The BJT Differential Amplifier
ECE 3050 Analog Electronics

The differential amplifier or diff amp is used in applications where it is desired to have an output voltage that is proportional to the difference between two input voltages. Fig. 1(a) shows the basic circuit diagram. The tail supply is modeled as a current source I'_Q having an output resistance R_Q. In the case of an ideal current source, R_Q is an open circuit. Often a diff amp is designed with a resistive tail supply. In this case, $I'_Q = 0$. There are two outputs shown. Either or both can be used. Often the difference voltage between the two outputs is used.

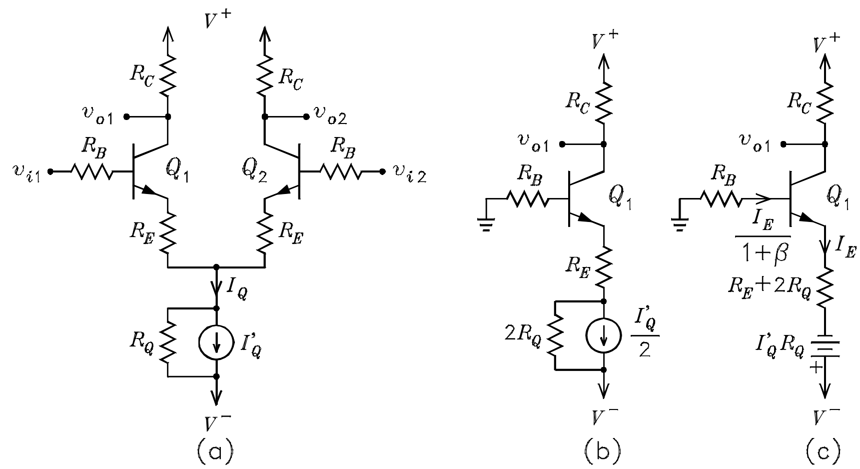

Figure 1: (a) Circuit diagram of the differential amplifier. (b) First equivalent bias circuit for Q_1. (b) Second equivalent bias circuit for Q_1.

The dc bias circuit is obtained by setting $v_{i1} = v_{i2} = 0$. The tail supply can be divided into two parallel current sources of value $I'_Q/2$, each having an output resistance $2R_Q$. By symmetry, no dc current flows between the two sides of the circuit so that the two sides can be separated. The circuit obtained for Q_1 is shown in Fig. 1(b). The circuit for Q_2 is identical. The circuit shown in Fig. 1(c) is obtained by making a Thévenin equivalent of the tail supply in Fig. 1(b). The bias equation for I_E is

$$0 - \left(V^- - I'_Q R_Q\right) = \frac{I_E}{1+\beta} R_B + V_{BE} + I_E \left(R_E + 2R_Q\right)$$

This can be solved I_E to obtain

$$I_E = \frac{-V^- + I'_Q R_Q - V_{BE}}{R_B/\left(1+\beta\right) + R_E + 2R_Q}$$

The dc collector-to-base voltage is given by

$$V_{CB} = V_C - V_B = \left(V^+ - \alpha I_E R_C\right) - \left(-\frac{I_E}{1+\beta} R_B\right) = V^+ - \alpha I_E R_C + \frac{I_E}{1+\beta} R_B$$

This must be greater than zero for the two BJTs to be biased in the active mode. The collector to emitter voltage is given by

$$V_{CE} = V_C - V_E = V_C - \left(V_B - V_{BE}\right) = V_{CB} + V_{BE}$$

It follows that r_e, r'_e, and r_0 for each transistor are given by

$$r_e = \frac{V_T}{I_E} \qquad r'_e = \frac{R_B + r_x}{1+\beta} + r_e \qquad r_0 = \frac{V_A + V_{CE}}{\alpha I_E}$$

To solve for the small-signal value of v_{o1}, we zero V^+, V^-, and I'_Q to form the ac signal circuit. Then the circuit seen looking out of the emitter of Q_1 is replaced by a Thévenin equivalent circuit. To obtain this, we first replace the circuit seen looking into the emitter of Q_2 with a Thévenin equivalent circuit. This circuit is shown in Fig. 2(a), where

$$v_{e2(oc)} = v_{i2} \frac{r_0 + R_C/(1+\beta)}{r'_e + r_0 + R_C/(1+\beta)} \qquad r_{ie2} = r'_e \frac{r_0 + R_C}{r'_e + r_0 + R_C/(1+\beta)}$$

The Thévenin voltage and resistance seen looking out of the emitter of Q_1 are given by

$$v_{te1} = v_{e2(oc)} \frac{R_Q}{R_Q + R_E + r_{ie2}} \qquad R_{te} = R_E + R_Q \| (R_E + r_{ie2})$$

The new circuit is shown in Fig. 2(b).

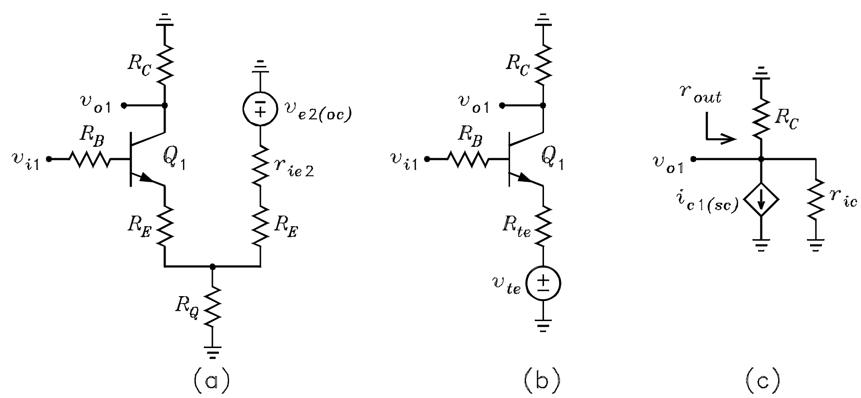

Figure 2: (a) Ac signal circuit for Q_1.(b) Circuit for calculating v_{o1} and r_{out}.

The circuit for calculating v_{o1} and r_{out} is shown in Fig. 2(c). We can write

$$v_{o1} = -i_{c1(sc)} \times r_{ic} \| R_C = (G_{mb} v_{i1} - G_{me} v_{te1}) \times r_{ic} \| R_C \qquad r_{out} = r_{ic} \| R_C$$

where

$$G_{mb} = \frac{\alpha}{r'_e + R_{te} \| r_0} \frac{r_0 - R_{te}/\beta}{r_0 + R_{te}} \qquad G_{me} = \frac{1}{R_{te} + r'_e \| r_0} \frac{\alpha r_0 + r'_e}{r_0 + r'_e} \qquad r_{ic} = \frac{r_0 + r'_e \| R_{te}}{1 - \alpha R_{te}/(r'_e + R_{te})}$$

When the above results are combined, we obtain

$$v_{o1} = -A_{v1} v_{i1} + A_{v2} v_{i2} = -A_{v1} \left(v_{i1} - \frac{A_{v2}}{A_{v1}} v_{i2} \right)$$

where A_{v1} and A_{v2} are the voltage gains given by

$$A_{v1} = G_{mb} \times r_{ic} \| R_C \qquad A_{v2} = G_{me} \times r_{ic} \| R_C \times \frac{R_Q}{R_Q + R_E + r_{ie2}} \frac{r_0 + R_C/(1+\beta)}{r'_e + r_0 + R_C/(1+\beta)}$$

By symmetry, v_{o2} is given by

$$v_{o2} = -v_{o1} = -A_{v1}\left(v_{i2} - \frac{A_{v2}}{A_{v1}}v_{i1}\right)$$

We see that the gains for v_{i1} and v_{i2} differ by the ratio A_{v2}/A_{v1}. This is given by

$$\frac{A_{v2}}{A_{v1}} = \frac{G_{me}}{G_{mb}} \frac{R_Q}{R_Q + R_E + r_{ie2}} \frac{r_0 + R_C/(1+\beta)}{r'_e + r_0 + R_C/(1+\beta)}$$

If this ratio is unity, v_{o1} and v_{o2} are proportional to the difference between the two input voltages. It can be seen that the ratio is exactly unity only if $R_Q \to \infty$ and $r_0 \to \infty$.

Let v_o be the differential output voltage. This is given by

$$v_o = v_{o1} - v_{o2} = -(A_{v1} + A_{v2})(v_{i1} - v_{i2}) = -A_{v1}\left(1 + \frac{A_{v2}}{A_{v1}}\right)(v_{i1} - v_{i2})$$

This is proportional to the difference between the two input voltages even if $R_Q < \infty$ and $r_0 < \infty$.

The r_0 approximations to the gains are obtained by letting $r_0 \to \infty$ except in the expression for r_{ic}. We obtain

$$A_{v1} \simeq G_m \times r_{ic} \| R_C \qquad A_{v2} = G_m \times r_{ic} \| R_C \times \frac{R_Q}{R_Q + R_E + r'_e}$$

where

$$G_m = \frac{\alpha}{r'_e + R_{te}} \qquad R_{te} = R_E + R_Q \| \left(R_E + r'_e\right)$$

The expression for r_{out} is the same except r_{ic} is calculated with $R_{te} = R_E + R_Q \| (R_E + r'_e)$.

The equivalent circuit seen looking into the base of Q_1 consists of the resistor r_{ib} in series with the voltage $v_{b1(oc)}$. These are given by

$$r_{ib} = r_x + (1+\beta) r_e + R_{te}\frac{(1+\beta) r_0 + R_C}{r_0 + R_{te} + R_C}$$

$$v_{b1(oc)} = v_{te1}\frac{r_0 + R_C}{R_{te} + r_0 + R_C} = v_{i2}\frac{r_0 + R_C/(1+\beta)}{r'_e + r_0 + R_C/(1+\beta)} \frac{r_0 + R_C}{R_{te} + r_0 + R_C}$$

The equivalent circuit is shown in Fig. 3(a). The circuit for Q_2 is shown in Fig. 3(b), where $v_{b2(oc)}$ is given by

$$v_{b2(oc)} = v_{te2}\frac{r_0 + R_C}{R_{te} + r_0 + R_C} = v_{i1}\frac{r_0 + R_C/(1+\beta)}{r'_e + r_0 + R_C/(1+\beta)} \frac{r_0 + R_C}{R_{te} + r_0 + R_C}$$

The two base equivalent circuits can be used to calculate the base currents i_{b1} and i_{b2}.

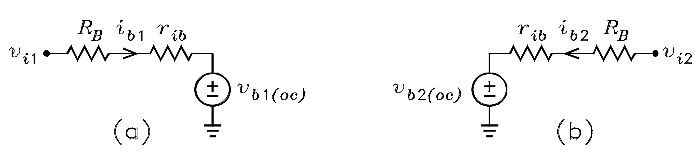

Figure 3: (a) Equivalent circuit for i_{b1}. (b) Equivalent circuit for i_{b2}.

Example 1 *It is given that* $I_Q' = 2\,\text{mA}$, $R_Q = 50\,\text{k}\Omega$, $R_B = 1\,\text{k}\Omega$, $R_E = 100\,\Omega$, $R_C = 10\,\text{k}\Omega$, $V^+ = 20\,\text{V}$, $V^- = -20\,\text{V}$, $V_T = 0.025\,\text{V}$, $r_x = 20\,\Omega$, $\beta = 99$, $V_{BE} = 0.65\,\text{V}$, *and* $V_A = 50\,\text{V}$. *Calculate the small-signal values for* v_{o1} *and* v_{o2}*, the output resistance* r_{out}*, and the equivalent input circuit seen by each source.*

Solution. First, we solve for the dc emitter current in each transistor. It is

$$I_E = \frac{-V^- + I_Q' R_Q - V_{BE}}{R_B/(1+\beta) + R_E + 2R_Q} = 1.192\,\text{mA}$$

To verify that the transistors are in the active mode, we calculate the collector-to-base voltage

$$V_{CB} = V^+ - \alpha I_E R_C + \frac{I_E}{1+\beta} R_B = 8.209\,\text{V}$$

Because this is greater than zero, both BJTs are in the active mode. The collector-to-emitter voltage is $V_{CE} = V_{CB} + V_{BE} = 8.859\,\text{V}$. Thus the resistances r_e, r_e', and r_0 are

$$r_e = \frac{V_T}{I_E} = 20.97\,\Omega \qquad r_e' = \frac{R_B + r_x}{1+\beta} + r_e = 31.17\,\Omega \qquad r_0 = \frac{V_A + V_{CE}}{\alpha I_E} = 49.87\,\text{k}\Omega$$

The resistances r_{ie} and R_{te} are

$$r_{ie} = r_e' \frac{r_0 + R_C}{r_e' + r_0 + R_C/(1+\beta)} = 37.32\,\Omega \qquad R_{te} = R_E + R_Q \| (R_E + r_{ie}) = 236.9\,\Omega$$

The transconductances G_{mb} and G_{me} and the resistance r_{ic} are

$$G_{mb} = \frac{\alpha}{r_e' + R_{te}\|r_0} \frac{r_0 - R_{te}/\beta}{r_0 + R_{te}} = \frac{1}{271}\,\text{S} \qquad G_{me} = \frac{1}{R_{te} + r_e'\|r_0} \frac{\alpha r_0 + r_e'}{r_0 + r_e'} = \frac{1}{270.8}\,\text{S}$$

$$r_{ic} = \frac{r_0 + r_e'\|R_{te}}{1 - \alpha R_{te}/(r_e' + R_{te})} = 398.9\,\Omega$$

It follows that the gains A_{v1} and A_{v2} are

$$A_{v1} = G_{mb} \times r_{ic}\|R_C = 36$$

$$A_{v2} = G_{me} \times r_{ic}\|R_C \times \frac{R_Q}{R_Q + R_E + r_{ie2}} \frac{r_0 + R_C/(1+\beta)}{r_e' + r_0 + R_C/(1+\beta)} = 35.91$$

Thus v_{o1} and v_{o2} are

$$v_{o1} = -36 v_{i1} + 35.91 v_{i2} \qquad v_{o2} = -36 v_{i2} + 35.91 v_{i1}$$

The differential output voltage is

$$v_o = v_{o1} - v_{o2} = 71.91\,(v_{i1} - v_{i2})$$

The output resistance is

$$r_{out} = r_{ic}\|R_C = 9.755\,\text{k}\Omega$$

In the equivalent input circuits, r_{ib}, $v_{b1(oc)}$, and $v_{b2(oc)}$ are

$$r_{ib} = r_x + (1+\beta)\, r_e + R_{te} \frac{(1+\beta)\, r_0 + R_C}{r_0 + R_{te} + R_C} = 19.82\,\text{k}\Omega$$

$$v_{b1(oc)} = v_{i2} \frac{r_0 + R_C/(1+\beta)}{r_e' + r_0 + R_C/(1+\beta)} \frac{r_0 + R_C}{R_{te} + r_0 + R_C} = 0.9953 v_{i2}$$

$$v_{b2(oc)} = v_{i1} \frac{r_0 + R_C/(1+\beta)}{r_e' + r_0 + R_C/(1+\beta)} \frac{r_0 + R_C}{R_{te} + r_0 + R_C} = 0.9953 v_{i1}$$