GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 4078 SPRING 1998
Problem Set #2

Assigned: 13 April 1998
Due Date: 20 April 1998 (MONDAY)

Quiz #1 will be given in class on 22-Apr-97 (Wed). Closed book, one page of notes allowed.

Reading: Chapter 2 (sections 2.6–end) and start Chapter 3 in Oppenheim & Schafer (1989).
In addition, **review** section 2.10 and Appendix A (A.1–A.3) for random signals.

The web site URL is: `www.ece.gatech.edu/users/mcclella/courses/ee4078.htm`

Turn in *only* the starred problems; some of them will be graded.

PROBLEM 2.1*:

Determine a simple formula for the DTFT of each of the following:

(a) Delayed pulse: $x_a[n] = \begin{cases} 4\frac{1}{2} & 13 < n < 23 \\ 0 & \text{elsewhere} \end{cases}$

(b) $x_b[n] = (\frac{3}{4})^{n-4}\, u[n-2]$

(c) $x_c[n] = \dfrac{\sin(\pi(n+2)/7)}{n+2}$

PROBLEM 2.2*:

Determine a simple formula for the inverse DTFT of each of the following:

(a) $X_a(e^{j\omega}) = \dfrac{e^{j13\omega}}{e^{j\omega} - \frac{1}{2}}$

(b) $X_b(e^{j\omega}) = \begin{cases} 0 & 2\pi/5 < |\omega| < 3\pi/5 \\ 1 & \text{elsewhere} \end{cases}$

(c) $X_c(e^{j\omega}) = 13\dfrac{\sin 4\frac{1}{2}\omega}{\sin \frac{1}{2}\omega}\, e^{-j\omega}$

(d) $X_d(e^{j\omega}) = 4\frac{1}{2} - \sin(13\omega)$

PROBLEM 2.3:

The *Dirichlet* function, sometimes called the“aliased sinc” function, occurs quite often in DSP.

(a) Show that $\displaystyle\sum_{n=0}^{L-1} e^{-j\omega n} = e^{-j\omega(L-1)/2}\, \frac{\sin(\omega L/2)}{\sin(\omega/2)}$

(b) Define the *Dirichlet* function as: $\mathrm{D}(\omega, L) = \dfrac{\sin(\omega L/2)}{\sin(\omega/2)}$. Plot $\mathrm{D}(\omega, 8)$ versus ω, and show that its period is 4π.

PROBLEM 2.4*:

A linear time-invariant system has the following impulse response:

$$h[n] = (-1)^n \frac{\sin(\pi n/13)}{\pi n}$$

(a) Determine a formula for the frequency response of the system, $H(e^{j\omega})$ and make a sketch.

(b) If the input to the LTI system is $x[n] = \sin(\pi n/4)$, determine the output.

PROBLEM 2.5*:

A stable linear time-invariant system has the following frequency response:

$$H(e^{j\omega}) = \frac{e^{-j\omega}}{e^{j\omega} - \frac{1}{2}}$$

(a) Sketch the magnitude and phase of $H(e^{j\omega})$.

(b) Determine a formula for the output of the system when the input is $x[n] = \delta[n-4]$.

(c) If the input to the LTI system is $x[n] = \sin(\pi(n-2)/5)u[n]$, determine the "steady state" value of the output as $n \to \infty$.

PROBLEM 2.6:

Define a *finite-length* impulse train by the formula:

$$p[n] = \sum_{\ell=0}^{4} (-1)^{\ell} \delta[n - 2\ell - 1] = \delta[n-1] - \delta[n-3] + \delta[n-5] - \delta[n-7] + \delta[n-9]$$

Let $P(e^{j\omega})$ denote the DTFT of $p[n]$.

(a) Determine the phase of $P(e^{j\omega})$.

(b) Compute the inverse DTFT of $V(e^{j\omega}) = e^{-j3\omega} P^*(e^{j\omega})$. Sketch $v[n]$.

(c) Compute the inverse DTFT of $C(e^{j\omega}) = |P(e^{j\omega})|^2$. Make a sketch of $c[n]$.

PROBLEM 2.7*:

Suppose that a speech signal is sampled at $f_s = 8000$Hz, and that a block of 400 points of the signal is collected for processing. These 400 points form a signal called $s[n]$. The DTFT of $s[n]$ is $S(e^{j\omega})$ and it contains peaks at

$$\omega = \left\{ \frac{\pi}{10}, \frac{\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{10} \right\}$$

What are the dominant frequency components (in Hertz) of the analog speech signal? and how many are there? Give your answer in Hertz or radians per second. EXPLAIN your answer by giving a formula that relates analog frequency to DTFT frequency.

PROBLEM 2.8:

Use the flip property and the conjugate property of the DTFT to prove the following:

(a) When $x[n]$ is purely imaginary and also odd, its DTFT is purely real.

(b) When $X(e^{j\omega}) = -X^*(e^{-j\omega})$, then $x[n]$ is purely imaginary.