

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 4078 SPRING 1998
Problem Set #1

Assigned: 1 April 1998

Due Date: 13 April 1998 (MONDAY)

Reading: Chapter 1 (all) and Chapter 2 (sections 2.1–2.6) in Oppenheim & Schaffer (1989).

In addition, **review** section 2.10 and Appendix A (A.1–A.3) for random signals.

Turn in *only* the starred problems; some of them will be graded.

PROBLEM 1.1*:

A *three-point weighted running average* is defined as follows:

$$y[n] = T\{x[n]\} = \frac{1}{3}(x[n] + 2x[n-2] + x[n-4])$$

where $x[n]$ is the input signal and $y[n]$ is the output signal.

- (a) Is this system causal? Prove or disprove.
- (b) Find the impulse response of this system, $h[n]$.
- (c) Determine the output $y[n]$ when the input is $x[n] = \delta[n+1] - 3\delta[n] + \delta[n-1]$. Plot $x[n]$ and $y[n]$.

PROBLEM 1.2:

For each of the following system transformations, determine whether or not the following properties hold:

(1) MEMORYLESS, (2) TIME-INVARIANT, (3) LINEAR, (4) CAUSAL

Justify your answers by giving a counter-example if the property is FALSE, or a proof if it is TRUE.

- (a) $y[n] = x[-n]$ (*Flip*)
- (b) $y[n] = x[3n]$ (*Compress*)
- (c) $y[n] = x[n-2]$ (*Delay by two*)

PROBLEM 1.3*:

(Review EE-3340) Assume that a random signal $x[n]$ is “Gaussian,” i.e., its *probability density function* (pdf) is

$$p_x(\xi) = Ae^{(-7\xi^2 - \xi)}$$

- (a) Determine the value for A so that $p_x(\xi)$ is a legitimate pdf.
- (b) Determine the mean and variance of the *stationary* random signal $x[n]$.
HINT: express $p_x(\xi)$ in the standard form for a Gaussian and extract the parameters.

PROBLEM 1.4*:

True or false? Is it possible to find two non-zero signals, $a[n]$ and $b[n]$, that satisfy $a[n] * b[n] = 0$?
If true, give a specific example.

PROBLEM 1.5*:

Compute the convolution sum $y[n] = x[n] * h[n]$ for the following pairs of signals: In all cases, make a sketch of $x[n]$, $h[n]$ and the result $y[n]$.

(a) $x[n] = (-\frac{1}{3})^n u[n]$ and $h[n] = 9\delta[n - 1] - \delta[n + 1]$

(b) $x[n] = u[n] - u[n - 6]$ and $h[n] = u[n + 2] - u[n - 2]$

(c) $x[n] = u[n - 2]$ and $h[n] = e^{j2\pi n/5} u[n]$

PROBLEM 1.6*:

A LTI system has an impulse response given by:

$$h[n] = 7 \left(\frac{1}{4}\right)^n u[n - 3]$$

- (a) If the input to this system is $x[n] = u[n]$, determine the output signal. Determine the numerical value of the output as $n \rightarrow \infty$.
- (b) If the input to this system is a periodic signal:

$$x[n] = \sin(2\pi n/5) \quad n = -\infty, \dots, -2, -1, 0, 1, 2, 3, \dots, \infty$$

Find the output of the system, $y[n] = h[n] * x[n]$, and then verify that $y[n]$ is periodic.

PROBLEM 1.7*:

Polynomial multiplication is *exactly* the same operation as convolution!

In this problem, the multiplication of two polynomials is treated in detail. Let $A(x)$ and $B(x)$ be two finite degree polynomials, and let $C(x)$ be their product: $C(x) = A(x)B(x)$

$$A(x) = a_0 + a_1x + a_2x^2 + \dots + a_px^p$$

$$B(x) = b_0 + b_1x + b_2x^2 + \dots + b_qx^q$$

$$C(x) = A(x)B(x) = \sum_{i=0}^{p+q} c_i x^i$$

- (a) Each coefficient of the product polynomial $C(x)$ is composed of pairwise products, $a_\ell b_k$, summed together. If the degree of $A(x)$ is $p = 3$ and the degree of $B(x)$ is $q = 7$, determine which index pairs (ℓ, k) contribute to c_3 ; and which terms make up c_4 .
- (b) State a general rule about these index pairs, applicable to the case of finding c_i , the coefficient of x^i for any p and q . NOTE: the objective in stating this rule is to be able to compute just one coefficient c_i without doing all the others. Base your rule on the convolution sum.
- (c) Use the general rule in the previous part to find c_{32} , c_{41} and c_{91} , when the product is formed from a 61st degree $A(x)$ with coefficients $a_\ell = (-1)^\ell$ for $\ell = 0, 1, \dots, 61$; and a 36th degree $B(x)$ with coefficients $b_k = 1$ for $k = 0, 1, \dots, 36$.

$$A(x) = 1 - x + x^2 - x^3 + x^4 - x^5 \dots + x^{60} - x^{61}$$

$$B(x) = 1 + x + x^2 + x^3 + x^4 + x^5 \dots + x^{35} + x^{36}$$