-invariant continuous-time system whose frequency response is $H(e^{j\omega T_s})$.

As an example, consider the 11-point moving averager

$$y[n] = \frac{1}{11}\sum_{k=0}^{10} x[n-k]$$

The frequency response of this discrete-time system is

$$H(e^{j\omega}) = \frac{\sin(\omega 11/2)}{11\sin(\omega/2)} e^{-j\omega 5}$$

The magnitude of this frequency response is shown in the top part of Fig. 2. Remember that $H(e^{j\omega})$ is periodic with period 2π. If this system is used for the discrete-time system in Fig. 1 with sampling frequency $f_s = 1000$, then the equivalent continuous-time frequency response is as shown in the bottom part of Fig. 2. Note that the frequency response of the overall system stops abruptly at $|f| = f_s/2 = 500$ Hz, since the ideal D/C converter does not reconstruct frequencies above $f_s/2$.

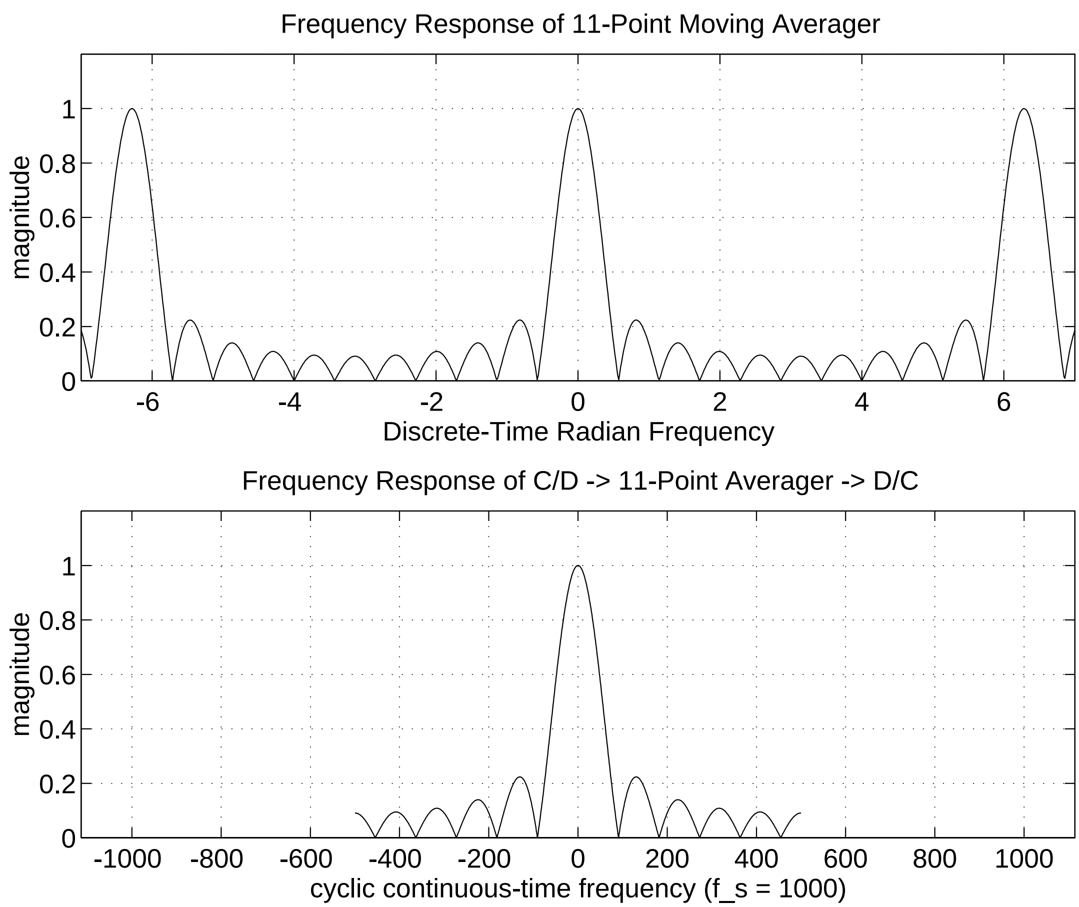

Figure 2: Frequency response of 11-point moving averager (top) and equivalent continuous-time frequency response (bottom) when the averager is used as the digital filter inside the system in Fig. 1. The sampling frequency is 1000 Hz.