The Common-Collector Amplifier

Basic Circuit

Fig. 1 shows the circuit diagram of a single stage common-collector amplifier. The object is to solve for the small-signal voltage gain, input resistance, and output resistance.

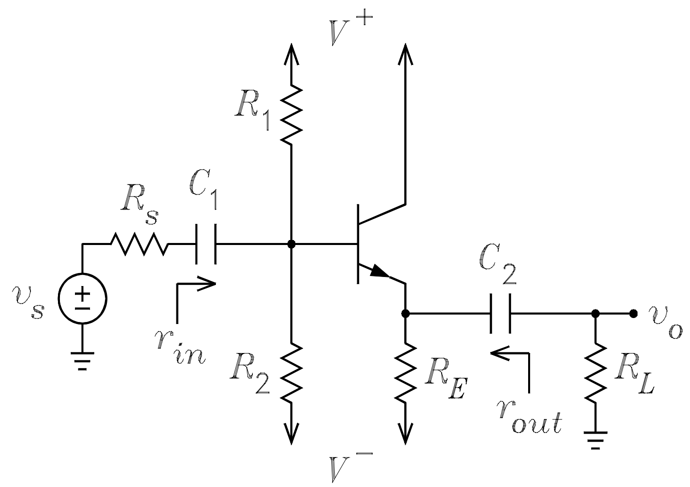

Figure 1: Common-collector amplifier.

DC Solution

(a) Replace the capacitors with open circuits. Look out of the 3 BJT terminals and make Thévenin equivalent circuits as shown in Fig. 2.

$$V_{BB} = \frac{V^+R_2 + V^-R_1}{R_1 + R_2} \qquad R_{BB} = R_1 \| R_2$$

$$V_{EE} = V^- \qquad R_{EE} = R_E \qquad V_{CC} = V^+ \qquad R_{CC} = R_C$$

(b) Make an "educated guess" for V_{BE}. Write the loop equation between the V_{BB} and the V_{EE} nodes.

$$V_{BB} - V_{EE} = I_B R_{BB} + V_{BE} + I_E R_{EE} = \frac{I_C}{\beta} R_{BB} + V_{BE} + \frac{I_C}{\alpha} R_{EE}$$

(c) Solve the loop equation for the currents.

$$I_C = \alpha I_E = \beta I_B = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/\beta + R_{EE}/\alpha}$$

(d) Verify that $V_{CB} > 0$ for the active mode.

$$V_{CB} = V_C - V_B = (V_{CC} - I_C R_{CC}) - (V_{BB} - I_B R_{BB}) = V_{CC} - V_{BB} - I_C (R_{CC} - R_{BB}/\beta)$$

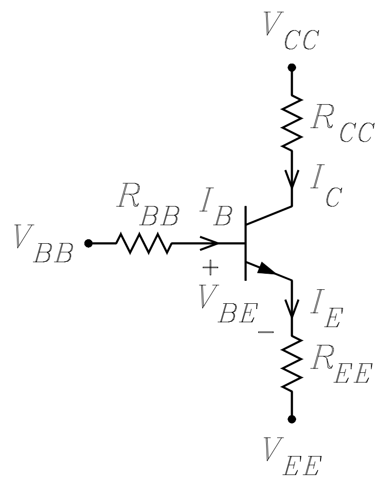

Figure 2: Bias circuit.

Small-Signal or AC Solutions

(a) Redraw the circuit with $V^+ = V^- = 0$ and all capacitors replaced with short circuits as shown in Fig. 3.

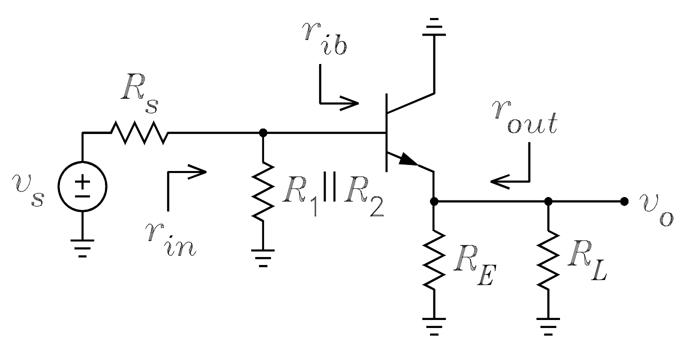

Figure 3: Signal circuit.

(b) Calculate g_m, r_π, r_e, and r_0 from the DC solution.

$$g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{V_T}{I_B} \qquad r_e = \frac{V_T}{I_E} \qquad r_0 = \frac{V_A + V_{CE}}{I_C}$$

(c) Replace the circuits looking out of the base with a Thévenin equivalent circuit as shown in Fig. 4.

$$v_{tb} = v_s \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \qquad R_{tb} = R_1 \| R_2$$

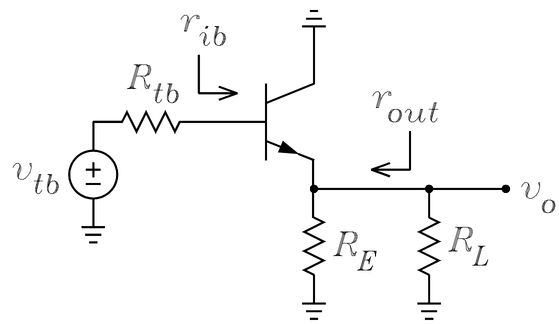

Figure 4: Signal circuit with Thévenin base circuit.

Exact Solution

(a) Replace the BJT in Fig. 4 with the Thévenin base emitter circuits as shown in Fig. 5. Solve for $v_{e(oc)}$.

$$v_{e(oc)} = v_{tb}\frac{r_0 + R_{tc}/\left(1+\beta\right)}{r_e' + r_0 + R_{tc}/\left(1+\beta\right)} \qquad R_{tc} = 0$$

Note that the Thévenin resistance R_{tc} looking out of the collector is zero in the original circuit. The exact solution gives the correct answer even if $R_{tc} \neq 0$.

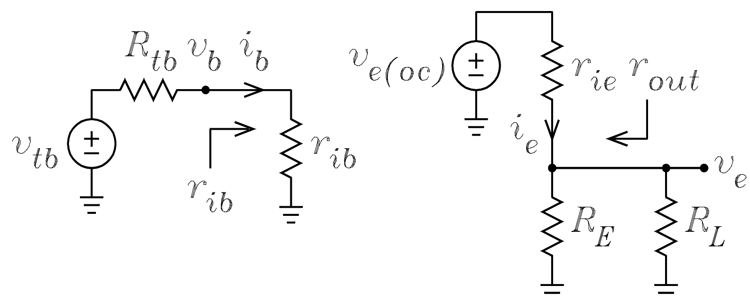

Figure 5: Base and emitter equivalent circuits.

(b) Solve for v_o.

$$v_o = v_{e(oc)}\frac{R_E \| R_L}{r_{ie} + R_E \| R_L} = v_s\frac{R_1\|R_2}{R_s + R_1\|R_2}\frac{r_0 + R_{tc}/\left(1+\beta\right)}{r_e' + r_0 + R_{tc}/\left(1+\beta\right)}\frac{R_E\|R_L}{r_{ie} + R_E\|R_L}$$

$$r_{ie} = r_e'\frac{r_0 + R_{tc}}{r_e' + r_0 + R_{tc}/\left(1+\beta\right)} \qquad r_e' = \frac{R_{tb} + r_x + r_\pi}{1+\beta} = \frac{R_{tb} + r_x}{1+\beta} + r_e$$

(c) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_s} = \frac{R_1\|R_2}{R_s + R_1\|R_2}\frac{r_0 + R_{tc}/\left(1+\beta\right)}{r_e' + r_0 + R_{tc}/\left(1+\beta\right)}\frac{R_E\|R_L}{r_{ie} + R_E\|R_L}$$

(d) Solve for r_{in}.

$$r_{in} = R_1\|R_2\|r_{ib} \qquad r_{ib} = r_x + r_\pi + R_{te}\frac{\left(1+\beta\right)r_0 + R_{tc}}{r_0 + R_{te} + R_{tc}} \qquad R_{te} = R_E\|R_L$$

(e) Solve for r_{out}.

$$r_{out} = r_{ie} \| R_E$$

(f) Special case for $r_0 = \infty$.

$$v_{e(oc)} = v_{tb} \qquad r_{ib} = r_x + r_\pi + (1+\beta) R_{te} \qquad r_{ie} = r'_e$$

Example 1 *For the CC amplifier in Fig. 1, it is given that* $R_S = 5\,\text{k}\Omega$, $R_1 = 120\,\text{k}\Omega$, $R_2 = 100\,\text{k}\Omega$, $R_E = 5.6\,\text{k}\Omega$, $R_L = 20\,\text{k}\Omega$, $V^+ = 15\,\text{V}$, $V^- = -15\,\text{V}$, $V_{BE} = 0.65\,\text{V}$, $\beta = 99$, $\alpha = 0.99$, $r_x = 20\,\Omega$, $V_A = 100\,\text{V}$ *and* $V_T = 0.025\,\text{V}$. *Solve for* A_v, r_{in}, *and* r_{out}.

Solution. Because the dc bias circuits are the same as for the common-emitter amplifier example, the bias values, r_e, g_m, and r_π are the same. Because V_{CE} is different, a new value of r_0 must be calculated. The collector-to-emitter voltage is given by

$$V_{CE} = V_C - V_E = V^+ - \left(V_{BB} - \frac{I_E}{1+\beta} R_{BB} - V_{BE} \right) = 17.01\,\text{V}$$

Thus r_0 has the value

$$r_0 = \frac{V_A + V_{CE}}{\alpha I_E} = 55.93\,\text{k}\Omega$$

In the signal circuit, the Thévenin voltage and resistance seen looking out of the base are given by

$$v_{tb} = \frac{R_1 \| R_2}{R_S + R_1 \| R_2} v_s = 0.916 v_s \qquad R_{tb} = R_S \| R_1 \| R_2 = 4.58\,\text{k}\Omega$$

The Thévenin resistances seen looking out of the emitter and the collector are

$$R_{te} = R_E \| R_3 = 4.375\,\text{k}\Omega \qquad R_{tc} = 0$$

Next, we calculate r'_e, $v_{e(oc)}$, r_{ie}, and r_{ib}, where $R_{tc} = 0$.

$$r'_e = \frac{R_{tb} + r_x}{1+\beta} + r_e = 57.83\,\Omega$$

$$v_{e(oc)} = \frac{r_0 + R_{tc}/(1+\beta)}{r'_e + r_0 + R_{tc}/(1+\beta)} v_{tb} = 0.999 v_{tb}$$

$$r_{ie} = r'_e \frac{r_0 + R_{tc}}{r'_e + r_0 + R_{tc}/(1+\beta)} = 57.77\,\Omega$$

$$r_{ib} = r_x + (1+\beta) r_e + R_{te} \frac{(1+\beta) r_0 + R_{tc}}{r_0 + R_{te} + R_{tc}} = 407\,\text{k}\Omega$$

The output voltage is given by

$$v_o = \frac{R_{te}}{r_{ie} + R_{te}} v_{e(oc)} = 0.987 \times 0.999 \times 0.916 v_s = 0.903 v_s$$

Thus the voltage gain is

$$A_v = 0.903$$

The input and output resistances are given by

$$r_{in} = R_1 \| R_2 \| r_{ib} = 48.1\,\text{k}\Omega \qquad r_{out} = r_{ie} \| R_E \| R_L = 57.02\,\Omega$$

Alternate Solutions

These solutions are exact because the Thévenin resistance R_{tc} looking out of the collector is zero. If $R_{tc} \neq 0$, replace r_0 with an open circuit in all formulas, i.e. let $r_0 = \infty$ In this case, the solutions are no longer exact, they are approximate.

Emitter Equivalent Circuit Solution

(a) After making the Thévenin equivalent circuits looking out of the base, replace the BJT with the emitter equivalent circuit as shown in Fig. 6.

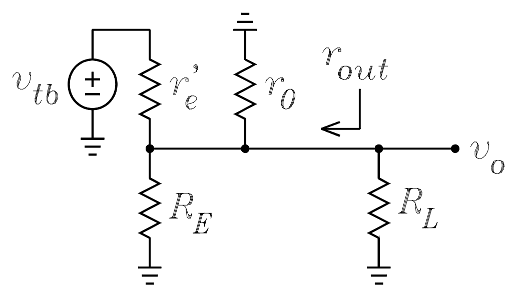

Figure 6: Simplified T model.

(b) Solve for r_e'.

$$r_e' = \frac{R_{tb} + r_x}{1+\beta} + r_e \qquad R_{tb} = R_S \| R_1 \| R_2$$

(c) Solve for v_o.

$$v_o = v_{tb} \frac{r_0 \| R_E \| R_L}{r_e' + r_0 \| R_E \| R_L} = v_s \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \frac{r_0 \| R_E \| R_L}{r_e' + r_0 \| R_E \| R_L}$$

(d) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_s} = \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \frac{r_0 \| R_E \| R_L}{r_e' + r_0 \| R_E \| R_L}$$

(e) Solve for r_{in}. Because the base node is absorbed, use the formula for r_{ib}.

$$r_{in} = R_1 \| R_2 \| r_{ib} \qquad r_{ib} = r_x + (1+\beta)(r_e + r_0 \| R_{te}) \qquad R_{te} = R_E \| R_L$$

(f) Solve for r_{out}.

$$r_{out} = r_0 \| r_e' \| R_E$$

Example 2 *Use the simplified T-model solutions to calculate the values of A_v, r_{in}, and r_{out} for Example 1.*

$$A_v = 0.916 \times 0.986 = 0.903$$

$$r_{in} = 48.51\,\mathrm{k}\Omega \qquad r_{out} = 57.18\,\Omega$$

π Model Solution

(a) After making the Thévenin equivalent circuits looking out of the base and emitter, replace the BJT with the π model as shown in Fig. 7.

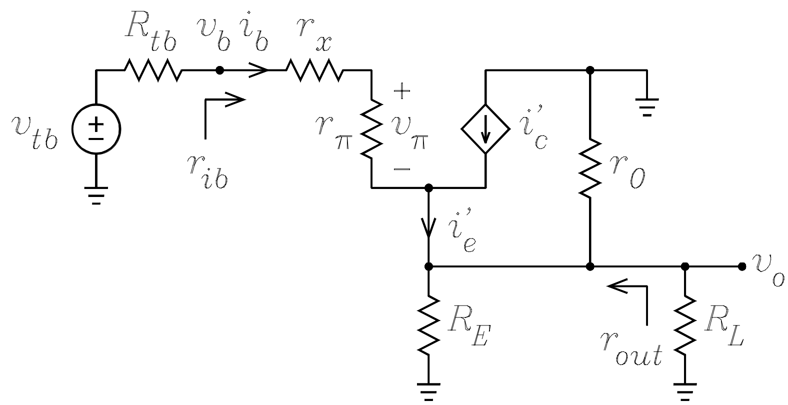

Figure 7: Hybrid-π model.

(b) Solve for i_e'.

$$v_{tb} = i_b\left(R_{tb} + r_x + r_\pi\right) + i_e' r_0 \| R_E \| R_L = \frac{i_e'}{1+\beta}\left(R_{tb} + r_x + r_\pi\right) + i_e' r_0 \| R_E \| R_L$$

$$\Longrightarrow \quad i_e' = \frac{v_{tb}}{\dfrac{R_{tb} + r_x + r_\pi}{1+\beta} + r_0 \| R_E \| R_L}$$

(c) Solve for v_o.

$$v_o = i_e' r_0 \| R_E \| R_L = \frac{v_{tb}}{\dfrac{R_{tb} + r_x + r_\pi}{1+\beta} + r_0 \| R_E \| R_L} r_0 \| R_E \| R_L$$

$$= v_s \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \frac{r_0 \| R_E \| R_L}{\dfrac{R_{tb} + r_x + r_\pi}{1+\beta} + r_0 \| R_E \| R_L}$$

(d) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_s} = \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \frac{r_0 \| R_E \| R_L}{\dfrac{R_{tb} + r_x + r_\pi}{1+\beta} + r_0 \| R_E \| R_L}$$

(e) Solve for r_{ib} and r_{in}.

$$v_b = i_b\left(r_x + r_\pi\right) + i_e' r_0 \| R_E \| R_L = i_b\left(r_x + r_\pi\right) + (1+\beta) i_b r_0 \| R_E \| R_L$$

$$= i_b\left[r_x + r_\pi + (1+\beta) r_0 \| R_E \| R_L\right]$$

$$r_{ib} = \frac{v_b}{i_b} = r_x + r_\pi + (1+\beta) r_0 \| R_E \| R_L$$

$$r_{in} = R_1 \| R_2 \| r_{ib}$$

(f) Solve for r_{out}.First, solve for the open-circuit output voltage. This is the output voltage with $R_L = \infty$.

$$v_{o(oc)} = v_{tb} \frac{r_0 \| R_E}{\dfrac{R_{tb} + r_x + r_\pi}{1+\beta} + r_0 \| R_E}$$

Next, solve for the short-circuit output current. This is the output current with $R_L = 0$. The output current is given by

$$i_o = \frac{v_o}{R_L} = \frac{v_{tb}}{\dfrac{R_{tb} + r_x + r_\pi}{1+\beta} + r_0 \| R_E \| R_L} \frac{r_0 \| R_E \| R_L}{R_L} = \frac{v_{tb}}{\dfrac{R_{tb} + r_x + r_\pi}{1+\beta} + r_0 \| R_E \| R_L} \frac{r_0 \| R_E}{R_L + r_0 \| R_E}$$

Now, let $R_L = 0$ to obtain

$$i_{o(sc)} = \frac{v_{tb}}{\dfrac{R_{tb} + r_x + r_\pi}{1+\beta}}$$

The output resistance is given by

$$r_{out} = \frac{v_{o(oc)}}{i_{o(sc)}} = \frac{r_0 \| R_E}{\dfrac{R_{tb} + r_x + r_\pi}{1+\beta} + r_0 \| R_E} \frac{R_{tb} + r_x + r_\pi}{1+\beta} = \left(\frac{R_{tb} + r_x + r_\pi}{1+\beta} \right) \| r_0 \| R_E$$

Note this is simply $r'_e \| r_0 \| R_E$, an answer that is obvious using the emitter equivalent circuit.

Example 3 *Use the π-model solutions to calculate the values of A_v, r_{in}, and r_{out} for Example 1.*

$$A_v = 0.916 \times 0.986 = 0.903$$
$$r_{in} = 48.51\,\mathrm{k}\Omega \qquad r_{out} = 57.18\,\Omega$$

T Model Solution

(a) After making the Thévenin equivalent circuits looking out of the base and emitter, replace the BJT with the T model as shown in Fig. 8.

(b) Solve for i'_e.

$$v_{tb} = i_b (R_{tb} + r_x) + i'_e (r_e + r_0 \| R_E \| R_L) = \frac{i'_e}{1+\beta} (R_{tb} + r_x) + i'_e (r_e + r_0 \| R_E \| R_L)$$

$$\Longrightarrow \quad i'_e = \frac{v_{tb}}{\dfrac{R_{tb} + r_x}{\beta} + r_e + r_0 \| R_E \| R_L}$$

(c) Solve for v_o.

$$v_o = i'_e r_0 \| R_E \| R_L = \frac{v_{tb}}{\dfrac{R_{tb} + r_x}{\beta} + r_e + r_0 \| R_E \| R_L} r_0 \| R_E \| R_L$$

$$= v_s \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \frac{r_0 \| R_E \| R_L}{\dfrac{R_{tb} + r_x}{\beta} + r_e + r_0 \| R_E \| R_L}$$

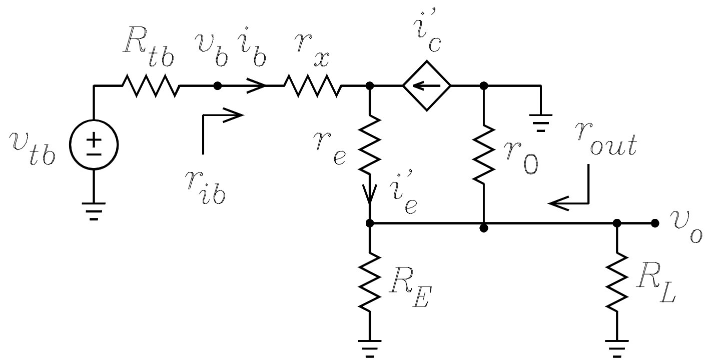

Figure 8: T model circuit.

(d) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_s} = \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \frac{r_0 \| R_E \| R_L}{\dfrac{R_{tb} + r_x}{\beta} + r_e + r_0 \| R_E \| R_L}$$

(e) Solve for r_{ib} and r_{in}.

$$\begin{aligned} v_b &= i_b r_x + i'_e (r_e + r_0 \| R_E \| R_L) = i_b r_x + (1+\beta)\, i_b\, (r_e + r_0 \| R_E \| R_L) \\ &= i_b \left[r_x + (1+\beta)(r_e + r_0 \| R_E \| R_L) \right] \end{aligned}$$

$$r_{ib} = \frac{v_b}{i_b} = r_x + (1+\beta)(r_e + r_0 \| R_E \| R_L)$$

$$r_{in} = R_1 \| R_2 \| r_{ib}$$

(f) Solve for r_{out}. First, solve for the open-circuit output voltage. This is the output voltage with $R_L = \infty$.

$$v_{o(oc)} = v_{tb} \frac{r_0 \| R_E}{\dfrac{R_{tb} + r_x}{\beta} + r_e + r_0 \| R_E}$$

Next, solve for the short-circuit output current. This is the output current with $R_L = 0$. The output current is given by

$$i_o = \frac{v_o}{R_L} = \frac{v_{tb}}{\dfrac{R_{tb} + r_x}{1+\beta} + r_e + r_0 \| R_E \| R_L} \frac{r_0 \| R_E \| R_L}{R_L} = \frac{v_{tb}}{\dfrac{R_{tb} + r_x}{1+\beta} + r_e r_0 \| R_E \| R_L} \frac{r_0 \| R_E}{R_L + r_0 \| R_E}$$

Now, let $R_L = 0$ to obtain

$$i_{o(sc)} = \frac{v_{tb}}{\dfrac{R_{tb} + r_x}{1+\beta} + r_e}$$

The output resistance is given by

$$r_{out} = \frac{v_{o(oc)}}{i_{o(sc)}} = \frac{r_0 \| R_E}{\dfrac{R_{tb} + r_x}{1+\beta} + r_e + r_0 \| R_E} \left(\frac{R_{tb} + r_x}{1+\beta} + r_e \right) = \left(\frac{R_{tb} + r_x}{1+\beta} + r_e \right) \| r_0 \| R_E$$

This is the same answer obtained from the emitter equivalent circuit.

Example 4 *Use the T-model solutions to calculate the values of A_v, r_{in}, and r_{out} for Example 1.*

$$A_v = 0.916 \times 0.986 = 0.903$$

$$r_{in} = 48.51\,\text{k}\Omega \qquad r_{out} = 57.18\,\Omega$$