Collection of Solved Feedback Amplifier Problems

This document contains a collection of solved feedback amplifier problems involving one or more active devices. The solutions make use of a graphical tool for solving simultaneous equations that is called the Mason Flow Graph (also called the Signal Flow Graph). When set up properly, the graph can be used to obtain by inspection the gain of a feedback amplifier, its input resistance, and its output resistance without solving simultaneous equations. Some background on how the equations are written and how the flow graph is used to solve them can be found at

http://users.ece.gatech.edu/~mleach/ece3050/notes/feedback/fdbkamps.pdf

The gain of a feedback amplifier is usually written in the form $A \div (1 + bA)$, where A is the gain with feedback removed and b is the feedback factor. In order for this equation to apply to the four types of feedback amplifiers, the input and output variables must be chosen correctly. For amplifiers that employ series summing at the input (alson called voltage summing), the input variable must be a voltage. In this case, the source is modeled as a Thévenin equivalent circuit. For amplifiers that employ shunt summing at the input (also called current summing), the input variable must be a current. In this case, the source is modeled as a Norton equivalent circuit. When the output sampling is in shunt with the load (also called voltage sampling), the output variable must be a voltage. When the output sampling is in series with the load (also called current sampling), the output variable must be a current. These conventions are followed in the following examples.

The quantity Ab is called the loop gain. For the feedback to be negative, the algebraic sign of Ab must be positive. If Ab is negative the feedback is positive and the amplifier is unstable. Thus if A is positive, b must also be positive. If A is negative, b must be negative. The quantity $(1 + Ab)$ is called the amount of feedback. It is often expressed in dB with the relation $20 \log (1 + Ab)$.

For series summing at the input, the expression for the input resistance is of the form $R_{IN} \times (1 + bA)$, where R_{IN} is the input resistance without feedback. For shunt summing at the input, the expression for the input resistance is of the form $R_{IN} \div (1 + bA)$. For shunt sampling at the output, the expression for the output resistance is of the form $R_O \div (1 + bA)$, where R_O is the output resistance without feedback. To calculate this in the examples, a test current source is added in shunt with the load. For series sampling at the output, the expression for the output resistance is of the form $R_O \times (1 + bA)$. To calculate this in the examples, a test voltage source is added in series with the load.

Most texts neglect the feedforward gain through the feedback network in calculating the forward gain A. When the flow graph is used for the analysis, this feedforward gain can easily be included in the analysis without complicating the solution. This is done in all of the examples here.

The dc bias sources in the examples are not shown. It is assumed that the solutions for the dc voltages and currents in the circuits are known. In addition, it is assumed that any dc coupling capacitors in the circuits are ac short circuits for the small-signal analysis.

Series-Shunt Example 1

Figure 1(a) shows the ac signal circuit of a series-shunt feedback amplifier. The input variable is v_1 and the output variable is v_2. The input signal is applied to the gate of M_1 and the feedback signal is applied to the source of M_1. Fig. 1(b) shows the circuit with feedback removed. A test current source i_t is added in shunt with the output to calculate the output resistance R_B. The feedback at the source of M_1 is modeled by a Thévenin equivalent circuit. The feedback factor or feedback ratio b is the coefficient of v_2 in this source, i.e. $b = R_1 / (R_1 + R_3)$. The circuit values are $g_m = 0.001\,\mathrm{S}$, $r_s = g_m^{-1} = 1\,\mathrm{k}\Omega$, $r_0 = \infty$, $R_1 = 1\,\mathrm{k}\Omega$, $R_2 = 10\,\mathrm{k}\Omega$, $R_3 = 9\,\mathrm{k}\Omega$, $R_4 = 1\,\mathrm{k}\Omega$, and $R_5 = 100\,\mathrm{k}\Omega$.

The following equations can be written for the circuit with feedback removed:

$$i_{d1} = G_{m1} v_a \qquad G_{m1} = \frac{1}{r_{s1} + R_1 \| R_3} \qquad v_a = v_1 - v_{ts1} \qquad v_{ts1} = \frac{R_1}{R_1 + R_3} v_2 \qquad i_{d2} = -g_m v_{tg2}$$

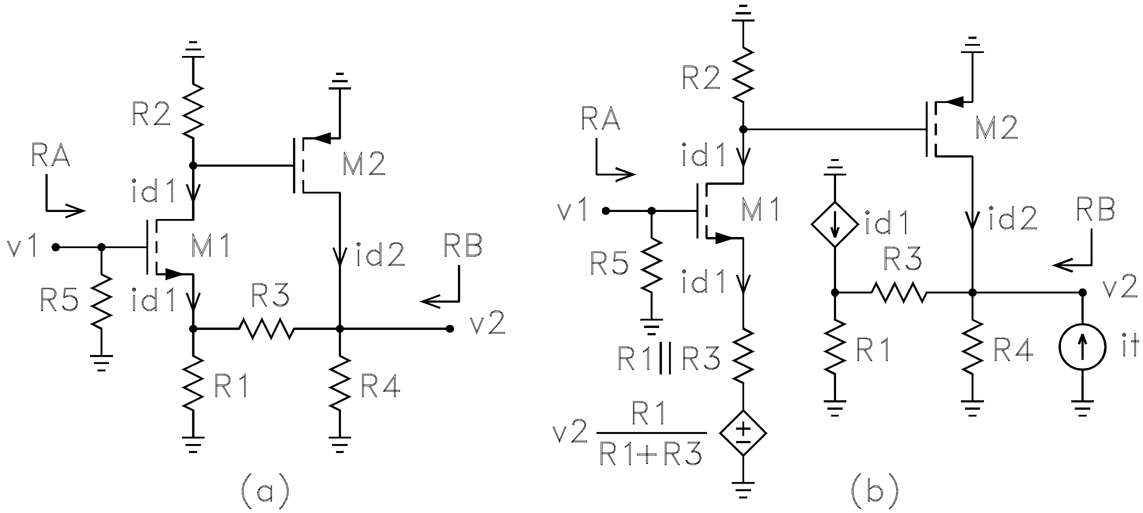

Figure 1: (a) Amplifier circuit. (b) Circuit with feedback removed.

$$v_{tg2} = -i_{d1} R_2 \qquad v_2 = i_{d2} R_C + i_t R_C + i_{d1} R_D \qquad R_C = R_4 \| (R_1 + R_3) \qquad R_D = \frac{R_1 R_4}{R_1 + R_3 + R_4}$$

The voltage v_a is the error voltage. The negative feedback tends to reduce v_a, making $|v_a| \to 0$ as the amount of feedback becomes infinite. When this is the case, setting $v_a = 0$ yields the voltage gain $v_2/v_1 = b^{-1} = 1 + R_3/R_1$. Although the equations can be solved algebraically, the signal-flow graph simplifies the solution.

Figure 2 shows the signal-flow graph for the equations. The determinant of the graph is given by

$$\Delta = 1 - G_{m1} \times [-R_2 \times (-g_{m2}) \times R_C + R_D] \times \frac{R_1}{R_1 + R_3} \times (-1)$$

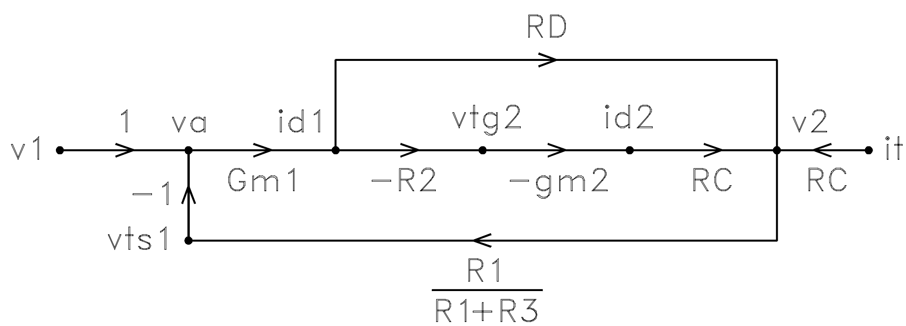

Figure 2: Signal-flow graph for the equations.

The voltage gain v_2/v_1 is calculated with $i_t = 0$. It is given by

$$\begin{aligned} \frac{v_2}{v_1} &= \frac{G_{m1} \times [-R_2 \times (-g_{m2}) \times R_C + R_D]}{\Delta} \\ &= \frac{\dfrac{1}{r_{s1} + R_1 \| R_3} \times (R_2 \times g_{m2} \times R_C + R_D)}{1 + \dfrac{1}{r_{s1} + R_1 \| R_3} \times (R_2 \times g_{m2} \times R_C + R_D) \times \dfrac{R_1}{R_1 + R_3}} \end{aligned}$$

This is of the form

$$\frac{v_2}{v_1} = \frac{A}{1 + Ab}$$

where

$$A = \frac{1}{r_{s1} + R_1 \| R_3} \times (R_2 \times g_{m2} \times R_C + R_D) = 4.83$$

$$b = \frac{R_1}{R_1 + R_3} = 0.1$$

Note that Ab is dimensionless. Numerical evaluation yields

$$\frac{v_2}{v_1} = \frac{4.83}{1 + 0.483} = 3.26$$

The output resistance R_B is calculated with $v_1 = 0$. It is given by

$$R_B = \frac{v_2}{i_t} = \frac{R_C}{\Delta} = \frac{R_C}{1 + Ab} = 613\,\Omega$$

Note that the feedback tends to decrease R_B. Because the gate current of M_1 is zero, the input resistance is $R_A = R_5 = 100\,\text{k}\Omega$.

Series-Shunt Example 2

A series-shunt feedback BJT amplifier is shown in Fig. 3(a). A test current source is added to the output to solve for the output resistance. Solve for the voltage gain v_2/v_1, the input resistance R_A, and the output resistance R_B. Assume $\beta = 100$, $r_\pi = 10\,\text{k}\Omega$, $\alpha = \beta/(1+\beta)$, $g_m = \beta/r_\pi$, $r_e = \alpha/g_m$, $r_0 = \infty$, $r_x = 0$, $R_1 = 1\,\text{k}\Omega$, $R_2 = 1\,\text{k}\Omega$, $R_3 = 2\,\text{k}\Omega$, $R_4 = 4\,\text{k}\Omega$, and $R_5 = 10\,\text{k}\Omega$. The circuit with feedback removed is shown in Fig. 3(b). The circuit seen looking out of the emitter of Q_1 is replaced with a Thévenin equivalent circuit made with respect to v_2. A test current source i_t is added to the output to solve for the output resistance.

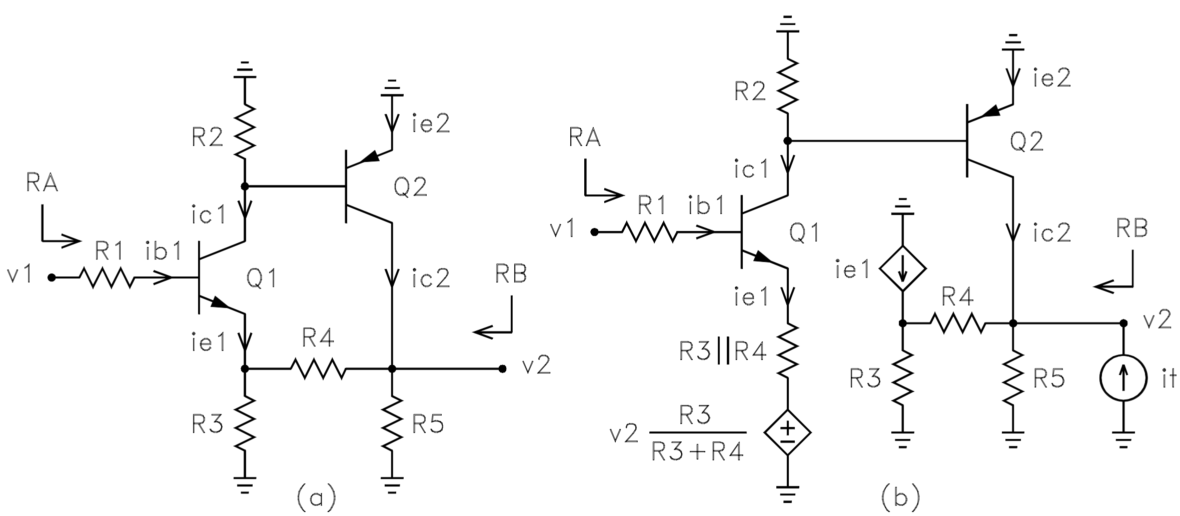

Figure 3: (a) Amplifier circuit. (b) Circuit with feedback removed.

For the circuit with feedback removed, we can write

$$i_{e1} = G_1 v_a \qquad v_a = v_1 - v_2 \frac{R_3}{R_3 + R_4} \qquad G_1 = \frac{1}{r'_{e1} + R_3 \| R_4} \qquad r'_{e1} = \frac{R_1}{1+\beta} + r_e \qquad i_{c1} = \alpha i_{e1}$$

$$i_{b1} = \frac{i_{c1}}{\beta} \qquad v_{tb2} = -i_{c1} R_2 \qquad i_{e2} = -G_2 v_{tb2} \qquad G_2 = \frac{1}{r'_{e2}} \qquad r'_{e2} = \frac{R_2}{1+\beta} + r_e$$

$$i_{c2} = \alpha i_{e2} \qquad v_2 = i_{c2} R_a + i_t R_a + i_{e1} R_b \qquad R_a = R_5 \| (R_3 + R_4) \qquad R_b = \frac{R_3 R_5}{R_3 + R_4 + R_5}$$

The equations can be solved algebraically or by a flow graph. The flow graph for the equations is shown in Fig. 4. The determinant is

$$\begin{aligned}
\Delta &= 1 - G_1 \times [\alpha \times -R_2 \times -G_2 \times \alpha \times R_a + R_b] \times \frac{-R_3}{R_3 + R_4} \\
&= 1 + G_1 \times [\alpha \times R_2 \times G_2 \times \alpha \times R_a + R_b] \times \frac{R_3}{R_3 + R_4} \\
&= 9.09
\end{aligned}$$

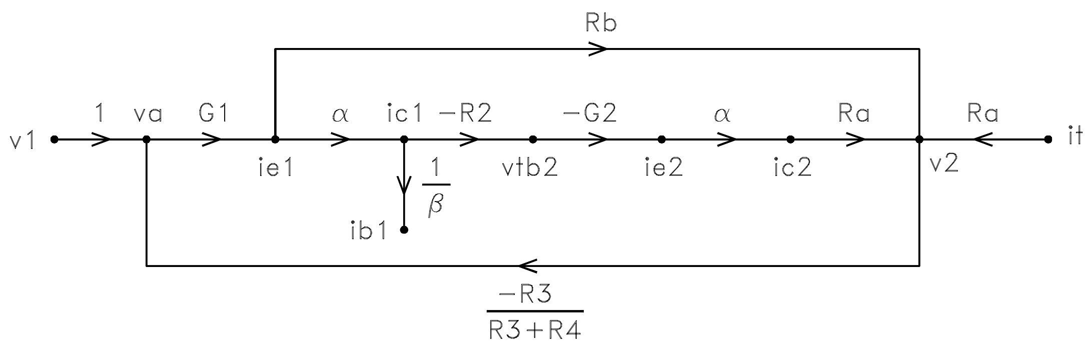

Figure 4: Signal-flow graph for the equations.

The voltage gain is

$$\begin{aligned}
\frac{v_2}{v_1} &= \frac{G_1 \times [\alpha \times -R_2 \times -G_2 \times \alpha \times R_a + R_b]}{\Delta} \\
&= \frac{G_1 \times [\alpha \times R_2 \times G_2 \times \alpha \times R_a + R_b]}{1 + G_1 \times [\alpha \times R_2 \times G_2 \times \alpha \times R_a + R_b] \times \dfrac{R_3}{R_3 + R_4}}
\end{aligned}$$

This is of the form

$$\frac{v_2}{i_1} = \frac{A}{1 + Ab}$$

where

$$\begin{aligned}
A &= G_1 \times [\alpha \times R_2 \times G_2 \times \alpha \times R_a + R_b] \\
&= \frac{1}{r'_{e1} + R_3 \| R_4} \left[\alpha \times R_2 \times \frac{1}{r'_{e2}} \times \alpha \times R_5 \| (R_3 + R_4) + \frac{R_3 R_5}{R_3 + R_4 + R_5} \right] \\
&= 24.27
\end{aligned}$$

$$b = \frac{R_3}{R_3 + R_4} = 0.333$$

Notice that the product Ab is positive. This must be true for the feedback to be negative.

Numerical evaluation of the voltage gain yields

$$\frac{v_2}{v_1} = \frac{A}{\Delta} = 2.67$$

The resistances R_A and R_B are given by

$$R_A = \left(\frac{i_{b1}}{v_1} \right)^{-1} = \left(\frac{G_1 \alpha / \beta}{\Delta} \right)^{-1} = \Delta \times \frac{\beta r'_{e1}}{\alpha} = \Delta \times \frac{\beta}{\alpha} (r'_{e1} + R_3 \| R_4) = 1.32\,\mathrm{M\Omega}$$

$$R_B = \frac{v_2}{i_t} = \frac{R_a}{\Delta} = \frac{R_5 \| (R_3 + R_4)}{\Delta} = 412.5\,\Omega$$

Series-Shunt Example 3

A series-shunt feedback BJT amplifier is shown in Fig. 5(a). Solve for the voltage gain v_2/v_1, the input resistance R_A, and the output resistance R_B. For J_1, assume $g_{m1} = 0.003\,\text{S}$, and $r_{01} = \infty$. For Q_2, assume $\beta_2 = 100$, $r_{\pi 2} = 2.5\,\text{k}\Omega$, $\alpha_2 = \beta_2/(1+\beta_2)$, $g_{m2} = \beta_2/r_{\pi 2}$, $r_{e2} = \alpha_2/g_{m2}$, $r_{02} = \infty$, $r_{x2} = 0$. The circuit elements are $R_1 = 1\,\text{M}\Omega$. $R_2 = 10\,\text{k}\Omega$, $R_3 = 1\,\text{k}\Omega$, $R_4 = 20\,\text{k}\Omega$, and $R_5 = 10\,\text{k}\Omega$. The circuit with feedback removed is shown in Fig. 5(b). The circuit seen looking out of the source of J_1 is replaced with a Thévenin equivalent circuit made with respect to v_2. A test current source i_t is added in shunt with the output to solve for the output resistance.

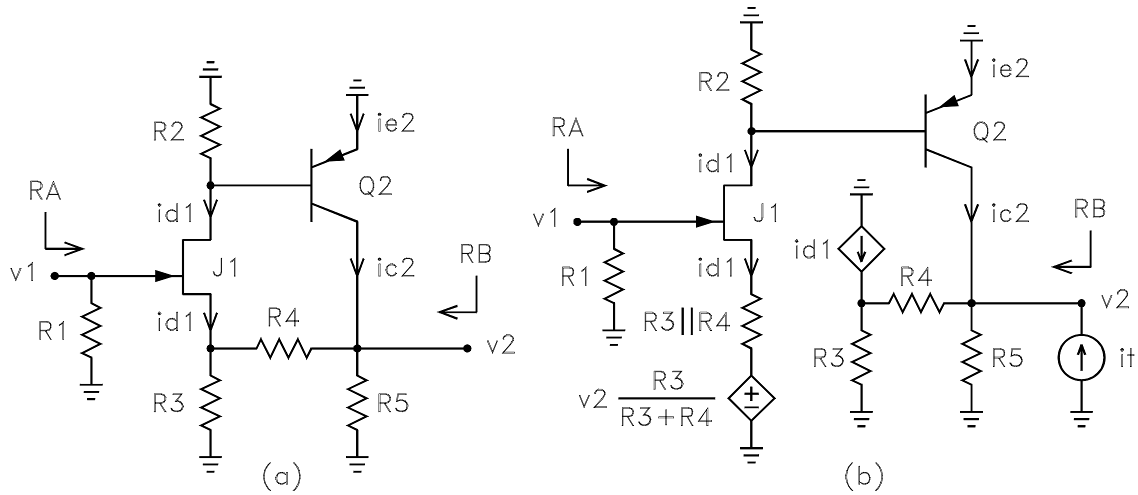

Figure 5: (a) Amplifier circuit. (b) Circuit with feedback removed.

For the circuit with feedback removed, we can write

$$v_a = v_1 - v_2 \frac{R_3}{R_3 + R_4} \qquad i_{d1} = G_1 v_a \qquad G_1 = \frac{1}{r_{s1} + R_3 \| R_4}$$

$$v_{tb2} = -i_{d1} R_2 \qquad i_{e2} = -G_2 v_{tb2} \qquad G_2 = \frac{1}{r'_{e2}} \qquad r'_{e2} = \frac{R_2}{1+\beta_2} + r_{e2}$$

$$i_{c2} = \alpha_2 i_{e2} \qquad v_2 = i_{c2} R_a + i_t R_a + i_{d1} R_b \qquad R_a = R_5 \| (R_3 + R_4) \qquad R_b = \frac{R_3 R_5}{R_3 + R_4 + R_5}$$

The equations can be solved algebraically or by a flow graph. The flow graph for the equations is shown in Fig. 6. The determinant is

$$\begin{aligned}
\Delta &= 1 - G_1 \times [-R_2 \times -G_2 \times \alpha \times R_a + R_b] \times \frac{-R_3}{R_3 + R_4} \\
&= 1 + G_1 \times [R_3 \times G_2 \times \alpha \times R_a + R_b] \times \frac{R_3}{R_3 + R_4} \\
&= 21.08
\end{aligned}$$

The voltage gain is

$$\begin{aligned}
\frac{v_2}{v_1} &= \frac{G_1 \times [-R_2 \times -G_2 \times \alpha_2 \times R_a + R_b]}{\Delta} \\
&= \frac{G_1 \times [R_2 \times G_2 \times \alpha_2 \times R_a + R_b]}{1 + G_1 \times [R_3 \times G_2 \times \alpha \times R_a + R_b] \times \dfrac{R_3}{R_3 + R_4}}
\end{aligned}$$

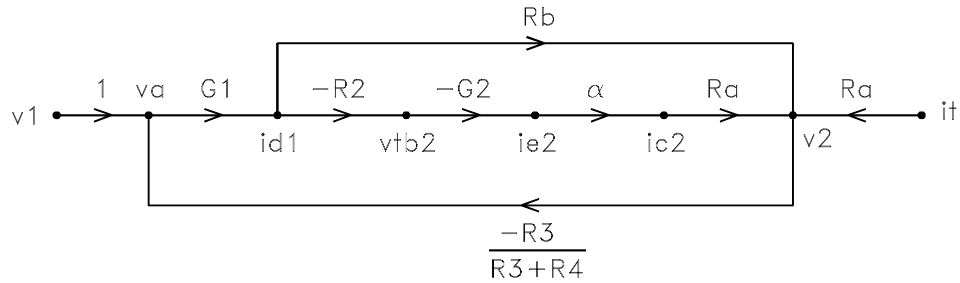

Figure 6: Signal-flow graph for the equations.

This is of the form

$$\frac{v_2}{i_1} = \frac{A}{\Delta} = \frac{A}{1 + Ab}$$

where

$$\begin{aligned} A &= G_1 \times [R_2 \times G_2 \times \alpha_2 \times R_a + R_b] \\ &= \frac{1}{r_{s1} + R_3 \| R_4} \left[R_2 \times \frac{1}{r'_{e1}} \times \alpha_1 \times R_5 \| (R_3 + R_4) + \frac{R_3 R_4}{R_3 + R_4 + R_5} \right] \\ &= 421.8 \end{aligned}$$

$$b = \frac{R_3}{R_3 + R_4} = 0.0467$$

Notice that the product Ab is positive. This must be true for the feedback to be negative.

Numerical evaluation of the voltage gain yields

$$\frac{v_2}{v_1} = \frac{421.8}{1 + 421.8 \times 0.0476} = 20$$

The resistances R_A and R_B are given by

$$R_A = R_1 = 1\,\mathrm{M}\Omega$$

$$R_B = \frac{v_2}{i_t} = \frac{R_a}{\Delta} = \frac{R_5 \| (R_3 + R_4)}{\Delta} = 321.3\,\Omega$$

Series-Shunt Example 4

A series-shunt feedback BJT amplifier is shown in Fig. 7(a). A test current source is added to the output to solve for the output resistance. Solve for the voltage gain v_2/v_1, the input resistance R_A, and the output resistance R_B. Assume $\beta = 50$, $r_\pi = 2.5\,\mathrm{k}\Omega$, $\alpha = \beta/(1+\beta)$, $g_m = \beta/r_\pi$, $r_e = \alpha/g_m$, $r_0 = \infty$, $r_x = 0$, $R_1 = 1\,\mathrm{k}\Omega$, $R_2 = 100\,\Omega$, $R_3 = 9.9\,\mathrm{k}\Omega$, $R_4 = 10\,\mathrm{k}\Omega$, and $R_5 = 10\,\mathrm{k}\Omega$. The circuit with feedback removed is shown in Fig. 8. The circuit seen looking out of the base of Q_2 is a Thévenin equivalent circuit made with respect to the voltage v_2. A test current source i_t is added in shunt with the output to solve for the output resistance.

The emitter eQuivalent circuit for calculating i_{e1} and i_{e2} is shown in Fig. 7(b). For this circuit and the circuit with feedback removed, we can write

$$i_{e1} = G_1 v_e \qquad v_e = v_1 - v_2 \frac{R_2}{R_2 + R_3} \qquad G_1 = \frac{1}{r_{e1} + r'_{e2}} \qquad r'_{e2} = \frac{R_2 \| R_3}{1 + \beta} + r_e \qquad i_{c1} = \alpha i_{e1}$$

$$i_{b1} = \frac{i_{c1}}{\beta} \qquad v_{tb3} = -i_{c1} R_1 \qquad i_{e3} = -G_2 v_{tb3} \qquad G_2 = \frac{1}{r'_{e3}} \qquad r'_{e3} = \frac{R_1}{1 + \beta} + r_e$$

$$i_{c3} = \alpha i_{e3} \qquad v_2 = i_{c3} R_a + i_t R_a - i_{b2} R_b \qquad R_a = R_4 \| (R_2 + R_3) \qquad R_b = \frac{R_2 R_4}{R_2 + R_3 + R_4}$$

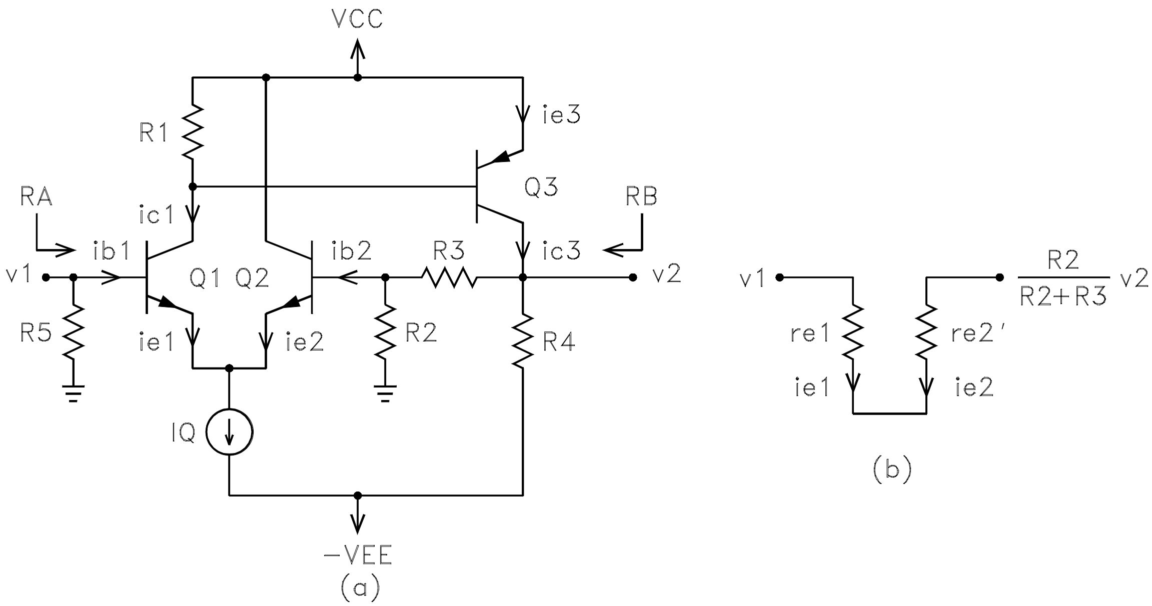

Figure 7: (a) Amplifier circuit. (b) Emitter equivalent circuit for calculating i_{e1} and i_{e2}.

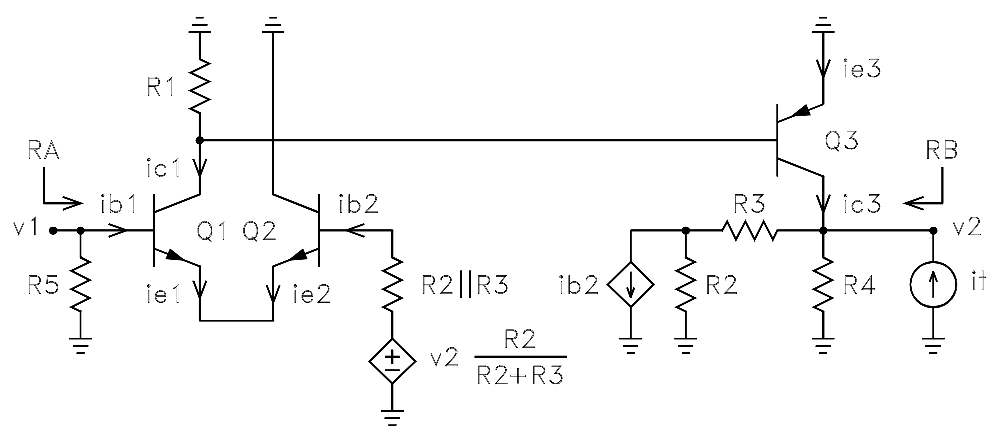

Figure 8: Circuit with feedback removed.

The equations can be solved algebraically or by a flow graph. The flow graph for the equations is shown in Fig. 9. The determinant is

$$\begin{aligned}\Delta &= 1 - G_1 \times \left(\alpha \times -R_1 \times -G_2 \times \alpha \times R_a - \frac{R_b}{1+\beta}\right) \times \frac{-R_2}{R_2+R_3} \\ &= 1 + G_1 \times \left(\alpha \times R_1 \times G_2 \times \alpha \times R_a - \frac{R_b}{1+\beta}\right) \times \frac{R_2}{R_2+R_3} \\ &= 8.004\end{aligned}$$

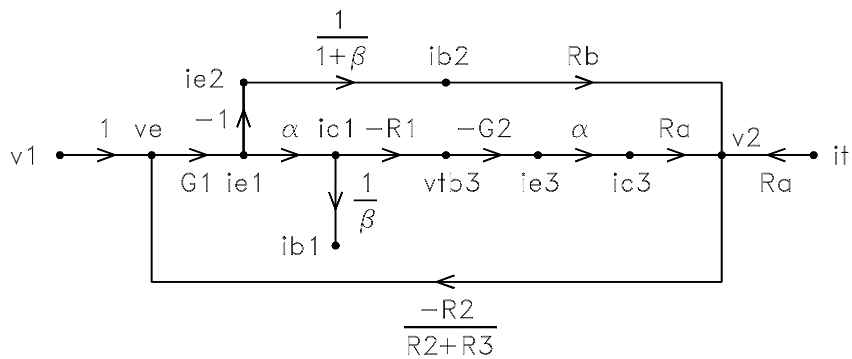

Figure 9: Flow graph for the equations.

The voltage gain is

$$\begin{aligned}\frac{v_2}{v_1} &= \frac{G_1 \times \left(\alpha \times -R_1 \times -G_2 \times \alpha \times R_a - \frac{R_b}{1+\beta}\right)}{\Delta} \\ &= \frac{G_1 \times \left(\alpha \times R_1 \times G_2 \times \alpha \times R_a - \frac{R_b}{1+\beta}\right)}{1 + G_1 \times \left(\alpha \times R_1 \times G_2 \times \alpha \times R_a - \frac{R_b}{1+\beta}\right) \times \frac{R_2}{R_2+R_3}}\end{aligned}$$

This is of the form

$$\frac{v_2}{i_1} = \frac{A}{1+Ab}$$

where

$$\begin{aligned}A &= G_1 \times \left(\alpha \times R_1 \times G_2 \times \alpha \times R_a - \frac{R_b}{1+\beta}\right) \\ &\quad \frac{1}{r_{e1}+r'_{e2}}\left[\alpha \times R_1 \times \frac{1}{r'_{e3}} \times \alpha \times R_4 \| (R_2+R_3) - \frac{1}{1+\beta}\frac{R_2 R_4}{R_2+R_3+R_4}\right] \\ &= 700.4\end{aligned}$$

$$b = \frac{R_2}{R_2+R_3} = 0.01$$

Notice that the product Ab is positive. This must be true for the feedback to be negative.

Numerical evaluation of the voltage gain yields

$$\frac{v_2}{v_1} = \frac{A}{\Delta} = 87.51$$

The resistances R_A and R_B are given by

$$R_A = R_5 \| \left(\frac{i_{b1}}{v_1}\right)^{-1} = R_5 \| \left(\frac{G_1\alpha/\beta}{\Delta}\right)^{-1} = R_5 \| \left[\Delta \times (1+\beta) \times (r_{e1}+r'_{e2})\right] = 8.032\,\mathrm{k}\Omega$$

$$R_B = \frac{v_2}{i_t} = \frac{R_a}{\Delta} = \frac{R_4 \| (R_2+R_3)}{\Delta} = 624.7\,\Omega$$

Shunt-Shunt Example 1

Figure 10(a) shows the ac signal circuit of a shunt-series feedback amplifier. The input variable is v_1 and the output variable is v_2. The input signal and the feedback signal are applied to the base of Q_1. A test current source i_t is added in shunt with the output to calculate the output resistance R_B. For the analysis to follow convention, the input source consisting of v_1 in series with R_1 must be converted into a Norton equivalent. This circuit is the current $i_1 = v_1/R_1$ in parallel with the resistor R_1. Fig. 10(b) shows the circuit with feedback removed and the source replaced with the Norton equivalent. The feedback at the base of Q_1 is modeled by a Norton equivalent circuit v_2/R_4 in parallel with the resistor R_4. The feedback factor or feedback ratio b is the negative of the coefficient of v_2 in this source, i.e. $b = -R_4^{-1}$. The circuit values are $\beta_1 = 100$, $g_{m1} = 0.05\,\mathrm{S}$, $r_{x1} = 0$, $r_{ib1} = \beta_1/g_{m1} = 2\,\mathrm{k\Omega}$, $r_{01} = \infty$, $g_{m2} = 0.001\,\mathrm{S}$, $r_{s2} = g_{m2}^{-1} = 1\,\mathrm{k\Omega}$, $r_{02} = \infty$, $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 1\,\mathrm{k\Omega}$, $R_3 = 10\,\mathrm{k\Omega}$, and $R_4 = 10\,\mathrm{k\Omega}$.

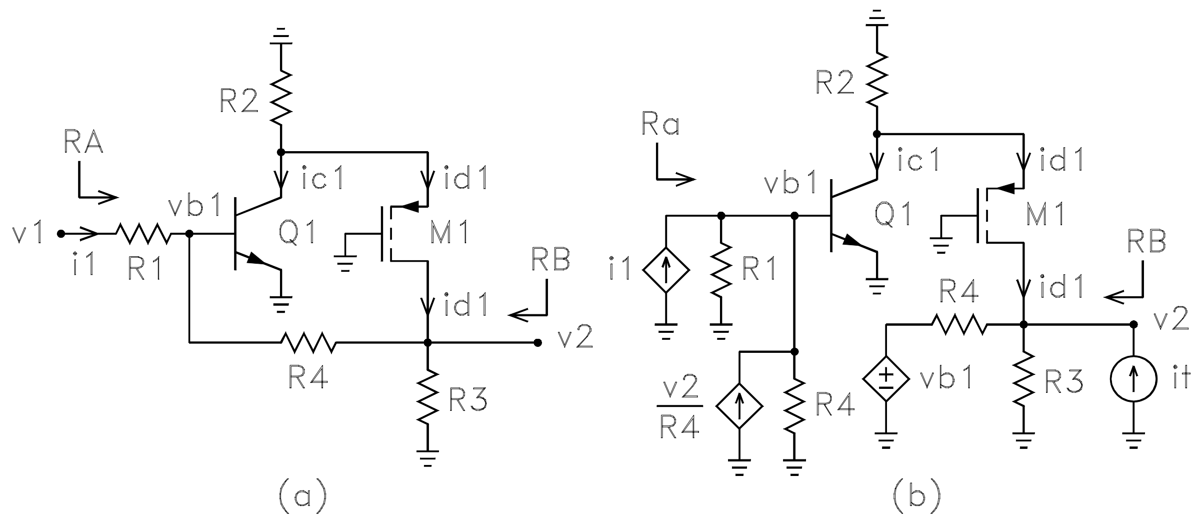

Figure 10: (a) Amplifier circuit. (b) Circuit with feedback removed.

The following equations can be written for the circuit with feedback removed:

$$v_{b1} = i_a R_b \qquad i_a = i_1 + \frac{v_2}{R_4} \qquad R_b = R_1 \| R_4 \| r_{ib1} \qquad i_{c1} = g_{m1} v_{b1}$$

$$i_{d1} = -i_{c1}\frac{R_2}{r_{s1} + R_2} \qquad v_2 = i_{d1} R_c + i_t R_c + v_{b1}\frac{R_3}{R_3 + R_4} \qquad R_c = R_3 \| R_4$$

The current i_a is the error current. The negative feedback tends to reduce i_a, making $|i_a| \to 0$ as the amount of feedback becomes infinite. When this is the case, setting $i_a = 0$ yields the current gain $v_2/i_1 = -R_4$.

Although the equations can be solved algebraically, the signal-flow graph simplifies the solution. Figure 11 shows the flow graph for the equations. The determinant of the graph is given by

$$\Delta = 1 - R_b \times \left[g_{m1} \times \frac{-R_2}{r_{s1} + R_2} \times R_c + \frac{R_3}{R_3 + R_4} \right] \times \frac{1}{R_4}$$

The transresistance gain is calculated with $i_t = 0$. It is given by

$$\frac{v_2}{i_1} = \frac{R_b \times \left(g_{m1} \times \dfrac{-R_2}{r_{s2} + R_2} \times R_c + \dfrac{R_3}{R_3 + R_4} \right)}{\Delta}$$

$$= -\frac{(R_1 \| R_4 \| r_{ib1}) \times \left[g_{m1} \times \dfrac{-R_2}{r_{s2} + R_2} \times (R_3 \| R_4) + \dfrac{R_3}{R_3 + R_4} \right]}{1 + \left[(R_1 \| R_4 \| r_{ib1}) \times g_{m1} \times \dfrac{-R_2}{r_{s2} + R_2} \times (R_3 \| R_4) + \dfrac{R_3}{R_3 + R_4} \right] \times \dfrac{-1}{R_4}}$$

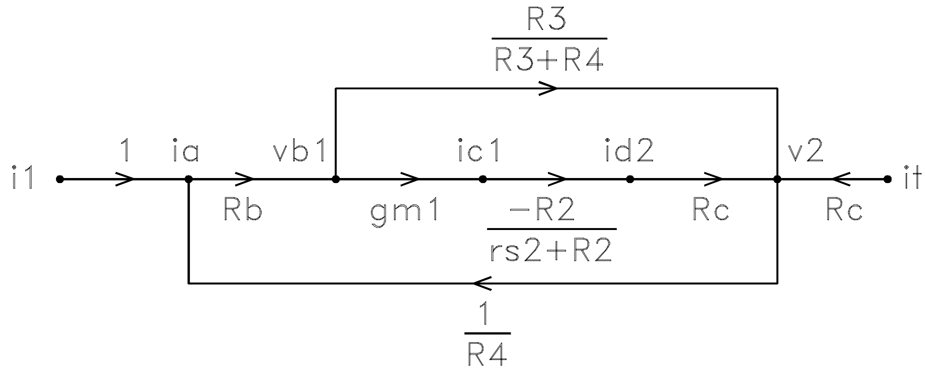

Figure 11: Signal-flow graph for the equations.

This is of the form

$$\frac{v_2}{i_1} = \frac{A}{1 + Ab}$$

where

$$A = (R_1 \| R_4 \| r_{ib1}) \times \left[g_{m1} \times \frac{-R_2}{r_{s2} + R_2} \times (R_3 \| R_4) + \frac{R_3}{R_3 + R_4} \right] = -77.81\,\mathrm{k}\Omega$$

$$b = -\frac{1}{R_4} = -10^{-4}\,\mathrm{S}$$

Note that Ab is dimensionless and positive. Numerical evaluation yields

$$\frac{v_2}{i_1} = \frac{-77.81 \times 10^3}{1 + (-77.81 \times 10^3) \times (-10^{-4})} = -8.861\,\mathrm{k}\Omega$$

The voltage gain is given by

$$\frac{v_2}{v_1} = \frac{v_2}{i_1} \times \frac{i_1}{v_1} = \frac{v_2}{i_1} \times \frac{1}{R_1} = -8.861$$

The resistance R_a is calculated with $i_t = 0$. It is given by

$$R_a = \frac{v_{b1}}{i_1} = \frac{R_b}{\Delta} = \frac{R_1 \| R_4 \| r_{ib1}}{1 + Ab} = 71.17\,\Omega$$

Note that the feedback tends to decrease R_a. The resistance R_A is calculated as follows:

$$R_A = R_1 + \left(R_a^{-1} - R_1^{-1} \right)^{-1} = 1.077\,\mathrm{k}\Omega$$

The resistance R_B is calculated with $i_1 = 0$. It is given by

$$R_B = \frac{v_2}{i_t} = \frac{R_c}{\Delta} = \frac{R_3 \| R_4}{1 + Ab} = 569.4\,\Omega$$

Shunt-Shunt Example 2

A shunt-shunt feedback JFET amplifier is shown in Fig. 12(a). Solve for the voltage gain v_2/v_1, the input resistance R_A, and the output resistance R_B. Assume $g_m = 0.005\,\mathrm{S}$, $r_s = g_m^{-1} = 200\,\Omega$, $r_0 = \infty$, $R_1 = 3\,\mathrm{k}\Omega$, $R_2 = 7\,\mathrm{k}\Omega$, $R_3 = 1\,\mathrm{k}\Omega$, $R_4 = 10\,\mathrm{k}\Omega$. The circuit with feedback removed is shown in Fig. 12(b) In this circuit, the source is replaced by a Norton equivalent circuit consisting of a current $i_1 = v_1/R_1$ in parallel with the resistor R_1. This is necessary for the feedback analysis to conform to convention for shunt-shunt feedback.. The circuit seen looking up into R_2 is replaced with a Norton equivalent circuit made with respect to v_2. A test current source i_t is added in shunt with the output to solve for the output resistance.

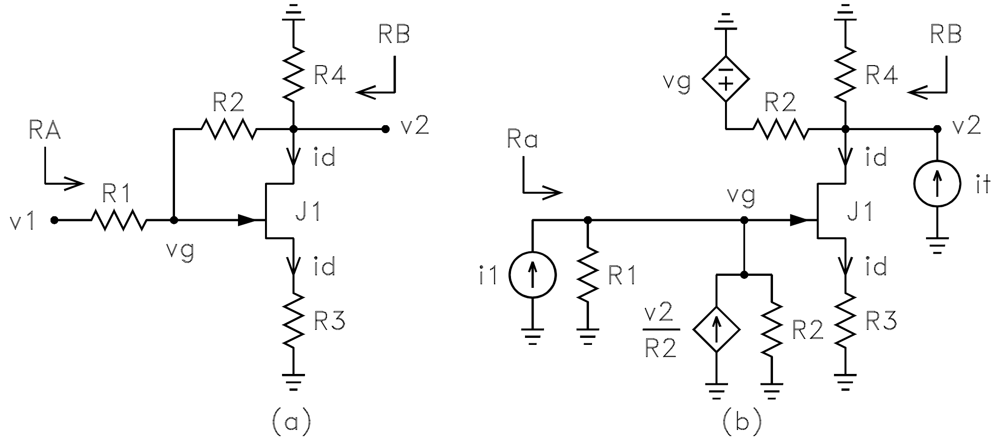

Figure 12: (a) Amplifier circuit. (b) Circuit with feedback removed.

For the circuit with feedback removed, we can write

$$v_g = i_1 R_b + \frac{v_2}{R_2} R_b \qquad R_b = R_1 \| R_2 \qquad i_d = G_m v_g \qquad G_m = \frac{1}{r_s + R_3}$$

$$v_2 = -i_d R_c + i_t R_c + v_g \frac{R_4}{R_2 + R_4} \qquad R_c = R_2 \| R_4$$

The equations can be solved algebraically or by a flow graph. The flow graph for the equations is shown in Fig. 13. The determinant is

$$\begin{aligned} \Delta &= 1 - R_b \times \left[G_m \times -R_c + \frac{R_4}{R_2 + R_4} \right] \times \frac{1}{R_2} \\ &= 1 + R_b \times \left[G_m \times R_c - \frac{R_4}{R_2 + R_4} \right] \times \frac{1}{R_2} \\ &= 1.853\,\Omega \end{aligned}$$

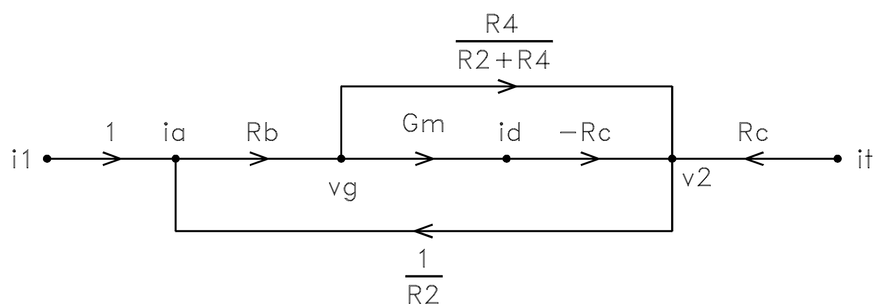

Figure 13: Signal-flow graph for the equations.

The transresistance gain is

$$\begin{aligned}\frac{v_2}{i_1} &= \frac{R_b \times \left[G_m \times -R_c + \dfrac{R_4}{R_2 + R_4}\right]}{\Delta} \\ &= \frac{-R_b \times \left[G_m \times R_c - \dfrac{R_4}{R_2 + R_4}\right]}{1 + R_b \times \left[G_m \times R_c + \dfrac{R_4}{R_2 + R_4}\right] \times \dfrac{1}{R_2}}\end{aligned}$$

This is of the form

$$\frac{v_2}{i_1} = \frac{A}{1 + Ab}$$

where

$$\begin{aligned}A &= -R_b \times \left[G_m \times R_c - \frac{R_4}{R_2 + R_4}\right] \\ &= -\left(R_1 \| R_2\right) \times \left[G_m \times R_2 \| R_4 - \frac{R_4}{R_2 + R_4}\right] \\ &= -5.971\,\mathrm{k}\Omega\end{aligned}$$

$$b = -\frac{1}{R_2} = -0.1429\,\mathrm{mS}$$

Note that the product Ab is dimensionless and positive. The latter must be true for the feedback to be negative. Numerical evaluation yields

$$\frac{v_2}{i_1} = \frac{A}{\Delta} = -3.22\,\mathrm{k}\Omega$$

The voltage gain is given by

$$\frac{v_2}{v_1} = \frac{v_2}{i_1} \times \frac{i_1}{v_1} = \frac{A}{\Delta} \times \frac{1}{R_1} = -1.074$$

The resistances R_a, R_A, and R_B are given by

$$R_a = \frac{v_g}{i_1} = \frac{R_b}{\Delta} = 1.13\,\mathrm{k}\Omega$$

$$R_A = R_1 + \left(\frac{1}{R_a} - \frac{1}{R_1}\right)^{-1} = 4.82\,\mathrm{k}\Omega$$

$$R_B = \frac{v_2}{i_t} = \frac{R_c}{\Delta} = 2.22\,\mathrm{k}\Omega$$

Shunt-Shunt Example 3

A shunt-shunt feedback BJT amplifier is shown in Fig. 14. The input variable is the v_1 and the output variable is the voltage v_2. The feedback resistor is R_2. The summing at the input is shunt because the input through R_1 and the feedback through R_2 connect in shunt to the same node, i.e. the v_{b1} node. The output sampling is shunt because R_2 connects to the output node. Solve for the voltage gain v_2/v_1, the input resistance R_A, and the output resistance R_B. Assume $\beta = 100$, $r_\pi = 2.5\,\mathrm{k}\Omega$, $g_m = \beta/r_\pi$, $\alpha = \beta/\left(1 + \beta\right)$, $r_e = \alpha/g_m$, $r_0 = \infty$, $r_x = 0$, $V_T = 25\,\mathrm{mV}$. The resistor values are $R_1 = 1\,\mathrm{k}\Omega$, $R_2 = 20\,\mathrm{k}\Omega$, $R_3 = 500\,\Omega$, $R_4 = 1\,\mathrm{k}\Omega$, and $R_5 = 5\,\mathrm{k}\Omega$.

The circuit with feedback removed is shown in Fig. 15. A test current source i_t is added in shunt with the output to solve for the output resistance R_B. In the circuit, the source is replaced by a Norton equivalent circuit consisting of a current

$$i_1 = \frac{v_1}{R_1}$$

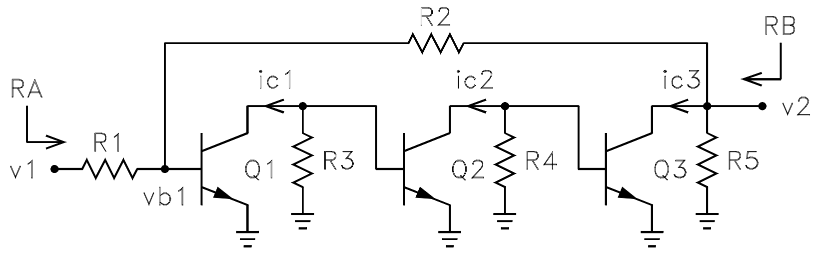

Figure 14: Amplifier circuit.

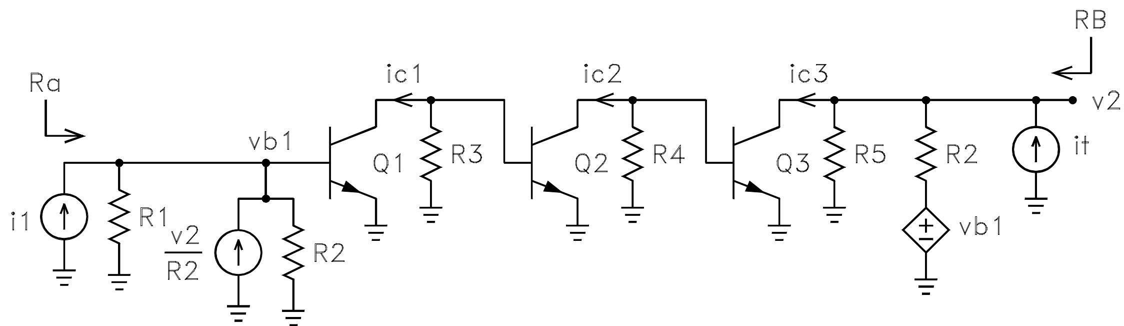

Figure 15: Circuit with feedback removed.

in parallel with the resistor R_1. This is necessary for the feedback analysis to conform to convention for shunt summing. The circuit seen looking into R_2 from the collector of Q_3 is replaced with a Thevenin equivalent circuit made with respect with v_{b1}.

For the circuit with feedback removed, we can write

$$i_e = i_1 + \frac{v_2}{R_2} \qquad v_{b1} = i_e R_b \qquad R_b = R_1 \| R_2 \| r_\pi \qquad i_{c1} = g_m v_{b1} \qquad v_{b2} = -i_{c1} R_c \qquad R_c = R_3 \| r_\pi$$

$$v_{b3} = -i_{c2} R_d \qquad R_d = R_4 \| r_\pi \qquad i_{c3} = g_m v_{b3} \qquad v_2 = (-i_{c3} + i_t) R_e + v_{b1} \frac{R_5}{R_2 + R_5} \qquad R_e = R_2 \| R_5$$

The equations can be solved algebraically or by a flow graph. The flow graph for the equations is shown in Fig. 16. The determinant is

$$\begin{aligned}
\Delta &= 1 - R_b \times \left(g_m \times -R_c \times g_m \times -R_d \times g_m \times -R_e + \frac{R_5}{R_2 + R_5} \right) \times \frac{1}{R_2} \\
&= 1 + R_b \times \left(g_m \times R_c \times g_m \times R_d \times g_m \times R_e - \frac{R_5}{R_2 + R_5} \right) \times \frac{1}{R_2} \\
&= 354.7
\end{aligned}$$

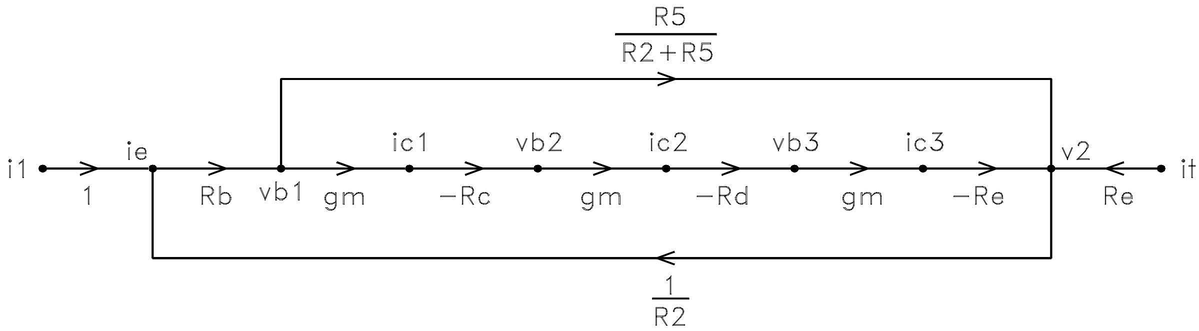

Figure 16: Signal-flow graph for the equations.

The transresistance gain is

$$\begin{aligned}
\frac{v_2}{i_1} &= \frac{R_b \times \left(g_m \times -R_c \times g_m \times -R_d \times g_m \times -R_e + \dfrac{R_2}{R_2 + R_5} \right)}{\Delta} \\
&= \frac{-R_b \times \left(g_m \times R_c \times g_m \times R_d \times g_m \times R_e - \dfrac{R_2}{R_2 + R_5} \right)}{1 + R_b \times \left(g_m \times R_c \times g_m \times R_d \times g_m \times R_e - \dfrac{R_2}{R_2 + R_5} \right) \times \dfrac{1}{R_2}} \\
&= \frac{-R_b \times \left(g_m \times R_c \times g_m \times R_d \times g_m \times R_e - \dfrac{R_2}{R_2 + R_5} \right)}{1 + \left[-R_b \times \left(g_m \times R_c \times g_m \times R_d \times g_m \times R_e - \dfrac{R_2}{R_2 + R_5} \right) \right] \times \dfrac{-1}{R_2}}
\end{aligned}$$

This is of the form

$$\frac{i_{e2}}{v_1} = \frac{A}{1 + Ab}$$

where A and b are given by

$$\begin{aligned}
A &= -R_b \times \left(g_m \times R_c \times g_m \times R_d \times g_m \times R_e - \frac{R_5}{R_2 + R_5} \right) \\
&= -R_1 \| R_2 \| r_\pi \times \left(g_m \times R_3 \| r_\pi \times g_m \times R_4 \| r_\pi \times g_m \times R_2 \| R_5 - \frac{R_5}{R_2 + R_5} \right) \\
&= -7.073\,\mathrm{M\Omega}
\end{aligned}$$

$$b = \frac{-1}{R_2} = -50\,\mu\mathrm{S}$$

Notice that the product Ab is dimensionless and positive. The latter must be true for the feedback to be negative.

Numerical evaluation of the transresistance gain yields

$$\frac{v_2}{i_1} = \frac{A}{\Delta} = -19.94\,\mathrm{k\Omega}$$

The resistances R_a and R_A are

$$R_a = \frac{v_{b1}}{i_1} = \frac{R_c}{\Delta} = 1.945\,\Omega$$

$$R_A = R_1 + \left(\frac{1}{R_a} - \frac{1}{R_1} \right)^{-1} = 1.002\,\mathrm{k\Omega}$$

The resistance R_B is

$$R_B = \frac{v_2}{i_t} = \frac{R_e}{\Delta} = 11.28\,\Omega$$

The voltage gain is

$$\frac{v_2}{v_{b1}} = \frac{v_2}{i_1} \times \left(\frac{v_{b1}}{i_1}\right)^{-1} = \left(\frac{A}{\Delta}\right) \times \frac{1}{R_A} = -19.91$$

Shunt-Shunt Example 4

A shunt-shunt feedback BJT amplifier is shown in Fig. 17. The input variable is the v_1 and the output variable is the voltage v_2. The feedback resistor is R_2. The summing at the input is shunt because the input through R_1 and the feedback through R_2 connect in shunt to the same node, i.e. the v_{e1} node. The output sampling is shunt because R_2 connects to the output node. Solve for the voltage gain v_2/v_1, the input resistance R_A, and the output resistance R_B. For Q_1 and Q_2, assume $\beta = 100$, $r_\pi = 2.5\,\mathrm{k}\Omega$, $g_m = \beta/r_\pi$, $\alpha = \beta/(1+\beta)$, $r_e = \alpha/g_m$, $r_0 = \infty$, $r_x = 0$, $V_T = 25\,\mathrm{mV}$. For J_3, assume $g_{m3} = 0.001\,\mathrm{S}$ and $r_{03} = \infty$. The resistor values are $R_1 = 1\,\mathrm{k}\Omega$, $R_2 = 100\,\mathrm{k}\Omega$, $R_3 = 10\,\Omega$, $R_4 = 30\,\mathrm{k}\Omega$, and $R_5 = 10\,\mathrm{k}\Omega$.

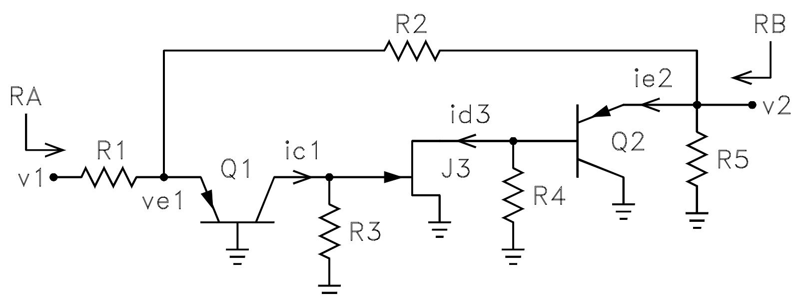

Figure 17: Amplifier circuit.

The circuit with feedback removed is shown in Fig. 18. A test current source i_t is added in shunt with the output to solve for the output resistance R_B. In the circuit, the source is replaced by a Norton equivalent circuit consisting of a current

$$i_1 = \frac{v_1}{R_1}$$

in parallel with the resistor R_1. This is necessary for the feedback analysis to conform to convention for shunt summing. The circuit seen looking into R_2 from the collector of Q_3 is replaced with a Thévenin equivalent circuit made with respect with v_{e1}.

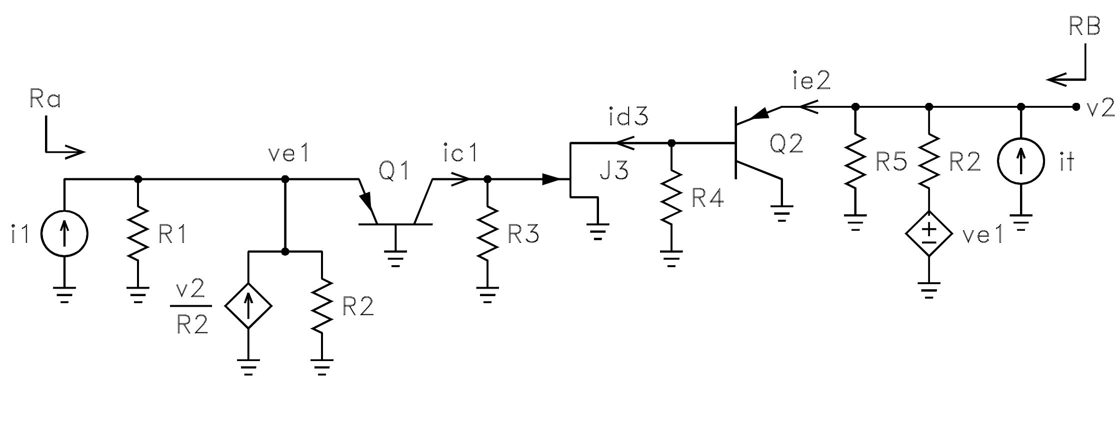

Figure 18: Circuit with feedback removed.

For the circuit with feedback removed, we can write

$$i_e = i_1 + \frac{v_2}{R_2} \qquad v_{e1} = i_e R_b \qquad R_b = R_1 \| R_2 \| r_{e1} \qquad i_{c1} = g_{m1} v_{e1} \qquad v_{tg3} = i_{c1} R_3$$

$$i_{d3} = g_{m3} v_{tg3} \qquad v_{tb2} = -i_{d3} R_4 \qquad i_{e2} = -G_1 v_{tb2} \qquad G_1 = \frac{1}{r'_{e2} + R_2 \| R_5} \qquad r'_{e2} = \frac{1}{R_4} + r_{e2}$$

$$v_2 = (-i_{e2} + i_t) R_c + v_{e1} \frac{R_5}{R_2 + R_5} \qquad R_c = R_2 \| R_5$$

The equations can be solved algebraically or by a flow graph. The flow graph for the equations is shown in Fig. 19. The determinant is

$$\begin{aligned} \Delta &= 1 - R_b \times \left(g_{m1} \times R_3 \times g_{m2} \times -R_4 \times -G_1 \times -R_c + \frac{R_5}{R_2 + R_5} \right) \times \frac{1}{R_2} \\ &= 1 + R_b \times \left(g_{m1} \times R_3 \times g_{m2} \times R_4 \times G_1 \times R_c - \frac{R_5}{R_2 + R_5} \right) \times \frac{1}{R_2} \\ &= 113.0 \end{aligned}$$

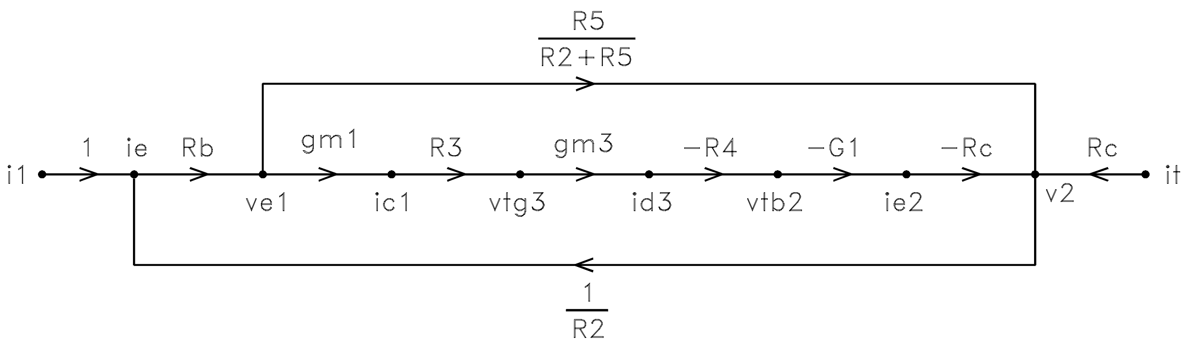

Figure 19: Signal-flow graph for the equations.

The transresistance gain is

$$\begin{aligned} \frac{v_2}{i_1} &= \frac{R_b \times \left(g_{m1} \times R_3 \times g_{m2} \times -R_4 \times -G_1 \times -R_c + \frac{R_2}{R_2 + R_5} \right)}{\Delta} \\ &= \frac{-R_b \times \left(g_{m1} \times R_3 \times g_{m2} \times R_4 \times G_1 \times R_c - \frac{R_2}{R_2 + R_5} \right)}{1 + R_b \times \left(g_{m1} \times R_3 \times g_{m2} \times R_4 \times G_1 \times R_c - \frac{R_2}{R_2 + R_5} \right) \times \frac{1}{R_2}} \\ &= \frac{-R_b \times \left(g_{m1} \times R_3 \times g_{m2} \times R_4 \times G_1 \times R_c - \frac{R_2}{R_2 + R_5} \right)}{1 + \left[-R_b \times \left(g_{m1} \times R_3 \times g_{m2} \times R_4 \times G_1 \times R_c - \frac{R_2}{R_2 + R_5} - \frac{R_2}{R_2 + R_5} \right) \right] \times \frac{-1}{R_2}} \end{aligned}$$

This is of the form

$$\frac{i_{e2}}{v_1} = \frac{A}{1 + Ab}$$

where A and b are given by

$$\begin{aligned} A &= -R_b \times \left(g_{m1} \times R_3 \times g_{m2} \times R_4 \times G_1 \times R_c - \frac{R_2}{R_2 + R_5} \right) \\ &= -R_1 \| R_2 \| r_{e1} \times \left(g_{m1} \times R_3 \times g_{m2} \times R_4 \times \frac{1}{r'_{e2} + R_2 \| R_5} \times R_2 \| R_5 - \frac{R_5}{R_2 + R_5} \right) \\ &= -11.2\,\mathrm{M}\Omega \end{aligned}$$

$$b = \frac{-1}{R_2} = -10\,\mu\text{S}$$

Notice that the product Ab is dimensionless and positive. The latter must be true for the feedback to be negative.

Numerical evaluation of the transresistance gain yields

$$\frac{v_2}{i_1} = \frac{A}{\Delta} = -99.11\,\text{k}\Omega$$

The resistances R_a and R_A are

$$R_a = \frac{v_{e1}}{i_1} = \frac{R_b}{\Delta} = 0.214\,\Omega$$

$$R_A = R_1 + \left(\frac{1}{R_a} - \frac{1}{R_1}\right)^{-1} = 1\,\text{k}\Omega$$

The resistance R_B is

$$R_B = \frac{v_2}{i_t} = \frac{R_c}{\Delta} = 80.49\,\Omega$$

The voltage gain is

$$\frac{v_2}{v_{e1}} = \frac{v_2}{i_1} \times \left(\frac{v_{e1}}{i_1}\right)^{-1} = \left(\frac{A}{\Delta}\right) \times \frac{1}{R_A} = -99.09$$

Series-Series Example 1

Figure 20(a) shows the ac signal circuit of a series-series feedback amplifier. The input variable is v_1 and the output variable is i_{d2}. The input signal is applied to the gate of M_1 and the feedback signal is applied to the source of M_1. Fig. 20(b) shows the circuit with feedback removed. A test voltage source v_t is added in series with the output to calculate the output resistance R_b. The feedback at the source of M_1 is modeled by a Thévenin equivalent circuit. The feedback factor or feedback ratio b is the coefficient of i_{d2} in this source, i.e. $b = R_5$. The circuit values are $g_m = 0.001\,\text{S}$, $r_s = g_m^{-1} = 1\,\text{k}\Omega$, $r_0 = \infty$, $R_1 = 50\,\text{k}\Omega$, $R_2 = 10\,\text{k}\Omega$, $R_3 = 1\,\text{k}\Omega$, $R_4 = 9\,\text{k}\Omega$, and $R_5 = 1\,\text{k}\Omega$.

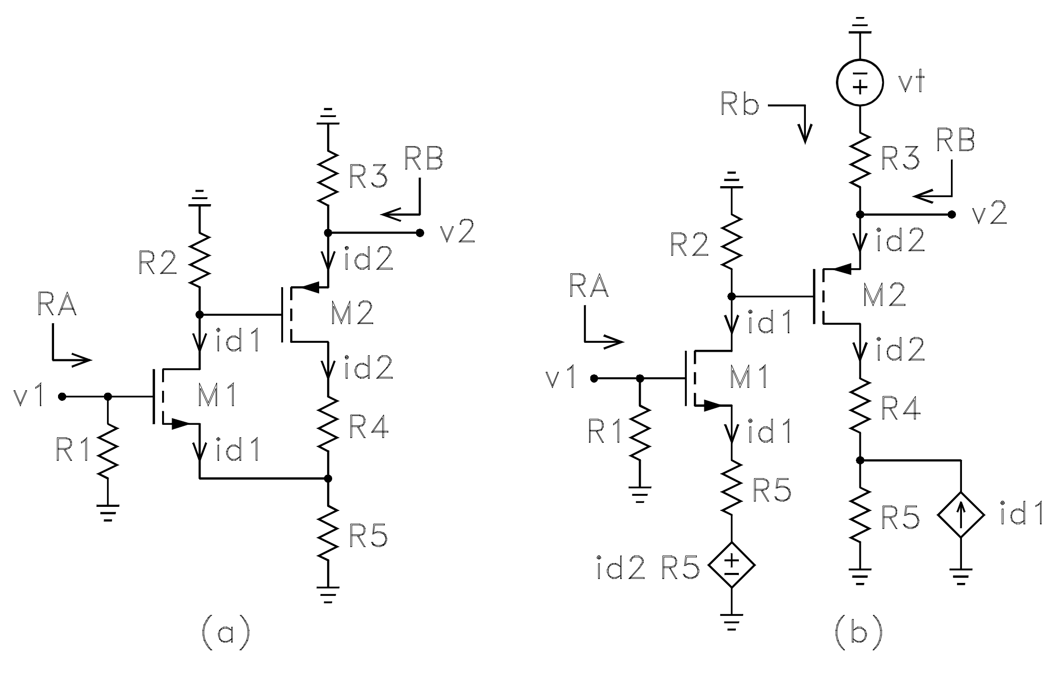

Figure 20: (a) Amplifier circuit. (b) Circuit with feedback removed.

The following equations can be written for the circuit with feedback removed:

$$i_{d1} = G_{m1} v_a \qquad G_{m1} = \frac{1}{r_{s1} + R_5} \qquad v_a = v_1 - v_{ts1} \qquad v_{ts1} = i_{d2} R_5$$

$$i_{d2} = G_{m2} v_b \qquad G_{m2} = \frac{1}{r_{s2} + R_3} \qquad v_b = v_t - v_{tg2} \qquad v_{tg2} = -i_{d1} R_2$$

The voltage v_a is the error voltage. The negative feedback tends to reduce v_a, making $|v_a| \to 0$ as the amount of feedback becomes infinite. When this is the case, setting $v_a = 0$ yields the transconductance gain $i_{d2}/v_1 = b^{-1} = R_5^{-1}$. Although the equations can be solved algebraically, the signal-flow graph simplifies the solution.

Figure 21 shows the signal-flow graph for the equations. The determinant of the graph is given by

$$\Delta = 1 - G_{m1} \times (-R_2) \times (-1) \times G_{m2} \times R_5 \times (-1)$$

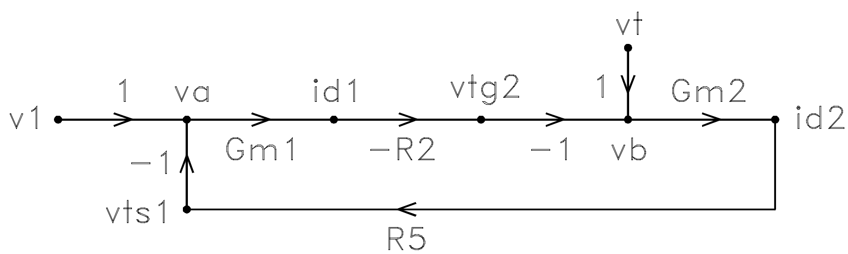

Figure 21: Flow graph for the equations.

The transconductance gain i_{d2}/v_1 is calculated with $v_t = 0$. It is given by

$$\begin{aligned} \frac{i_{d2}}{v_1} &= \frac{G_{m1} \times (-R_2) \times (-1) \times G_{m2}}{\Delta} \\ &= \frac{\dfrac{1}{r_{s1} + R_5} \times R_2 \times \dfrac{1}{r_{s1} + R_5}}{1 + \dfrac{1}{r_{s1} + R_5} \times R_2 \times \dfrac{1}{r_{s1} + R_5} \times R_5} \end{aligned}$$

This is of the form

$$\frac{i_{d2}}{v_1} = \frac{A}{1 + Ab}$$

where

$$A = G_{m1} \times (-R_2) \times (-1) \times G_{m2} = \frac{1}{r_{s1} + R_5} \times R_2 \times \frac{1}{r_{s2} + R_3} = 2.5 \times 10^{-3}\,\mathrm{S}$$

$$b = R_5 = 1000\,\Omega$$

Note that bA is dimensionless. Numerical evaluation yields

$$\frac{i_{d2}}{v_1} = \frac{2.5 \times 10^{-3}}{1 + 1000 \times 2.5 \times 10^{-3}} = 7.124 \times 10^{-4}\,\mathrm{S}$$

The resistance R_b is calculated with $v_1 = 0$. It is given by

$$R_b = \left(\frac{i_{d2}}{v_t}\right)^{-1} = \left(\frac{G_{m2}}{\Delta}\right)^{-1} = (1 + bA)(r_{s2} + R_3) = 7\,\mathrm{k}\Omega$$

Note that the feedback tends to increase R_b. The resistance R_B is calculated as follows:

$$R_B = (R_b - R_3) \| R_3 = 857.1\,\Omega$$

Because the gate current of M_1 is zero, the input resistance is $R_A = R_1 = 50\,\mathrm{k}\Omega$.

Series-Series Example 2

A series-series feedback BJT amplifier is shown in Fig. 22. The input variable is the voltage v_1 and the output variable is the voltage v_2. The feedback is from i_{e2} to the emitter of Q_1. Because the feedback does not connect to the input node, the input summing is series. The output sampling is series because the feedback is proportional to the current that flows in series with the output rather than the output voltage. Solve for the transconductance gain i_{c3}/v_1, the voltage gain v_2/v_1, the input resistance R_A, and the output resistance R_B. Assume $\beta = 100$, $I_{C1} = 0.6\,\text{mA}$, $I_{C2} = 1\,\text{mA}$, $I_{C3} = 4\,\text{mA}$, $\alpha = \beta/(1+\beta)$, $g_m = I_C/V_T$, $r_e = \alpha V_T/I_C$, $r_0 = \infty$, $r_x = 0$, $V_T = 25\,\text{mV}$, $R_1 = 100\,\Omega$, $R_2 = 9\,\text{k}\Omega$, $R_3 = 5\,\text{k}\Omega$, $R_4 = 600\,\Omega$, $R_5 = 640\,\Omega$, and $R_6 = 100\,\Omega$. The circuit with feedback removed is shown in Fig. 23.

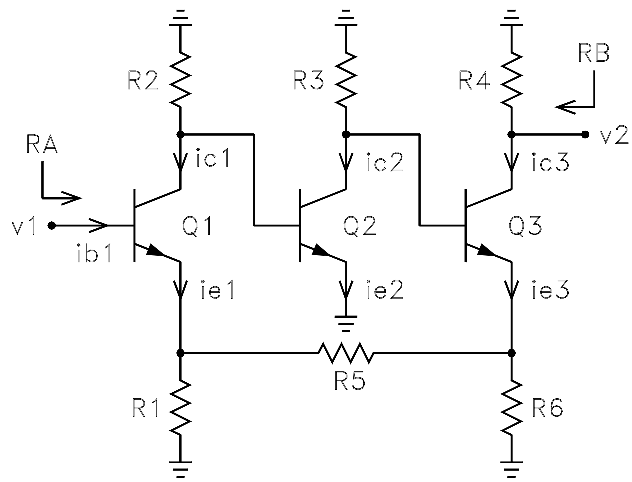

Figure 22: Amplifier circuit.

The circuit looking out of the emitter of Q_1 is a Thévenin equivalent made with respect to the current i_{e3}. The output current is proportional to this current, i.e. $i_{c3} = \alpha i_{e3}$. Because $r_0 = \infty$ for Q_3, the feedback does not affect the output resistance seen looking down through R_4 because it is infinite. For a finite r_0, a test voltage source can be added in series with R_4 to solve for this resistance. It would be found that a finite r_0 for Q_3 considerably complicates the circuit equations and the flow graph.

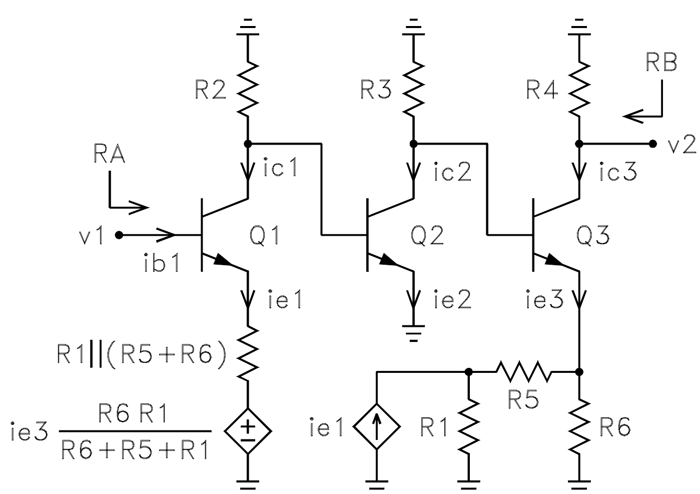

Figure 23: Circuit with feedback removed.

For the circuit with feedback removed, we can write

$$v_e = v_1 - i_{e3}\frac{R_6 R_1}{R_6+R_5+R_1} \qquad i_{e1} = G_1 v_e \qquad G_1 = \frac{1}{r_{e1} + R_1 \| (R_5+R_6)} \qquad i_{c1} = \alpha i_{e1}$$

$$i_{b1} = \frac{i_{c1}}{\beta} \qquad v_{tb2} = -i_{c1}R_2 \qquad i_{e2} = G_2 v_{tb2} \qquad G_2 = \frac{1}{r'_{e2}} \qquad r'_{e2} = \frac{R_2}{1+\beta} + r_e$$

$$i_{c2} = \alpha i_{e2} \qquad v_{tb3} = -i_{c2}R_3 \qquad i_{e3} = G_3 v_{tb3} - k_1 i_{e1} \qquad G_3 = \frac{1}{r'_{e3} + R_6 \| (R_1+R_5)}$$

$$k_1 = \frac{R_1}{R_1+R_5+r'_{e3}\|R_6}\frac{R_6}{R_6+r'_{e3}} \qquad i_{c3} = \alpha i_{e3} \qquad v_2 = -i_{c2}R_4$$

The equations can be solved algebraically or by a flow graph. The flow graph for the equations is shown in Fig. 24. The determinant is

$$\begin{aligned}\Delta &= 1 - \left\{ G_1 \times [(\alpha \times -R_2 \times G_2 \times \alpha \times -R_3 \times G_3) - k_1] \times \frac{-R_6 R_1}{R_6+R_5+R_1} \right\} \\ &= 1 + G_1 \times (\alpha \times R_2 \times G_2 \times \alpha \times R_a - k_1) \times \frac{R_6 R_1}{R_6+R_5+R_1} \\ &= 251.5\end{aligned}$$

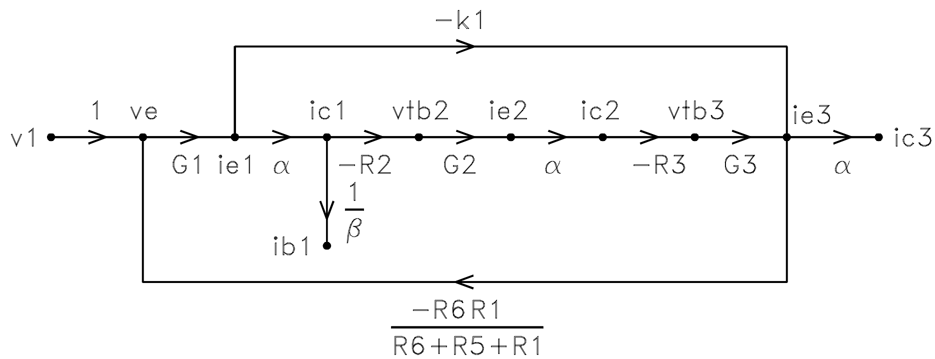

Figure 24: Signal-flow graph for the circuit.

The transconductance gain is

$$\begin{aligned}\frac{i_{c3}}{v_1} &= \frac{G_1 \times (\alpha \times -R_2 \times G_2 \times \alpha \times -R_3 \times G_3 - k_1) \times \alpha}{\Delta} \\ &= \frac{G_1 \times (\alpha \times R_2 \times G_2 \times \alpha \times R_3 \times G_3 - k_1) \times \alpha}{1 + [G_1 \times (\alpha \times R_2 \times G_2 \times \alpha \times R_a - k_1)] \times \dfrac{R_6 R_1}{R_6+R_5+R_1}} \\ &= \frac{G_1 \times [(\alpha \times R_2 \times G_2 \times \alpha \times R_3 \times G_3) - k_1] \times \alpha}{1 + [G_1 \times (\alpha \times R_2 \times G_2 \times \alpha \times R_a - k_1) \times \alpha] \times \dfrac{R_6 R_1}{R_6+R_5+R_1} \times \dfrac{1}{\alpha}}\end{aligned}$$

This is of the form

$$\frac{i_{c3}}{v_1} = \frac{A}{1+Ab}$$

where A and b are given by

$$\begin{aligned}A &= G_1 \times [(\alpha \times R_2 \times G_2 \times \alpha \times R_3 \times G_3) - k_1] \times \alpha \\ &= \frac{1}{r_{e1} + R_1\|(R_5+R_6)} \times \left[\left(\alpha \times R_2 \times \frac{1}{r'_{e2}} \times \alpha \times R_3 \times \frac{1}{r'_{e3} + R_6\|(R_1+R_5)} \times \alpha \right) \right. \\ &\quad \left. - \frac{R_1}{R_1+R_5+r'_{e3}\|R_6}\frac{R_6}{R_6+r'_{e3}} \right] \times \alpha \\ &= 20.83\,\mathrm{S}\end{aligned}$$

$$b = \frac{R_6 R_1}{R_6 + R_5 + R_1} \times \frac{1}{\alpha} = 12.02\,\Omega$$

Notice that the product Ab is dimensionless and positive. The latter must be true for the feedback to be negative.

Numerical evaluation of the transconductance gain yields

$$\frac{i_{c3}}{v_1} = \frac{A}{\Delta} = 0.083$$

The voltage gain is given by

$$\frac{v_2}{v_1} = \frac{i_{c3}}{v_1} \times \frac{v_2}{i_{c3}} = \frac{A}{\Delta} \times -R_4 = -49.7$$

The resistances R_A and R_B are given by

$$R_A = \left(\frac{i_{b1}}{v_1}\right)^{-1} = \left(\frac{G_1 \alpha/\beta}{\Delta}\right)^{-1} = \frac{\Delta \times (1+\beta)}{G_1} = \Delta \times (1+\beta) \times [r_{e1} + R_1 \| (R_5 + R_6)] = 3.285\,\mathrm{M}\Omega$$

$$R_B = R_4 = 600\,\Omega$$

Series-Series Example 3

A series-series feedback BJT amplifier is shown in Fig. 25(a). The input variable is the voltage v_1 and the output variable is the voltage v_2. The feedback is from i_{e2} to i_{c2} to the emitter of Q_1. Because the feedback does not connect to the input node, the input summing is series. Because the feedback does not sample the output voltage, the sampling is series. That is, the feedback network samples the current in series with the outpu. Solve for the transconductance gain i_{e2}/v_1, the voltage gain v_2/v_1, the input resistance R_A, and the output resistance R_B. Assume $\beta = 100$, $r_\pi = 2.5\,\mathrm{k}\Omega$, $\alpha = \beta/(1+\beta)$, $r_e = \alpha/g_m$, $r_0 = \infty$, $r_x = 0$, $V_T = 25\,\mathrm{mV}$, $R_1 = 100\,\Omega$, $R_2 = 1\,\mathrm{k}\Omega$, $R_3 = 20\,\mathrm{k}\Omega$, and $R_4 = 10\,\mathrm{k}\Omega$.

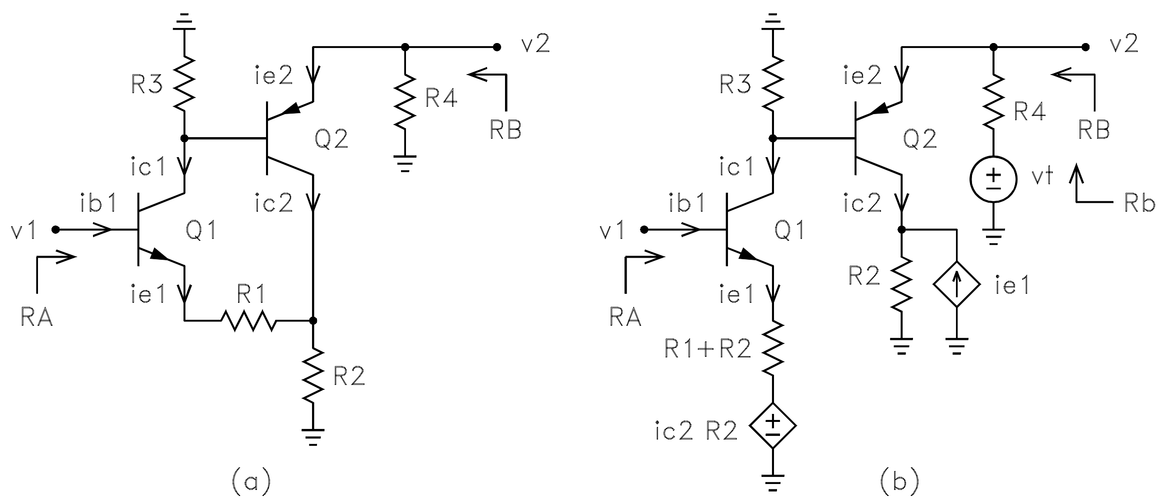

Figure 25: (a) Amplifier circuit. (b) Circuit with feedback removed.

The circuit with feedback removed is shown in Fig. 25(b). The circuit seen looking out of the emitter of Q_1 is replaced with a Thévenin equivalent circuit made with respect with i_{c2}. The output current i_{e2} is proportional to this current, i.e. $i_{e2} = \alpha i_{c2}$. A test voltage source v_t is added in series with the output to solve for the output resistance. The resistance seen by the test source is labeled R_b.

For the circuit with feedback removed, we can write

$$v_e = v_1 - i_{c2} R_2 \qquad i_{e1} = G_1 v_e \qquad G_1 = \frac{1}{r_e + R_1 + R_2} \qquad i_{c1} = \alpha i_{e1} \qquad i_{b1} = \frac{i_{c1}}{\beta}$$

$$v_{tb2} = -i_{c1} R_3 \qquad i_{e2} = G_2 (v_t - v_{tb2}) \qquad G_2 = \frac{1}{r'_{e2} + R_4} \qquad r'_{e2} = \frac{R_3}{1+\beta} + r_e \qquad i_{c2} = \alpha i_{e2}$$

The equations can be solved algebraically or by a flow graph. The flow graph for the equations is shown in Fig. 26. The determinant is

$$\begin{aligned} \Delta &= 1 - (G_1 \times \alpha \times -R_3 \times -G_2 \times \alpha \times -R_2) \\ &= 1 + G_1 \times \alpha \times R_2 \times G_2 \times \alpha \times R_2 \\ &= 1.181 \end{aligned}$$

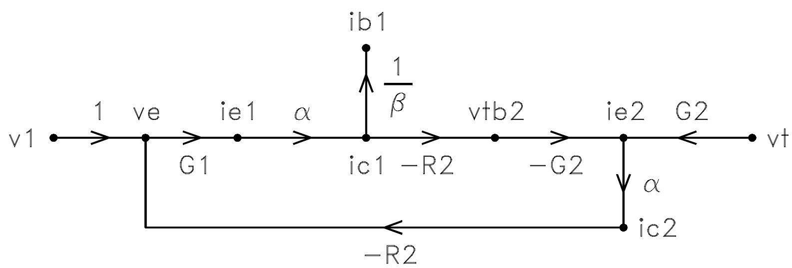

Figure 26: Signal-flow graph for the equations.

The transconductance gain is

$$\begin{aligned} \frac{i_{e2}}{v_1} &= \frac{G_1 \times \alpha \times -R_3 \times -G_2}{\Delta} \\ &= \frac{(G_1 \times \alpha \times R_3 \times G_2)}{1 + (G_1 \times \alpha \times R_3 \times G_2) \times \alpha \times R_2} \end{aligned}$$

This is of the form

$$\frac{i_{e2}}{v_1} = \frac{A}{1+Ab}$$

where A and b are given by

$$\begin{aligned} A &= G_1 \times \alpha \times R_3 \times G_2 \\ &= \frac{1}{r_{e1} + R_1 + R_2} \times \alpha \times R_3 \times \frac{1}{r'_{e2} + R_4} \\ &= 0.9117\,\text{mS} \end{aligned}$$

$$b = \alpha R_2 = 1.98\,\text{k}\Omega$$

Notice that the product Ab is dimensionless and positive. The latter must be true for the feedback to be negative.

Numerical evaluation of the transconductance gain yields

$$\frac{i_{e2}}{v_1} = \frac{A}{\Delta} = 0.325\,\text{mS}$$

The voltage gain is given by

$$\frac{v_2}{v_1} = \frac{i_{e2}}{v_1} \times \frac{v_2}{i_{e2}} = \frac{A}{\Delta} \times -R_4 = -3.25$$

The resistances R_A and R_B are given by

$$R_A = \left(\frac{i_{b1}}{v_1}\right)^{-1} = \left(\frac{G_1 \alpha / \beta}{\Delta}\right)^{-1} = \Delta \times (1+\beta)(r_e + R_1 + R_2) = 602\,\text{k}\Omega$$

$$R_b = \left(\frac{i_{e2}}{v_t}\right)^{-1} = \left(\frac{G_{m2}}{\Delta}\right)^{-1} = \Delta \times (R_4 + r'_{e2}) = 28.68\,\text{k}\Omega$$

$$R_B = (R_b - R_4) \| R_4 = 6.513\,\text{k}\Omega$$

Series-Series Example 4

A series-series feedback BJT amplifier is shown in Fig. 27(a). The input variable is the voltage v_1 and the output variable is the current i_{c2}. The feedback is from i_{c2} to i_{e2} to the gate of J_1. The input summing is series because the feedback does not connect to the same node that the source connects. The output sampling is series because the feedback is proportional to the output current i_{c2}. Solve for the transconductance gain i_{c2}/v_1, the voltage gain v_2/v_1, the input resistance R_A, and the output resistance R_B. For J_1, assume that $g_{m1} = 0.001\,\mathrm{S}$, $r_{s1} = g_{m1}^{-1} = 1000\,\Omega$, and $r_{01} = \infty$. For Q_2, assume $\beta_2 = 100$, $r_{\pi 2} = 2.5\,\mathrm{k}\Omega$, $\alpha_2 = \beta_2/\left(1+\beta_2\right)$, $r_{e2} = \alpha_2/g_{m2}$, $r_{02} = \infty$, $r_{x2} = 0$, $V_T = 25\,\mathrm{mV}$. The resistor values are $R_1 = 1\,\mathrm{k}\Omega$, $R_2 = 10\,\mathrm{k}\Omega$, $R_3 = 1\,\mathrm{k}\Omega$, $R_4 = 10\,\mathrm{k}\Omega$, $R_5 = 1\,\mathrm{k}\Omega$, and $R_6 = 10\,\mathrm{k}\Omega$.

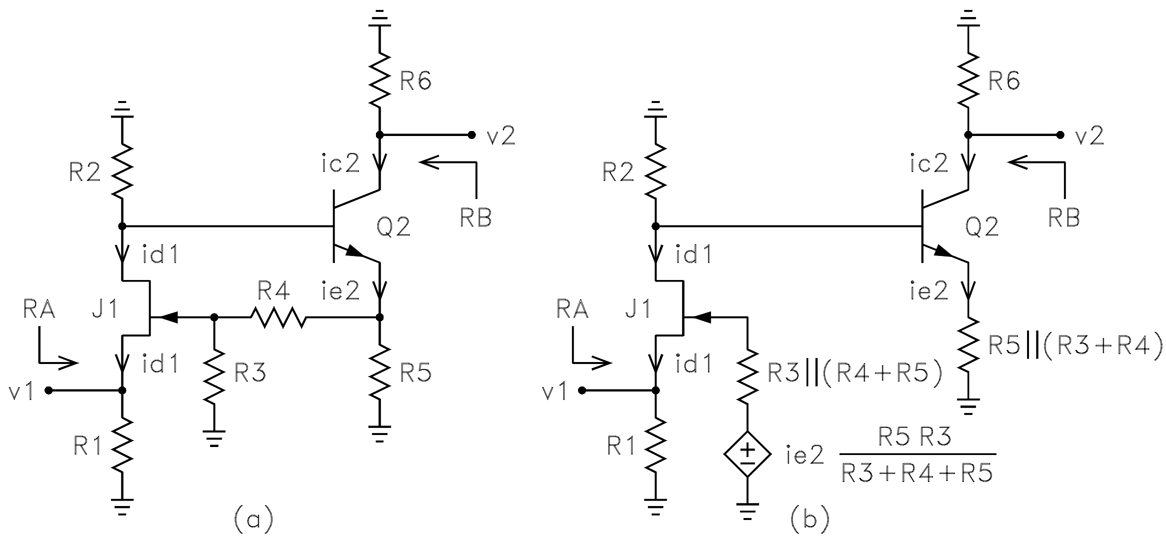

Figure 27: (a) Amplifier circuit. (b) Circuit with feedback removed.

The circuit with feedback removed is shown in Fig. 27(b). The circuit seen looking out of the emitter of Q_1 is replaced with a Thévenin equivalent circuit made with respect with i_{e2}. The output current is proportional to this current, i.e. $i_{c2} = \alpha_2 i_{e2}$. Because $r_{02} = \infty$, the feedback does not affect the output resistance seen looking down through R_6 because it is infinite. For a finite r_{02}, a test voltage source can be added in series with R_6 to solve for this resistance. It would be found that a finite r_{02} considerably complicates the circuit equations and the flow graph.

For the circuit with feedback removed, we can write

$$i_{d1} = g_{m1} v_e \qquad v_e = i_{e2}\frac{R_5 R_3}{R_3 + R_4 + R_5} - v_1 \qquad v_{tb2} = -i_{d1} R_2 \qquad i_{e2} = G_1 v_{tb2}$$

$$G_1 = \frac{1}{r'_{e2} + R_5 \| \left(R_3 + R_4\right)} \qquad r'_{e2} = \frac{R_2}{1+\beta_2} + r_{e2} \qquad i_{c2} = \alpha_2 i_{e2}$$

The equations can be solved algebraically or by a flow graph. The flow graph for the equations is shown in Fig. 28. The determinant is

$$\begin{aligned}
\Delta &= 1 - \left(g_{m1} \times -R_2 \times G_1 \times \frac{R_5 R_3}{R_3 + R_4 + R_5}\right) \\
&= 1 + g_{m1} \times R_2 \times G_1 \times \frac{R_5 R_3}{R_3 + R_4 + R_5} \\
&= 1.801
\end{aligned}$$

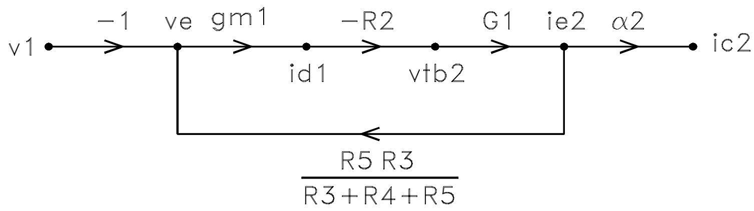

Figure 28: Signal-flow graph for the equations.

The transconductance gain is

$$\begin{aligned}\frac{i_{e2}}{v_1} &= \frac{-1\times g_{m1}\times -R_2\times G_1\times\alpha_2}{\Delta}\\ &= \frac{g_{m1}\times R_2\times G_1\times\alpha_2}{1+g_{m1}\times R_2\times G_1\times\dfrac{R_5R_3}{R_3+R_4+R_5}}\\ &= \frac{(g_{m1}\times R_2\times G_1\times\alpha_2)}{1+(g_{m1}\times R_2\times G_1\times\alpha_2)\times\dfrac{R_5R_3}{R_3+R_4+R_5}\times\dfrac{1}{\alpha_2}}\end{aligned}$$

This is of the form

$$\frac{i_{e2}}{v_1}=\frac{A}{1+Ab}$$

where A and b are given by

$$\begin{aligned}A &= g_{m1}\times R_2\times G_1\times\alpha_2\\ &= g_{m1}\times R_2\times\frac{1}{r'_{e2}+R_5\|(R_3+R_4)}\times\alpha_2\\ &= 9.516\,\text{mS}\end{aligned}$$

$$b=\frac{R_5R_3}{R_3+R_4+R_5}\times\frac{1}{\alpha_2}=84.17\,\Omega$$

Notice that the product Ab is dimensionless and positive. The latter must be true for the feedback to be negative.

Numerical evaluation of the transconductance gain yields

$$\frac{i_{c2}}{v_1}=\frac{A}{\Delta}=5.284\,\text{mS}$$

The voltage gain is given by

$$\frac{v_2}{v_1}=\frac{i_{c2}}{v_1}\times\frac{v_2}{i_{c2}}=\frac{A}{\Delta}\times -R_6=-52.84$$

The resistances R_A and R_B are given by

$$R_A=R_1\|\left(\frac{-i_{d1}}{v_1}\right)^{-1}=R_1\|\left(\frac{g_{m1}}{\Delta}\right)^{-1}=R_1\|\left(\frac{\Delta}{g_{m1}}\right)=643\,\Omega$$

$$R_B=R_6=10\,\text{k}\Omega$$

Series-Series Example 5

A series-series feedback BJT amplifier is shown in Fig. 29. The input variable is the current i_1 and the output variable is the current i_{e2}. The feedback path is the path from i_{e2} to i_{c2} to i_{e3} to i_{c3} to the emitter of Q_1. The input summing is series because the feedback does not connect to the input node. The output sampling is series because the feedback is proportional to the output current i_{e2} and not the output voltage v_2. Solve for the current gain gain i_{e2}/i_1, the transresistance gain v_2/i_1, the input resistance R_A, and the output resistance R_B. Assume $\beta = 100$, $r_\pi = 2.5\,\text{k}\Omega$, $g_m = \beta/r_\pi$, $\alpha = \beta/(1+\beta)$, $r_e = \alpha/g_m$, $r_0 = \infty$, $r_x = 0$, $V_T = 25\,\text{mV}$. The resistor values are $R_1 = 1\,\text{k}\Omega$, $R_2 = 100\,\Omega$, $R_3 = 10\,\text{k}\Omega$, $R_4 = 100\,\Omega$, $R_5 = 1\,\text{k}\Omega$, and $R_6 = 10\,\text{k}\Omega$.

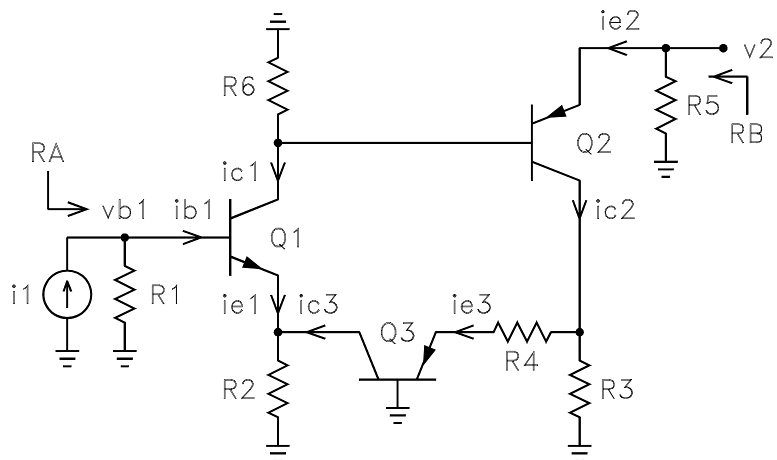

Figure 29: Amplifier circuit.

The circuit with feedback removed is shown in Fig. 30. The source is replaced with a Thevenin equivalent circuit consisting of a voltage

$$v_1 = i_1 R_1$$

in series with the resistor R_1. This is necessary for the feedback analysis to conform to convention for series summing at the input. The circuit seen looking out of the emitter of Q_1 is replaced with a Thévenin equivalent circuit made with respect with i_{c3}. The latter is proportional to the output current i_{e2}. The relation is

$$\frac{i_{c3}}{i_{e2}} = \frac{i_{c3}}{i_{e3}} \times \frac{i_{e3}}{i_{c2}} \times \frac{i_{c2}}{i_{e2}} = \alpha \times \frac{R_3}{R_3 + R_4 + r_{e3}} \times \alpha$$

Note that r_{e3} in this equation is the small-signal resistance seen looking into the emitter of Q_3.

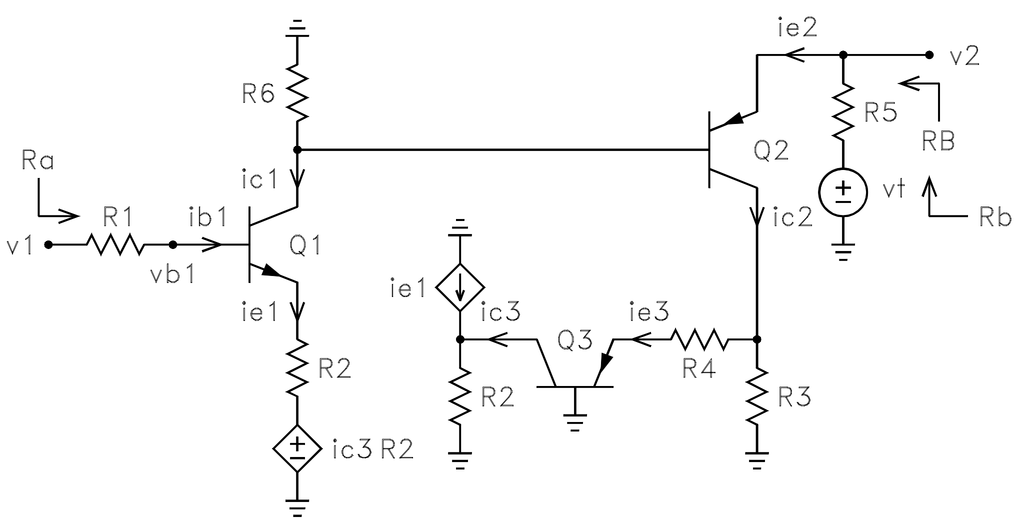

Figure 30: Circuit with feedback removed.

For the circuit with feedback removed, we can write

$$v_e = v_1 - i_{c3}R_2 \qquad i_{e1} = G_1 v_e \qquad G_1 = \frac{1}{r'_{e1} + R_2} \qquad r'_{e1} = \frac{R_1}{1+\beta} + r_e \qquad i_{c1} = \alpha i_{e1} \qquad i_{b1} = \frac{i_{c1}}{\beta}$$

$$v_{tb2} = -i_{c1}R_6 \qquad i_{e2} = G_2\left(v_t - v_{tb2}\right) \qquad G_2 = \frac{1}{r'_{e2} + R_5} \qquad r'_{e2} = \frac{R_6}{1+\beta} + r_e$$

$$i_{c2} = \alpha i_{e2} \qquad i_{e3} = i_{c2}\frac{R_3}{R_3 + R_4 + r_e} \qquad i_{c3} = \alpha i_{e3}$$

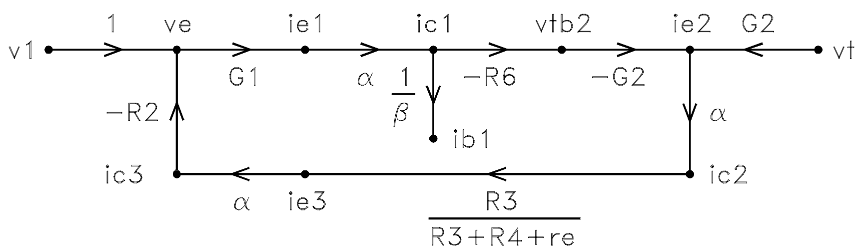

Figure 31: Signal-flow graph for the equations.

The equations can be solved algebraically or by a flow graph. The flow graph for the equations is shown in Fig. 31. The determinant is

$$\begin{aligned}
\Delta &= 1 - G_1 \times \alpha \times -R_6 \times -G_2 \times \alpha \times \frac{R_3}{R_3 + R_4 + r_e} \times \alpha \times R_2 \\
&= 1 + G_1 \times \alpha \times R_6 \times G_2 \times \alpha \times \frac{R_3}{R_3 + R_4 + r_e} \times \alpha \times R_2 \\
&= 7.335
\end{aligned}$$

The transconductance gain is

$$\begin{aligned}
\frac{i_{e2}}{v_1} &= \frac{1 \times G_1 \times \alpha \times -R_6 \times -G_2}{\Delta} \\
&= \frac{G_1 \times \alpha \times R_6 \times G_2}{1 + G_1 \times \alpha \times R_6 \times G_2 \times \alpha \times \frac{R_3}{R_3 + R_4 + r_e} \times \alpha \times R_2}
\end{aligned}$$

This is of the form

$$\frac{i_{e2}}{v_1} = \frac{A}{1 + Ab}$$

where A and b are given by

$$\begin{aligned}
A &= G_1 \times \alpha \times R_6 \times G_2 \\
&= \frac{1}{r'_{e1} + R_2} \times \alpha \times R_6 \times \frac{1}{r'_{e2} + R_5} \\
&= 65.43\,\mathrm{mS}
\end{aligned}$$

$$b = \alpha \times \frac{R_3}{R_3 + R_4 + r_e} \times \alpha \times R_2 = 96.82\,\Omega$$

Notice that the product Ab is dimensionless and positive. The latter must be true for the feedback to be negative.

Numerical evaluation of the transconductance gain yields

$$\frac{i_{e2}}{v_1} = \frac{A}{\Delta} = 8.92\,\mathrm{mS}$$

The current gain is given by

$$\frac{i_{e2}}{i_1} = \frac{i_{e2}}{v_1} \times \frac{v_1}{i_1} = \frac{A}{\Delta} \times R_1 = 8.92$$

The transresistance gain is

$$R_m = \frac{v_2}{i_1} = \frac{i_{e2}}{i_1} \times \frac{v_2}{i_{e2}} = \frac{A}{\Delta} \times R_1 \times -R_5 = -8.92\,\mathrm{k\Omega}$$

The resistances R_a and R_A

$$R_a = \left(\frac{i_{b1}}{v_1}\right)^{-1} = \left(\frac{G_1 \alpha/\beta}{\Delta}\right)^{-1} = \Delta \times [R_1 + r_\pi + (1+\beta) R_2] = 113.3\,\mathrm{k\Omega}$$

$$R_A = \frac{v_{b1}}{i_1} = (R_a - R_1) \| R_1 = 991.2\,\Omega$$

The voltage gain is

$$\frac{v_2}{v_{b1}} = \frac{v_2}{i_1} \times \left(\frac{v_{b1}}{i_1}\right)^{-1} = \left(\frac{A}{\Delta} \times R_1\right) \times \frac{1}{R_A} = -990$$

The resistances R_b and R_B are

$$R_b = \left(\frac{i_{e2}}{v_t}\right)^{-1} = \left(\frac{G_2}{\Delta}\right)^{-1} = \Delta \times (r'_{e2} + R_5) = 10.98\,\mathrm{k\Omega}$$

$$R_B = (R_b - R_5) \| R_5 = 889.5\,\Omega$$

Shunt-Series Example 1

Figure 32(a) shows the ac signal circuit of a shunt-series feedback amplifier. The input variable is v_1 and the output variable is i_{d2}. The input signal and the feedback signal are applied to the source of M_1. A test voltage source v_t is added in series with the output to calculate the output resistance R_b. For the analysis to follow convention, the input source consisting of v_1 in series with R_1 must be converted into a Norton equivalent. This circuit is the current

$$i_1 = \frac{v_1}{R_1}$$

in parallel with the resistor R_1. Fig. 32(b) shows the circuit with feedback removed and the source replaced with the Norton equivalent. A test source v_t is added in series with the output to calculate the resistance R_b. The feedback at the source of M_1 is modeled by a Norton equivalent circuit i_{d2} in parallel with the resistor R_4. The feedback is from the output current i_{d2} to the source of M_1. The circuit values are $g_m = 0.001\,\mathrm{S}$, $r_s = g_m^{-1} = 1\,\mathrm{k\Omega}$, $r_0 = \infty$, $R_1 = 10\,\mathrm{k\Omega}$, $R_2 = 20\,\mathrm{k\Omega}$, $R_3 = 1\,\mathrm{k\Omega}$, $R_4 = 1\,\mathrm{k\Omega}$, and $R_5 = 1\,\mathrm{k\Omega}$.

The following equations can be written for the circuit with feedback removed:

$$v_{s1} = i_a R_c \qquad i_a = i_1 + i_{d2} \qquad R_c = R_1 \| R_4 \| r_{s1} \qquad i_{d1} = -g_{m1} v_{s1}$$

$$i_{d2} = G_{m2} v_b \qquad G_{m2} = \frac{1}{r_{s2} + R_3} \qquad v_b = v_t - v_{tg2} \qquad v_{tg2} = -i_{d1} R_2$$

The current i_a is the error current. The negative feedback tends to reduce i_a, making $|i_a| \to 0$ as the amount of feedback becomes infinite. When this is the case, setting $i_a = 0$ yields the current gain $i_{d2}/i_1 = -1$.

Although the equations can be solved algebraically, the signal-flow graph simplifies the solution. Fig. 33 shows the flow graph for the equations. The determinant of the graph is given by

$$\Delta = 1 - R_c \times (-g_{m1}) \times (-R_2) \times (-1) \times G_{m2} \times 1$$

The current gain is calculated with $v_t = 0$. It is given by

$$\begin{aligned}\frac{i_{d2}}{i_1} &= \frac{R_c \times (-g_{m1}) \times (-R_2) \times (-1) \times G_{m2}}{\Delta} \\ &= -\frac{R_1 \| R_4 \| r_{s1} \times g_{m1} \times R_2 \times \dfrac{1}{r_{s2} + R_3}}{1 + R_c \times g_{m1} \times R_2 \times \dfrac{1}{r_{s2} + R_3} \times 1}\end{aligned}$$

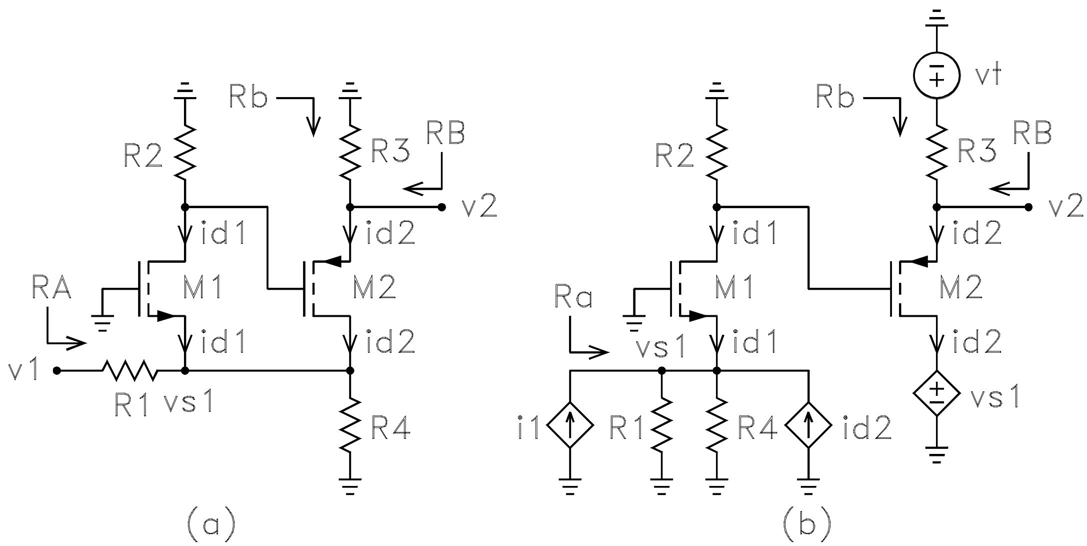

Figure 32: (a) Amplifier circuit. (b) Circuit with feedback removed.

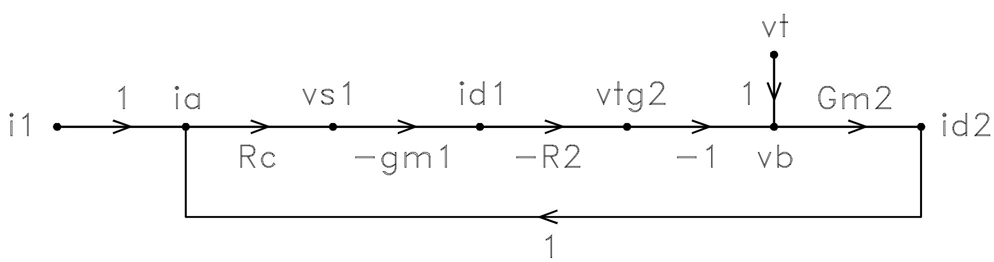

Figure 33: Flow graph for the equations.

This is of the form

$$\frac{i_{d2}}{i_1} = \frac{A}{1 + Ab}$$

where

$$A = R_c \times (-g_{m1}) \times (-R_2) \times (-1) \times G_{m2} = -(R_1 \| R_4 \| r_{s1}) \times g_{m1} \times R_2 \times \frac{1}{r_{s2} + R_3} = -0.3333$$

$$b = -1$$

Note that Ab is dimensionless. Numerical evaluation yields

$$\frac{i_{d2}}{i_1} = \frac{2.5 \times 10^{-3}}{1 + 1000 \times 2.5 \times 10^{-3}} = -0.7692$$

The voltage gain is given by

$$\frac{v_2}{v_1} = \frac{i_{d2}}{i_1} \times \frac{i_1}{v_1} \times \frac{v_2}{i_{d2}} = \frac{i_{d2}}{i_1} \times \frac{1}{R_1} \times (-R_3) = 0.7692$$

The resistance R_a is calculated with $v_t = 0$. It is given by

$$R_a = \frac{v_{s1}}{i_1} = \frac{R_c}{\Delta} = \frac{R_1 \| R_4 \| r_{s1}}{1 + 1 \times R_c \times g_{m1} \times R_2 \times \dfrac{1}{r_{s2} + R_3}} = 76.92\,\Omega$$

Note that the feedback tends to decrease R_a. The resistance R_A is calculated as follows:

$$R_A = R_1 + \left(R_a^{-1} - R_1^{-1}\right)^{-1} = 1.083\,\mathrm{k\Omega}$$

The resistance R_b is calculated with $i_1 = 0$. It is given by

$$R_b = \left(\frac{i_{d2}}{v_t}\right)^{-1} = \left(\frac{G_{m2}}{\Delta}\right)^{-1} = (1 + Ab)(r_{s2} + R_3) = 8.667\,\mathrm{k\Omega}$$

Note that the feedback tends to increase R_b. The resistance R_B is calculated as follows:

$$R_B = (R_b - R_3) \| R_3 = 884.6\,\Omega$$

Shunt-Series Example 2

A shunt-series feedback BJT amplifier is shown in Fig. 34(a). The input variable is the voltage v_1 and the output variable is the current i_{e2}. The feedback is from i_{e2} to i_{c2} to the source of M_1. The input summing is shunt because the feedback connects to the same node that the source connects. The output sampling is series because the feedback is proportional to the output current i_{e2}. Solve for the voltage gain v_2/v_1, the input resistance R_A, and the output resistance R_B. For M_1, assume that $g_{m1} = 0.001\,\mathrm{S}$, $r_{s1} = g_{m1}^{-1} = 1000\,\Omega$, and $r_{01} = \infty$. For Q_2, assume $\beta_2 = 100$, $r_{\pi 2} = 2.5\,\mathrm{k\Omega}$, $\alpha_2 = \beta_2 / (1 + \beta_2)$, $r_{e2} = \alpha_2 / g_{m2}$, $r_{02} = \infty$, $r_{x2} = 0$, $V_T = 25\,\mathrm{mV}$. The resistor values are $R_1 = 10\,\mathrm{k\Omega}$, $R_2 = 100\,\mathrm{k\Omega}$, $R_3 = 100\,\mathrm{k\Omega}$, $R_4 = 10\,\mathrm{k\Omega}$, and $R_5 = 1\,\mathrm{k\Omega}$. The circuit with feedback removed is shown in Fig. 34(b). The source is replaced with a Norton equivalent circuit. The current i_1 is given by

$$i_1 = \frac{v_1}{R_1}$$

The circuit seen looking into R_2 from the v_{s1} node is replaced with a Norton equivalent circuit made with respect with i_{c2}. The output current is proportional to this current, i.e. $i_{c2} = \alpha_2 i_{e2}$. A test voltage source v_t is added in series with i_{e2} to calculate the resistance R_b.

For the circuit with feedback removed, we can write

$$v_{s1} = i_a R_c \qquad i_a = i_1 - \frac{R_4}{R_2 + R_4} i_{c2} \qquad R_c = R_1 \| (R_2 + R_4) \| r_{s1} \qquad i_{d1} = g_{m1} v_{s1} \qquad v_{tb2} = i_{d1} R_3$$

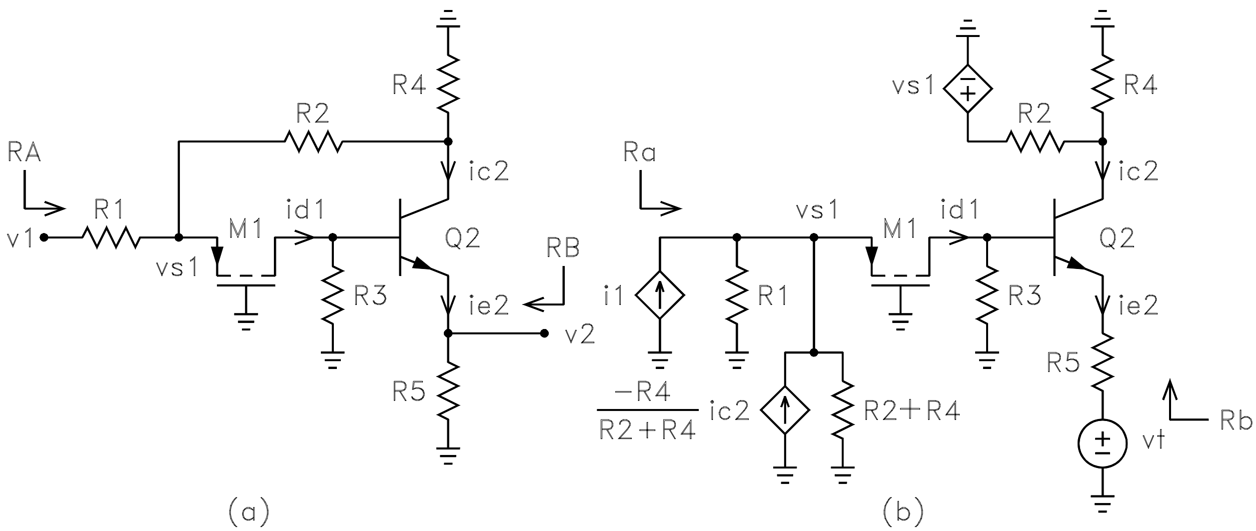

Figure 34: (a) Shunt-series amplifier. (b) Amplifier with feedback removed.

$$i_{e2} = G_1 v_{tb2} - \frac{v_t}{R_d} \qquad G_1 = \frac{1}{r'_{e2} + R_5} \qquad r'_{e2} = \frac{R_3}{1+\beta_2} + r_{e2} \qquad R_d = R_5 + r'_{e2} \qquad i_{c2} = \alpha_2 i_{e2}$$

The equations can be solved algebraically or by a flow graph. The flow graph for the equations is shown in Fig. 35. The determinant is

$$\begin{aligned}
\Delta &= 1 - \left(1 \times R_c \times g_{m1} \times R_3 \times G_1 \times \alpha_2 \times \frac{-R_4}{R_2 + R_4}\right) \\
&= 1 + R_c \times g_{m1} \times R_3 \times G_1 \times \alpha_2 \times \frac{R_4}{R_2 + R_4} \\
&= 5.025
\end{aligned}$$

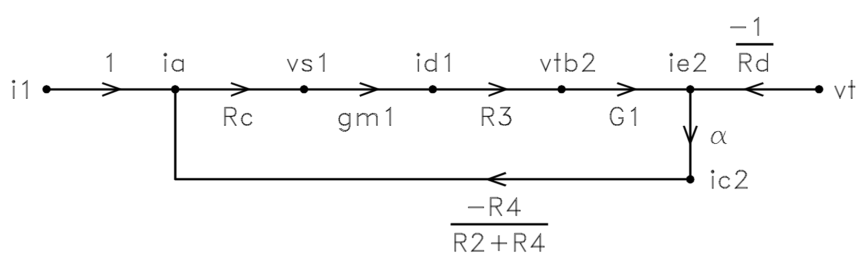

Figure 35: Signal-flow graph for the equations.

The current gain is

$$\begin{aligned}
\frac{i_{e2}}{i_1} &= \frac{R_c \times g_{m1} \times R_3 \times G_1}{\Delta} \\
&= \frac{R_c \times g_{m1} \times R_3 \times G_1}{1 + R_c \times g_{m1} \times R_3 \times G_1 \times \alpha_2 \times \dfrac{R_4}{R_2 + R_4}} \\
&= \frac{\left(R_1 \| \left(R_2 + R_4\right) \| r_{s1} \times g_{m1} \times R_3 \times \dfrac{1}{r'_{e2} + R_4}\right)}{1 + \left(R_1 \| \left(R_2 + R_4\right) \| r_{s1} \times g_{m1} \times R_3 \times \dfrac{1}{r'_{e2} + R_4}\right) \times \alpha_2 \times \dfrac{R_4}{R_2 + R_4}}
\end{aligned}$$

This is of the form

$$\frac{i_{e2}}{i_1} = \frac{A}{1 + Ab}$$

where A and b are given by

$$\begin{aligned} A &= R_c \times g_{m1} \times R_3 \times G_1 \\ &= R_1 \| (R_2 + R_4) \| r_{s1} \times g_{m1} \times R_3 \times \frac{1}{r'_{e2} + R_4} \\ &= 44.75 \end{aligned}$$

$$b = \alpha_2 \times \frac{R_4}{R_2 + R_4} = 0.09$$

Notice that the product Ab is dimensionless and positive. The latter must be true for the feedback to be negative.

Numerical evaluation of the current gain yields

$$\frac{i_{e2}}{i_1} = \frac{A}{\Delta} = 8.90$$

The resistances R_a and R_A are

$$R_a = \frac{v_{s1}}{i_1} = \frac{R_c}{\Delta} = 179.3\,\Omega$$

$$R_A = R_1 + \left(R_a^{-1} - R_1^{-1}\right)^{-1} = 10.13\,\mathrm{k}\Omega$$

The resistance R_b and R_B are

$$R_b = \left(\frac{-i_{e2}}{v_t}\right)^{-1} = \left(\frac{1}{\Delta}\frac{1}{R_d}\right)^{-1} = \Delta R_d = 10.12\,\mathrm{k}\Omega$$

$$R_B = (R_b - R_5) \| R_5 = 901.3\,\Omega$$

The voltage gain is given by

$$\frac{v_2}{v_1} = \frac{i_{e2}}{i_1} \times \frac{v_2}{i_{e2}} \times \frac{i_1}{v_1} = \frac{A}{\Delta} \times R_2 \times \frac{1}{R_1} = 89.0$$

Shunt-Series Example 3

A shunt-series feedback BJT amplifier is shown in Fig. 36(a). The input variable is the voltage v_1 and the output variable is the current i_{c2}. The feedback is from i_{c2} to i_{e2} to i_{c3} to the emitter of Q_1. The input summing is shunt because the feedback connects to the same node that the source connects. The output sampling is series because the feedback is proportional to the output current i_{c2}. Solve for the voltage gain v_2/v_1, the input resistance R_A, and the output resistance R_B. Assume $\beta = 100$, $r_\pi = 2.5\,\mathrm{k}\Omega$, $\alpha = \beta/(1+\beta)$, $r_e = \alpha/g_m$, $r_0 = \infty$, $r_x = 0$, $V_T = 25\,\mathrm{mV}$. The resistor values are $R_1 = R_3 = 1\,\mathrm{k}\Omega$ and $R_2 = R_4 = R_5 = 10\,\mathrm{k}\Omega$.

The circuit with feedback removed is shown in Fig. 36(b). The source is replaced with a Norton equivalent circuit consisting of the current

$$i_1 = \frac{v_1}{R_1}$$

in parallel with the resistor R_1. The feedback is modeled by a Norton equivalent circuit consisting of the current i_{c3}. Because $r_{03} = \infty$, the output resistance of this source is an open circuit. The output current is proportional to this current. Because $r_{02} = \infty$, the feedback does not affect the output resistance seen looking down through R_5 because it is infinite. For a finite r_{02}, a test voltage source can be added in series with R_5 to solve for this resistance. It would be found that a finite r_{02} considerably complicates the circuit equations and the flow graph.

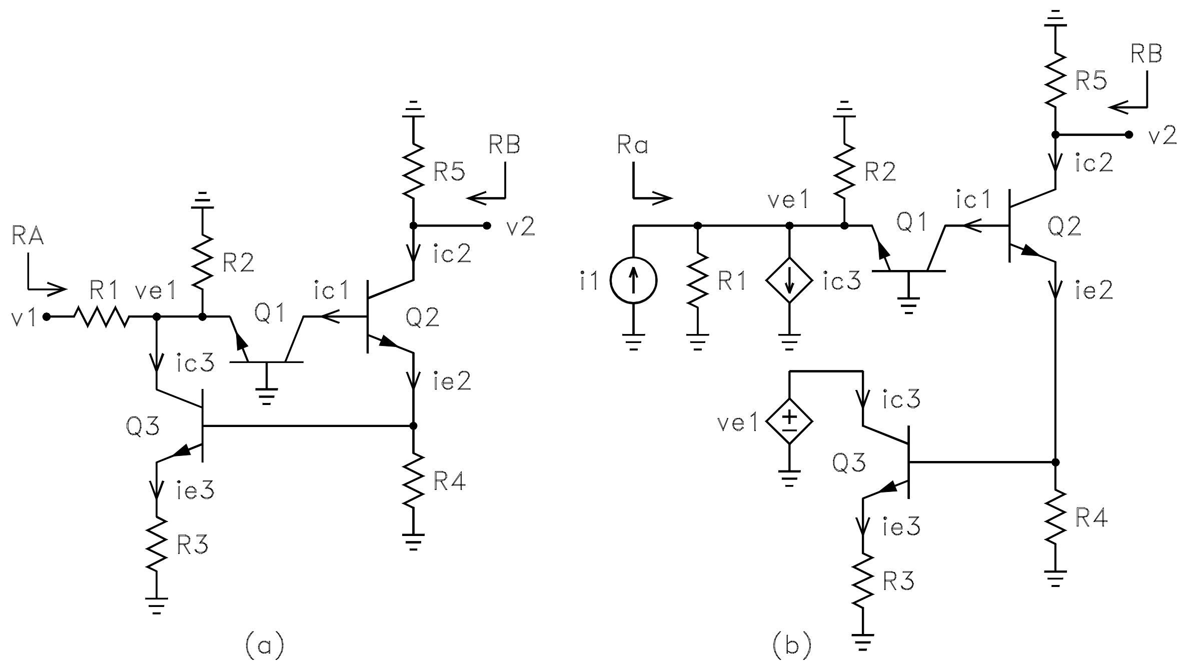

Figure 36: (a) Amplifier circuit. (b) Circuit with feedback removed.

For the circuit with feedback removed, we can write

$$i_a = i_1 - i_{c3} \qquad v_{e1} = i_a R_b \qquad R_b = R_1 \| R_2 \| r_{e1} \qquad i_{c1} = -g_{m1} v_{e1} \qquad i_{c2} = -\beta_2 i_{c1} \qquad i_{e2} = \frac{i_{e2}}{\alpha_2}$$

$$v_{tb3} = i_{e2} R_4 \qquad i_{e3} = G_1 v_{tb3} \qquad G_1 = \frac{1}{r'_{e3} + R_3} \qquad r'_{e3} = \frac{R_4}{1 + \beta_3} + r_{e3} \qquad i_{c3} = \alpha_3 i_{e3}$$

The equations can be solved algebraically or by a flow graph. The flow graph for the equations is shown in Fig. 37. The determinant is

$$\begin{aligned} \Delta &= 1 - \left(R_b \times -g_{m1} \times -\beta_1 \times \frac{1}{\alpha_2} \times R_4 \times G_1 \times \alpha_3 \times -1 \right) \\ &= 1 + R_b \times g_{m1} \times \beta_1 \times \frac{1}{\alpha_2} \times R_4 \times G_1 \times \alpha_3 \\ &= 858.7 \end{aligned}$$

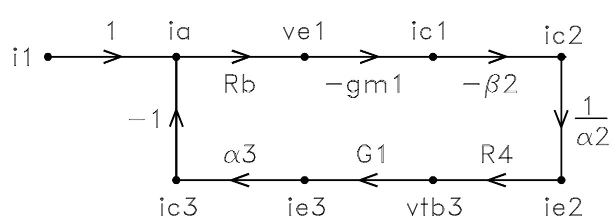

Figure 37: Signal-flow graph for the equations.

The transconductance gain is

$$\begin{aligned}\frac{i_{c2}}{i_1} &= \frac{1 \times R_b \times -g_{m1} \times -\beta_2}{\Delta} \\ &= \frac{R_b \times g_{m1} \times \beta_2}{1 + R_b \times g_{m1} \times \beta_2 \times \dfrac{1}{\alpha_2} \times R_4 \times G_1 \times \alpha_3} \\ &= \frac{(R_b \times g_{m1} \times \beta_2)}{1 + (R_b \times g_{m1} \times \beta_2) \times \left(\dfrac{1}{\alpha_2} \times R_4 \times G_1 \times \alpha_3\right)}\end{aligned}$$

This is of the form

$$\frac{i_{c2}}{i_1} = \frac{A}{1 + Ab}$$

where A and b are given by

$$\begin{aligned}A &= R_b \times g_{m1} \times \beta_2 \\ &= R_1 \| R_2 \| r_{e1} \times g_{m1} \times \beta_2 \\ &= 96.39\end{aligned}$$

$$b = \frac{1}{\alpha_2} \times R_4 \times G_1 \times \alpha_3 = \frac{1}{\alpha_2} \times R_4 \times \frac{1}{r'_{e3} + R_3} \times \alpha_3 = 8.899$$

Notice that the product Ab is dimensionless and positive. The latter must be true for the feedback to be negative.

Numerical evaluation of the transconductance gain yields

$$\frac{i_{c2}}{i_1} = \frac{A}{\Delta} = 0.112$$

The voltage gain is given by

$$\frac{v_2}{v_1} = \frac{i_1}{v_1} \times \frac{i_{c2}}{i_1} \times \frac{v_2}{i_{c2}} = \frac{1}{R_1} \times \frac{A}{\Delta} \times -R_5 = -1.122$$

The resistances R_a, R_A, and R_B are given by

$$R_a = \frac{R_b}{\Delta} = 0.028\,\Omega \qquad R_A = R_1 + \left(R_a^{-1} - R_1^{-1}\right)^{-1} = 1\,\mathrm{k}\Omega \qquad R_B = R_5 = 10\,\mathrm{k}\Omega$$

Shunt-Series Example 4

Figure 38(a) shows the ac signal circuit of a shunt-series feedback amplifier. The input variable is v_1 and the output variable is i_{d2}. The input signal and the feedback signal are applied to the gate of M_1. For the analysis to follow convention, the input source consisting of v_1 in series with R_1 must be converted into a Norton equivalent. The feedback is from the output current i_{d2} to the source of M_2 and to the gate of M_1. The circuit values are $g_m = 0.001\,\mathrm{S}$, $r_s = g_m^{-1} = 1\,\mathrm{k}\Omega$, $r_0 = \infty$, $R_1 = 1\,\mathrm{k}\Omega$, $R_2 = 100\,\mathrm{k}\Omega$, $R_3 = 10\,\mathrm{k}\Omega$, $R_4 = 1\,\mathrm{k}\Omega$, $R_5 = 1\,\mathrm{k}\Omega$, and $R_6 = 100\,\Omega$.

The circuit with feedback removed is shown in Fig. 38(b). The source is replaced with a Norton equivalent circuit consisting of the current

$$i_1 = \frac{v_1}{R_1}$$

in parallel with the resistor R_1. The feedback is modeled by a Norton equivalent circuit consisting of the current $k_1 i_{d2}$. The output current is proportional to this current. Because $r_{02} = \infty$, the feedback does not affect the output resistance seen looking up from signal ground into the lower terminal of R_3 because it is infinite. For a finite r_{02}, a test voltage source can be added in series with R_3 to solve for this resistance. It would be found that a finite r_{02} considerably complicates the circuit equations and the flow graph.

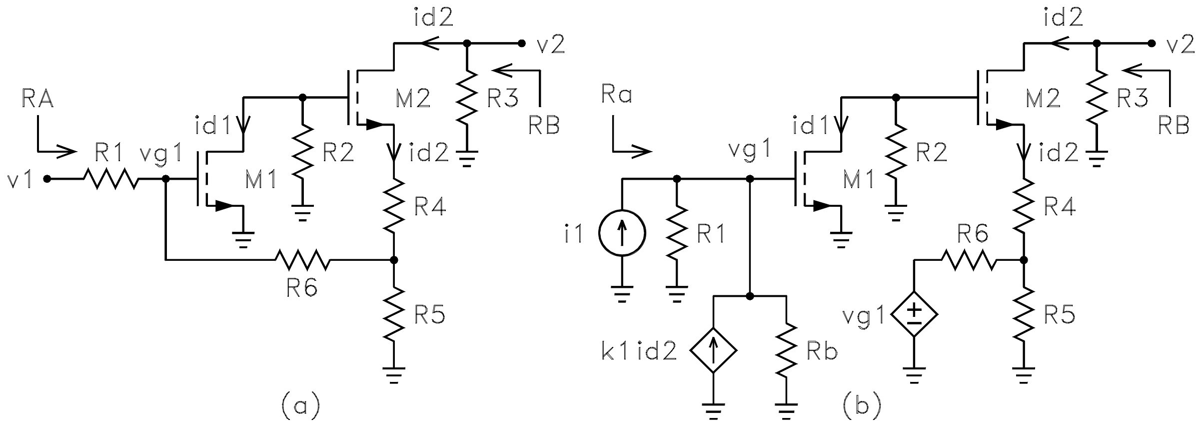

Figure 38: (a) Amplifier circuit. (b) Circuit with feedback removed.

The following equations can be written for the circuit with feedback removed:

$$i_a = i_1 + k_1 i_{d2} \qquad k_1 = \frac{R_5}{R_5 + R_6} \qquad v_{g1} = i_a R_c \qquad R_c = R_1 \| R_b \qquad R_b = R_5 + R_6$$

$$i_{d1} = g_{m1} v_{g1} \qquad i_{d2} = G_1 \left(v_{tg2} - v_{ts2}\right) \qquad v_{tg2} = -i_{d1} R_2 \qquad v_{ts2} = k_2 v_{g1}$$

$$k_2 = \frac{R_5}{R_5 + R_6} \qquad G_1 = \frac{1}{r_{s2} + R_{ts2}} \qquad R_{ts2} = R_4 + R_5 \| R_6$$

The current i_a is the error current. The negative feedback tends to reduce i_a, making $|i_a| \to 0$ as the amount of feedback becomes infinite. When this is the case, setting $i_a = 0$ yields the current gain $i_{d2}/i_1 = -1/k_1$.

Although the equations can be solved algebraically, the signal-flow graph simplifies the solution. Fig. 39 shows the flow graph for the equations. The determinant of the graph is given by

$$\begin{aligned} \Delta &= 1 - (R_c \times g_{m1} \times -R_2 \times G_1 \times k_1) \\ &= 1 + R_c \times g_{m1} \times R_2 \times G_1 \times k_1 \end{aligned}$$

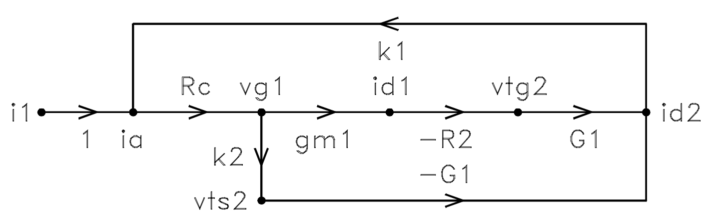

Figure 39: Signal-flow graph for the equations.

The current gain is given by

$$\begin{aligned} \frac{i_{d2}}{i_1} &= \frac{R_c \times g_{m1} \times -R_2 \times G_1}{\Delta} \\ &= \frac{-\left(R_c \times g_{m1} \times R_2 \times \dfrac{1}{r_{s2} + R_4 + R_5 \| R_6}\right)}{1 + \left[-\left(R_c \times g_{m1} \times R_2 \times \dfrac{1}{r_{s2} + R_4 + R_5 \| R_6}\right)\right] \times (-k_1)} \end{aligned}$$

This is of the form

$$\frac{i_{d2}}{i_1} = \frac{A}{1 + Ab}$$

where

$$A = -\left(R_c \times g_{m1} \times R_2 \times \frac{1}{r_{s2} + R_4 + R_5 \| R_6}\right) = -25.02$$

$$b = -k_1 = -0.909$$

Note that Ab is dimensionless. Numerical evaluation yields

$$\frac{i_{d2}}{i_1} = \frac{-25.02}{1 + (-25.02) \times (-0.909)} = -1.054$$

The voltage gain is given by

$$\frac{v_2}{v_1} = \frac{i_{d2}}{i_1} \times \frac{i_1}{v_1} \times \frac{v_2}{i_{d2}} = \frac{i_{d2}}{i_1} \times \frac{1}{R_1} \times -R_3 = -10.54$$

The resistance R_a is

$$R_a = \frac{v_{s1}}{i_1} = \frac{R_c}{\Delta} = \frac{R_1 \| (R_5 + R_6)}{\Delta} = 22.03\,\Omega$$

Note that the feedback tends to decrease R_a. The resistance R_A is

$$R_A = R_1 + \left(R_a^{-1} - R_1^{-1}\right)^{-1} = 1.023\,\text{k}\Omega$$

The resistance R_B is

$$R_B = R_3 = 10\,\text{k}\Omega$$

This is not a function of the feedback because r_{02} has been assumed to be infinite.

Shunt-Series Example 5

Figure 40(a) shows the ac signal circuit of a shunt-series feedback amplifier. The input variable is v_1 and the output variable is i_{d2}. The input signal and the feedback signal are applied to the base Q_1. For the analysis to follow convention, the input source consisting of v_1 in series with R_1 must be converted into a Norton equivalent. The feedback is from the output current i_{c2} to the current i_{e2} to the current i_{e3} to the current i_{c3}. The resistor values are $R_1 = 1\,\text{k}\Omega$, $R_2 = 10\,\text{k}\Omega$, $R_3 = 10\,\text{k}\Omega$, and $R_4 = 10\,\text{k}\Omega$. Assume $\beta = 100$, $r_\pi = 2.5\,\text{k}\Omega$, $\alpha = \beta/(1+\beta)$, $r_e = \alpha/g_m$, $r_0 = \infty$, $r_x = 0$, $V_T = 25\,\text{mV}$.

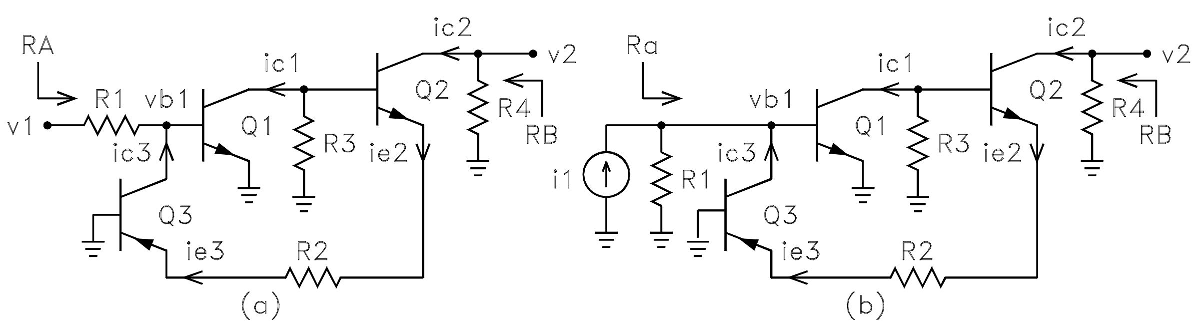

Figure 40: (a) Amplifier circuit. (b) Circuit with the the source replaced with a Norton equivalent.

The circuit with feedback removed is shown in Fig. 40(b). The source is replaced with a Norton equivalent circuit consisting of the current

$$i_1 = \frac{v_1}{R_1}$$

in parallel with the resistor R_1. The feedback is modeled by a Norton equivalent circuit consisting of the current i_{c3}. The output current is proportional to this current. Because $r_{02} = \infty$, the feedback does not affect the output resistance seen looking up from signal ground into the lower terminal of R_4 because it is infinite. For a finite r_{02}, a test voltage source can be added in series with R_4 to solve for this resistance. It would be found that a finite r_{02} considerably complicates the circuit equations and the flow graph.

The following equations can be written for the circuit with feedback removed:

$$i_a = i_1 + i_{c3} \qquad v_{b1} = i_a R_b \qquad R_b = R_1 \| r_{\pi 1} \qquad i_{c1} = g_{m1} v_{b1} \qquad v_{tb2} = -i_{c1} R_3 \qquad i_{e2} = G_1 v_{tb2}$$

$$G_1 = \frac{1}{r'_{e2} + R_2 + r_{e3}} \qquad r'_{e2} = \frac{R_3}{1+\beta_2} + r_{e2} \qquad i_{c2} = \alpha_2 i_{e2} \qquad i_{e3} = i_{e2} \qquad i_{c3} = \alpha_3 i_{e3}$$

The current i_a is the error current. The negative feedback tends to reduce i_a, making $|i_a| \to 0$ as the amount of feedback becomes infinite. When this is the case, setting $i_a = 0$ yields the current gain $i_{d2}/i_1 = -1/k_1$.

Although the equations can be solved algebraically, the signal-flow graph simplifies the solution. Fig. 41 shows the flow graph for the equations. The determinant of the graph is given by

$$\begin{aligned} \Delta &= 1 - (R_b \times g_{m1} \times -R_3 \times G_1 \times 1 \times \alpha_3 \times 1) \\ &= 1 + R_b \times g_{m1} \times R_3 \times G_1 \times \alpha_3 \\ &= 28.88 \end{aligned}$$

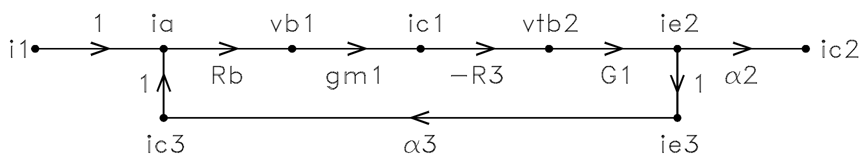

Figure 41: Signal-flow graph for the equations.

The current gain is given by

$$\begin{aligned} \frac{i_{d2}}{i_1} &= \frac{1 \times R_b \times g_{m1} \times -R_3 \times G_1 \times \alpha_2}{\Delta} \\ &= \frac{-(R_b \times g_{m1} \times R_3 \times G_1 \times \alpha_2)}{1 + [-(R_b \times g_{m1} \times R_3 \times G_1 \times \alpha_2)] \times \left(-\dfrac{\alpha_3}{\alpha_2}\right)} \end{aligned}$$

This is of the form

$$\frac{i_{d2}}{i_1} = \frac{A}{1+Ab}$$

where

$$A = -(R_b \times g_{m1} \times R_3 \times G_1 \times \alpha_2) = -27.88$$

$$b = -\frac{\alpha_3}{\alpha_2} = -1$$

Note that Ab is dimensionless and the product is positive. The latter is a result of the feedback being negative. Numerical evaluation yields

$$\frac{i_{d2}}{i_1} = \frac{-27.88}{1 + (-27.88) \times (-1)} = -0.965$$

The voltage gain is given by

$$\frac{v_2}{v_1} = \frac{i_{d2}}{i_1} \times \frac{i_1}{v_1} \times \frac{v_2}{i_{d2}} = \frac{i_{d2}}{i_1} \times \frac{1}{R_1} \times -R_4 = 9.654$$

The resistance R_a is

$$R_a = \frac{v_{s1}}{i_1} = \frac{R_b}{\Delta} = \frac{R_1 \| r_{\pi 1}}{\Delta} = 24.74\,\Omega$$

Note that the feedback tends to decrease R_a. The resistance R_A is

$$R_A = R_1 + \left(R_a^{-1} - R_1^{-1}\right)^{-1} = 1.025\,\mathrm{k}\Omega$$

The resistance R_B is

$$R_B = R_4 = 10\,\mathrm{k}\Omega$$

This is not a function of the feedback because r_{02} has been assumed to be infinite.