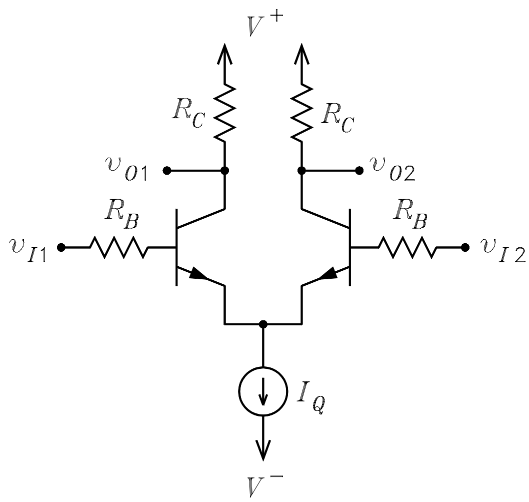
ECE3040 – Assignment 8

Symbolic answers given for problems below depend on the small-signal model used, i.e. the pi model, the T model, or the simplified T model, and the approach taken to solve for the currents. If you are familiar with the models, it should be obvious how the answers were obtained. To obtain better practice, you should also try to solve the problems using different models and approaches.

1. The figure shows a differential amplifier. It is given that $V^+ = 15\,\mathrm{V}$, $V^- = -15\,\mathrm{V}$, and $R_B = 1\,\mathrm{k}\Omega$. For each BJT, $\beta = 99$, $r_0 = \infty$, and $V_T = 25\,\mathrm{mV}$.

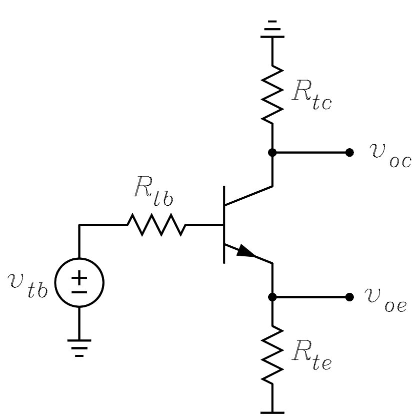
   (a) Solve for R_C and I_Q such that $V_C = V^+/2$ for each BJT and $v_{o1} = -v_{o2} = -100\,(v_{i1} - v_{i2})$. [$I_Q = 2.5\,\mathrm{mA}$ and $R_C = 6.061\,\mathrm{k}\Omega$]

   (b) Let a resistor R_E be inserted in series with the emitter of each BJT. Solve for the value of R_E which will make $v_{o1} = -v_{o2} = -50\,(v_{i1} - v_{i2})$. [$R_E = 30\,\Omega$]



2. The figure shows a BJT phase splitter. The BJT is biased at the emitter current I_E. If $r_0 = \infty$, show that

$$v_{oc} = \frac{-\alpha R_{tc}}{\dfrac{R_{tb}}{1+\beta} + R_{te} + \dfrac{V_T}{I_E}} v_{tb} \qquad v_{oc} = \frac{+R_{te}}{\dfrac{R_{tb}}{1+\beta} + R_{te} + \dfrac{V_T}{I_E}} v_{tb}$$

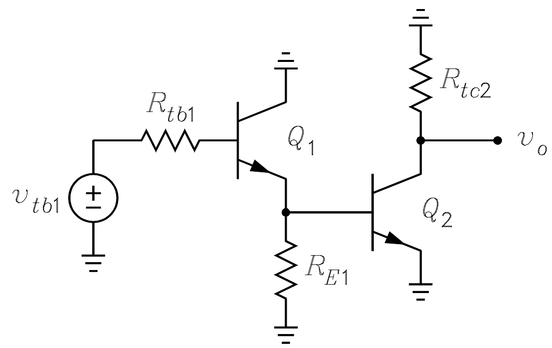


3. The figure shows a CC/CE amplifier. Show that v_o/v_{tb1} can be written

$$\frac{v_o}{v_{tb1}} = \frac{v_{e1}}{v_{tb1}} \times \frac{v_{b2}}{v_{e1}} \times \frac{i'_{c2}}{v_{b2}} \times \frac{v_o}{i'_{c2}}$$

where

$$\frac{v_{b2}}{v_{tb1}} = \frac{r_{01} \| R_{E1} \| r_{\pi 2}}{\dfrac{R_{tb1}}{1+\beta_1} + r_{01} \| R_{E1} \| r_{\pi 2} + r_{e1}} \qquad \frac{v_{b2}}{v_{e1}} = 1 \qquad \frac{i'_{c2}}{v_{b2}} = g_{m2} \qquad \frac{v_o}{i'_{c2}} = -\left(r_{02} \| R_{tc2}\right)$$

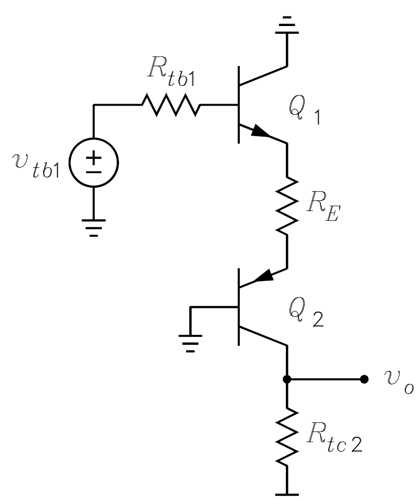


4. The figure shows a CC/CB stage, sometimes called an emitter coupled pair. For each BJT assume $r_0 = \infty$. Show that v_o/v_{tb1} can be written

$$\frac{v_o}{v_{tb1}} = \frac{i'_{e1}}{v_{tb1}} \times \frac{i'_{e2}}{i'_{e1}} \times \frac{i'_{c2}}{i'_{e2}} \times \frac{v_o}{i'_{c2}}$$

where

$$\frac{i'_{e1}}{v_{tb1}} = \frac{1}{\dfrac{R_{tb1}}{1+\beta_1} + r_{e1} + R_E + r_{e2}} \qquad \frac{i'_{e2}}{i'_{e1}} = -1 \qquad \frac{i'_{c2}}{i'_{e2}} = \alpha_2 \qquad \frac{v_o}{i'_{c2}} = -R_{tc2}$$



5. If r_{01} is included in 4, show that

$$\frac{i'_{e1}}{v_{tb1}} = \frac{1}{\dfrac{R_{tb1}}{1+\beta_1} + r_{e1} + r_{01} \| (R_E + r_{e2})} \qquad \frac{i'_{e2}}{i'_{e1}} = \frac{-r_{01}}{r_{01} + R_E + r_{e2}}$$

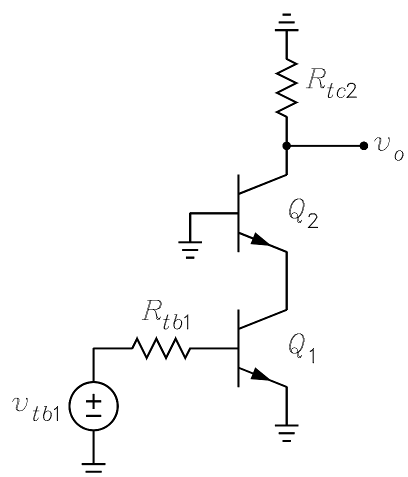
$$\frac{i'_{c2}}{i'_{e2}} = \alpha_2 \qquad \frac{v_o}{i'_{c2}} = -R_{tc2}$$

6. The figure shows a CE/CB or cascode amplifier. If $r_{02} = \infty$, show that

$$\frac{v_o}{v_{tb1}} = \frac{i_{b1}}{v_{tb1}} \times \frac{i'_{c1}}{i_{b1}} \times \frac{i'_{e2}}{i'_{c1}} \times \frac{i'_{c2}}{i'_{e2}} \times \frac{v_o}{i'_{c2}}$$

where

$$\frac{i_{b1}}{v_{tb1}} = \frac{1}{R_{tb1} + r_{\pi 1}} \qquad \frac{i'_{c1}}{i_{b1}} = \beta_1 \qquad \frac{i'_{e2}}{i'_{c1}} = \frac{r_{01}}{r_{01} + r_{e2}} \qquad \frac{i'_{c2}}{i'_{e2}} = \alpha_2 \qquad \frac{v_o}{i'_{c2}} = -R_{tc2}$$



7. A differential amplifier has a differential gain of -100 and a common-mode gain of -0.01. That is, its output voltage is given by

$$v_o = -100 v_{id} - 0.01 v_{icm}$$

where v_{id} and v_{icm}, respectively, are the differential and common-mode components of the input voltages given by

$$v_{id} = v_{i1} - v_{i2} \qquad v_{icm} = \frac{v_{i1} + v_{i2}}{2}$$

The input voltages are given by $v_{i1} = 0.1 \sin \omega_1 t - 0.01 \sin \omega_2 t$ and $v_{i2} = 0.1 \sin \omega_1 t + 0.01 \sin \omega_2 t$.

(a) Show that the differential and common-mode input voltages are given by

$$v_{id} = -0.02 \sin \omega_2 t \qquad v_{icm} = 0.1 \sin \omega_1 t$$

(b) Show that the output voltage is given by

$$v_o = -0.001 \sin \omega_1 t + 2 \sin \omega_2 t$$

(c) If the ω_1 term is an unwanted interference signal and the ω_2 term is a desired signal, the input signal-to-noise ratio is $|v_{id}/v_{icm}| = 0.02/0.1 = 0.2$ or $-14\,\text{dB}$, which is pretty low. What is the signal-to-noise ratio at the output? Answer: 2000 or 66 dB.

(d) What is the improvement in the signal-to-noise ratio between the input and the output? Answer: 80 dB. Note: This problem illustrates how a common-mode noise can be reduced by a large amount with a differential amplifier input stage.

(e) The common-mode rejection ratio is the ratio of the differential gain to the common-mode gain, expressed in dB. What is the $CMRR$? Answer: 80 dB.

8. The figure shows a complementary CC amplifier. Each BJT has the saturation current $I_S = 2 \times 10^{-12}$ A.

(a) If cutin is defined as the base-emitter voltage at which the collector current is 0.1 mA, determine the cutin voltage for the two transistors. [$V_\gamma = 0.443\,\text{V}$]

(b) If $r_0 = \infty$ and either transistor is in its active mode, use the T model to show that the slope of the v_O versus v_I curve is given by

$$\frac{\Delta v_O}{\Delta v_I} = \frac{v_O}{v_O + V_T}$$

where V_T is the thermal voltage.