Chapter 2
Characteristics of Physical Op-Amps

In the preceding chapter, the op-amp is treated as an ideal circuit element. Because the waveform of the output voltage from a physical op-amp is never exactly the same as the waveform that would be expected from an ideal op-amp, this assumption is never true in practice. Although most op-amp circuits can be designed by assuming that the op-amps are ideal, the circuits never perform exactly as predicted because of the non-ideal characteristics of the op-amps. Some of these characteristics are discussed in this chapter. In addition, a linear controlled-source model of the op-amp is developed which can be used in computer simulation programs such as SPICE.

2.1 Effects of Finite Gain and Bandwidth

2.1.1 Open-Loop Transfer Function

In our analysis of op-amp circuits this far, we have considered the op-amps to have an infinite gain and an infinite bandwidth. This is not true for physical op-amps. In this section, we examine the effects of a non-infinite gain and non-infinite bandwidth on the inverting and the non-inverting amplifier circuits. Fig. 2.1 shows the circuit symbol of an op-amp having an open-loop voltage-gain transfer function $A(s)$. The output voltage is given by

$$V_o = A(s)(V_+ - V_-) \tag{2.1}$$

where complex variable notation is used. We assume here that $A(s)$ can be modeled by a single-pole low-pass transfer function of the form

$$A(s) = \frac{A_0}{1 + s/\omega_0} \tag{2.2}$$

where A_0 is the dc gain constant and ω_0 is the pole frequency. Most general purpose op-amps have a voltage-gain transfer function of this form for frequencies such that $|A(j\omega)| \geq 1$.

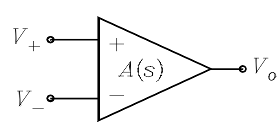

Figure 2.1: Op-amp symbol.

2.1.2 Gain-Bandwidth Product

Figure 2.2 shows the Bode magnitude plot for $A(j\omega)$. The radian gain-bandwidth product is defined as the frequency ω_x for which $|A(j\omega)| = 1$. It is given by

$$\omega_x = \omega_0\sqrt{A_0^2 - 1} \simeq A_0\omega_0 \tag{2.3}$$

where we assume that $A_0 >> 1$. This equation illustrates why ω_x is called a gain-bandwidth product. It is given by the product of the dc gain constant A_0 and the radian bandwidth ω_0. It is commonly specified in Hz with the symbol f_x, where $f_x = \omega_x/2\pi$. Many general purpose op-amps have a gain-bandwidth product $f_x \simeq 1\,\text{MHz}$ and a dc gain constant $A_0 \simeq 2\times10^5$. It follows from Eq. (2.3) that the corresponding pole frequency in the voltage-gain transfer function for the general purpose op-amp is $f_0 \simeq 1\times10^6/\left(2\times10^5\right) = 5\,\text{Hz}$.

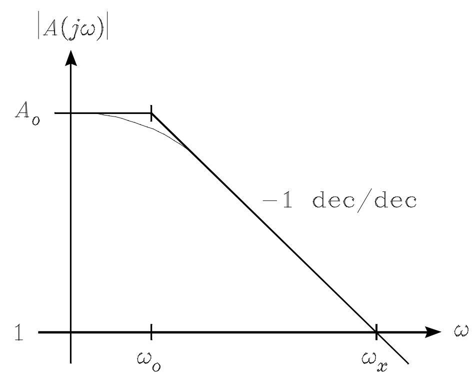

Figure 2.2: Bode plot of $|A(j\omega)|$.

2.1.3 Non-Inverting Amplifier

Figure 2.3 shows the circuit diagram of a non-inverting amplifier. For this circuit, we can write by inspection

$$V_o = A(s)(V_i - V_-) \tag{2.4}$$

$$V_- = V_o\frac{R_1}{R_1 + R_F} \tag{2.5}$$

Simultaneous solution for the voltage-gain transfer function yields

$$\frac{V_o}{V_i} = \frac{A(s)}{1 + A(s)R_1/(R_1+R_F)} = \frac{1 + R_F/R_1}{1 + (1 + R_F/R_1)/A(s)} \tag{2.6}$$

For $s = j\omega$ and $|(1 + R_F/R_1)/A(j\omega)| << 1$, this reduces to $V_o/V_i \simeq (1 + R_F/R_1)$. This is the gain which would be predicted if the op-amp is assumed to be ideal.

When Eq. (2.2) is used for $A(s)$, it is straightforward to show that Eq. (2.6) can be written

$$\frac{V_o}{V_i} = \frac{A_{0f}}{1 + s/\omega_{0f}} \tag{2.7}$$

where A_{0f} is the gain constant with feedback and ω_{0f} is the radian pole frequency with feedback. These are given by

$$A_{0f} = \frac{A_0}{1 + A_0R_1/(R_1+R_F)} = \frac{1 + R_F/R_1}{1 + (1 + R_F/R_1)/A_0} \tag{2.8}$$

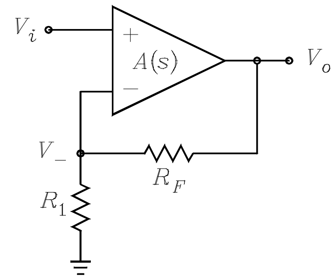

Figure 2.3: Non-inverting amplifier.

$$\omega_{0f} = \omega_0 \left(1 + \frac{A_0 R_1}{R_1 + R_F}\right) \tag{2.9}$$

It follows from these two equations that the radian gain-bandwidth product of the non-inverting amplifier with feedback is given by $A_{0f}\omega_{0f} = A_0\omega_0 = \omega_x$. This is the same as for the op-amp without feedback. Fig. 2.4 shows the Bode magnitude plots for both V_o/V_i and $A(j\omega)$. The figure shows that the break frequency on the plot for V_o/V_i lies on the negative-slope asymptote of the plot for $A(j\omega)$.

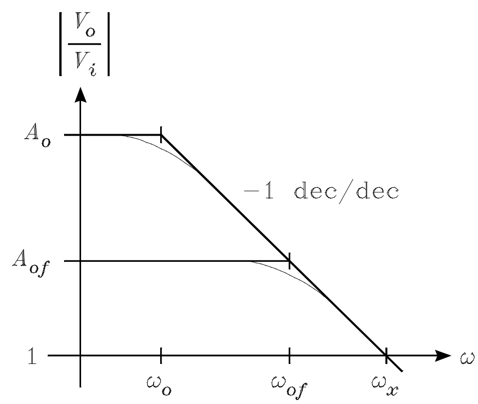

Figure 2.4: Bode plot for $|V_o/V_i|$.

Example 1 *At very low frequencies, an op-amp has the frequency independent open-loop gain* $A(s) = A_0 = 2 \times 10^5$*. The op-amp is to be used in a non-inverting amplifier. The theoretical gain is calculated assuming that the op-amp is ideal. What is the highest theoretical gain that gives an error between the theoretical gain and the actual gain that is less than* 1%*?*

Solution. The theoretical gain is given by $(1 + R_F/R_1)$. The actual gain is always less than the theoretical gain. For an error less than 1%, we can use Eq. (2.6) to write

$$1 - 0.01 < \frac{1}{1 + (1 + R_F/R_1)/(2 \times 10^5)}$$

This can be solved for the upper bound on the theoretical gain to obtain

$$1 + \frac{R_F}{R_1} < 2 \times 10^5 \left(\frac{1}{0.99} - 1\right) = 2020$$

Example 2 *An op-amp has a gain-bandwidth product of* 1 MHz. *The op-amp is to be used in a non-inverting amplifier circuit. Calculate the highest gain that the amplifier can have if the half-power or* -3 dB *bandwidth is to be* 20 kHz *or more.*

Solution. The minimum bandwidth occurs at the highest gain. For a bandwidth of 20 kHz, we can write $A_{0f} \times 20 \times 10^3 = 10^6$. Solution for A_{0f} yields $A_{0f} = 50$.

Example 3 *Two non-inverting op-amp amplifiers are operated in cascade. Each amplifier has a gain of* 10. *If each op-amp has a gain-bandwidth product of* 1 MHz, *calculate the half-power or* -3 dB *bandwidth of the cascade amplifier.*

Solution. Each amplifier by itself has a pole frequency of $10^6/10 = 100$ kHz, corresponding to a radian frequency $\omega_{0f} = 2\pi \times 100,000$. The cascade combination has the voltage-gain transfer function given by

$$\frac{V_o}{V_i} = 100\left(\frac{1}{1 + s/\omega_{0f}}\right)^2$$

The half-power frequency is obtained by setting $s = j\omega$ and solving for the frequency for which $|V_o/V_i|^2 = 100^2/2$. If we let $x = \omega/\omega_{0f}$, the resulting equation is

$$100^2\left(\frac{1}{1 + x^2}\right)^2 = \frac{100^2}{2}$$

This equation reduces to $1 + x^2 = \sqrt{2}$. Solution for x yields $x = 0.644$. It follows that the half-power frequency is 0.644×100 kHz $= 64.4$ kHz.

2.1.4 Inverting Amplifier

Figure 2.5(a) shows the circuit diagram of an inverting amplifier. Fig. 2.5(b) shows an equivalent circuit which can be used to solve for V_-. By inspection, we can write

$$V_o = -A(s)\,V_- \tag{2.10}$$

$$V_- = \left(\frac{V_i}{R_1} + \frac{V_o}{R_F}\right)(R_1 \| R_F) \tag{2.11}$$

These equations can be solved for the voltage-gain transfer function to obtain

$$\frac{V_o}{V_i} = \frac{-(1/R_1)\,A(s)\,(R_1\|R_F)}{1 + (1/R_F)\,A(s)\,(R_1\|R_F)} = \frac{-R_F/R_1}{1 + (1 + R_F/R_1)/A(s)} \tag{2.12}$$

For $s = j\omega$ and $|(1 + R_F/R_1)/A(j\omega)| << 1$, the voltage-gain transfer function reduces to $V_o/V_i \simeq -R_F/R_1$. This is the gain which would be predicted if the op-amp is assumed to be ideal.

When Eq. (2.2) is used for $A(s)$, it is straightforward to show that the voltage-gain transfer function reduces to

$$\frac{V_o}{V_i} = \frac{-A_{0f}}{1 + s/\omega_{0f}} \tag{2.13}$$

where A_{0f} is the gain constant with feedback and ω_{0f} is the radian pole frequency with feedback. These are given by

$$A_{0f} = \frac{(1/R_1)\,A_0\,(R_1\|R_F)}{1 + (1/R_F)\,A_0\,(R_1\|R_F)} = \frac{R_F/R_1}{1 + (1 + R_F/R_1)/A_0} \tag{2.14}$$

$$\omega_{0f} = \omega_0\left(1 + A_0\frac{R_1\|R_F}{R_F}\right) = \omega_0\left(1 + A_0\frac{R_1}{R_1 + R_F}\right) \tag{2.15}$$

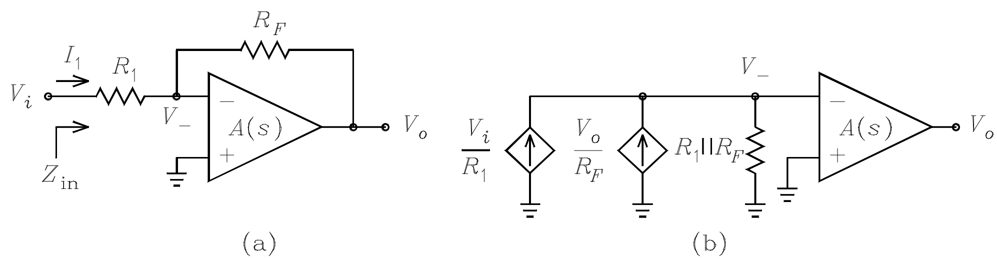

Figure 2.5: (a) Inverting amplifier. (b) Equivalent circuit for calculating V_o.

Note that A_{0f} is defined here as a positive quantity so that the negative sign for the inverting gain is retained in the transfer function for V_o/V_i.

Let the radian gain-bandwidth product of the inverting amplifier with feedback be denoted by ω_x'. It follows from Eqs. (2.14) and (2.15) that this is given by $\omega_x' = A_{0f}\omega_{0f} = \omega_x R_F / (R_F + R_1)$. This is less than the gain-bandwidth product of the op-amp without feedback by the factor $R_F / (R_F + R_1)$. Fig. 2.6 shows the Bode magnitude plots for V_o/V_i and for $A(j\omega)$. The frequency labeled ω_{0f}' is the break frequency for the non-inverting amplifier with the same gain magnitude as the inverting amplifier. The non-inverting amplifier with the same gain has a bandwidth that is greater by the factor $(1 + R_1/R_F)$. The bandwidth of the inverting and the non-inverting amplifiers is approximately the same if $R_1/R_F << 1$. This is equivalent to the condition that $A_{0f} >> 1$.

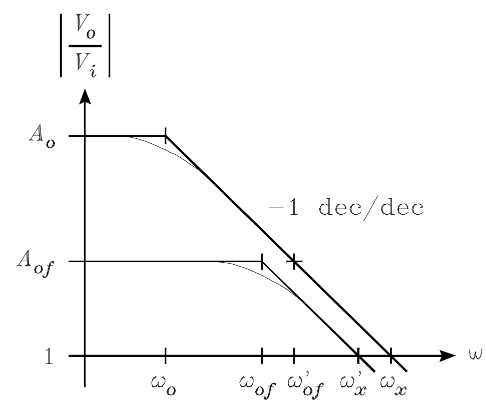

Figure 2.6: Bode plot for $|V_o/V_i|$.

Example 4 *An op-amp has a gain-bandwidth product of* 1 MHz. *Compare the bandwidths of an inverting and a non-inverting amplifier which use the op-amp for* $A_{0f} = 1$, 2, 5, *and* 10.

Solution. The non-inverting amplifier has a bandwidth of f_x/A_{0f}. The inverting amplifier has a bandwidth of $(f_x/A_{0f}) \times R_F / (R_1 + R_F)$. If we approximate A_{0f} of the inverting amplifier by $A_{0f} \simeq R_F/R_1$, its bandwidth reduces to $f_x / (1 + A_{0f})$. The calculated bandwidths of the two amplifiers are summarized in the following table.

A_{0f}	Non-Inverting	Inverting
1	1 MHz	500 kHz
2	500 kHz	333 kHz
5	200 kHz	167 kHz
10	100 kHz	91 kHz

For the case of the ideal op-amp, the V_- input to the inverting amplifier is a virtual ground so that the input impedance Z_{in} is resistive and equal to R_1. For the op-amp with finite gain and bandwidth, the V_- terminal is not a virtual ground so that the input impedance differs from R_1. We use the circuit in Fig. 2.5(a) to solve for the input impedance as follows:

$$Z_{in} = \frac{V_i}{I_1} = \frac{V_i}{(V_i - V_-)/R_1} = \frac{R_1}{1 - (V_-/V_o)(V_o/V_i)} \tag{2.16}$$

To put this into the desired form, we let $V_-/V_o = -1/A(s)$ and use Eq. (2.12) for V_o/V_i. The equation for Z_{in} reduces to

$$Z_{in} = R_1 + \frac{R_F}{1 + A(s)} = R_1 + \left[\frac{1}{R_F} + \left(\frac{R_F}{A_0} + \frac{R_F}{A_0\omega_0}s\right)^{-1}\right]^{-1} \tag{2.17}$$

where Eq. (2.2) has been used. It follows from this equation that Z_{in} consists of the resistor R_1 in series with an impedance that consists of the resistor R_F in parallel with the series combination of a resistor R_2 and an inductor L given by

$$R_2 = \frac{R_F}{A_0} \tag{2.18}$$

$$L = \frac{R_F}{A_0\omega_0} \tag{2.19}$$

The equivalent circuit for Z_{in} is shown in Fig. 2.7(a). If $A_0 \to \infty$, it follows that $R_2 \to 0$ and $L_2 \to 0$ so that $Z_{in} \to R_1$. The impedance transfer function for Z_{in} is of the form of a high-pass shelving transfer function given by

$$Z_{in}(s) = R_{DC}\frac{1 + s/\omega_z}{1 + s/\omega_p} \tag{2.20}$$

where R_{DC} is the dc resistance, ω_p is the pole frequency, and ω_z is the zero frequency. These are given by

$$R_{DC} = R_1 + \frac{R_F}{1 + A_0} \tag{2.21}$$

$$\omega_p = \frac{R_2 + R_F}{L} = \omega_0(1 + A_0) \tag{2.22}$$

$$\omega_z = \frac{R_2 + R_F \| R_1}{L} = \omega_0\left(1 + \frac{A_0 R_1}{R_1 + R_F}\right) \tag{2.23}$$

The Bode magnitude plot for Z_{in} is shown in Fig. 2.7(b).

Example 5 *At very low frequencies, an op-amp has the frequency independent open-loop gain $A(s) = A_0 = 2 \times 10^5$. The op-amp is to be used in an inverting amplifier with a gain of -1000. What is the required ratio R_F/R_1? For the value of R_F/R_1, how much larger is the input resistance than R_1?*

Solution. By Eq. (2.12), we have

$$-1000 = -\frac{R_F/R_1}{1 + (1 + R_F/R_1)/(2 \times 10^5)}$$

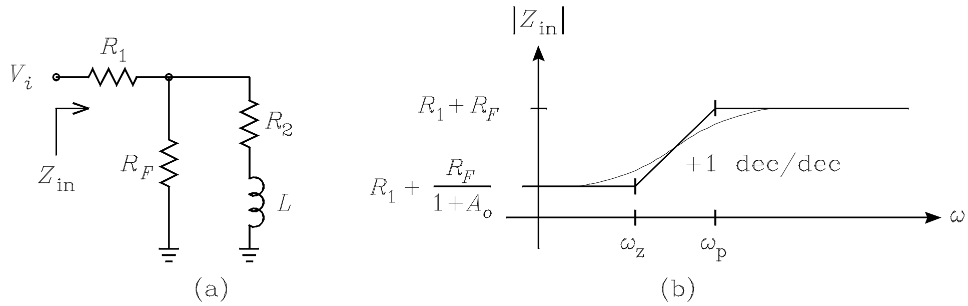

Figure 2.7: (a) Equivalent input impedance. (b) Bode plot for $|Z_{in}|$.

This can be solved for R_F/R_1 to obtain

$$\frac{R_F}{R_1} = \frac{2 \times 10^5 + 1}{(2 \times 10^5/1000) - 1} = 1005$$

By Eq. (2.17), the input resistance can be written

$$R_{in} = R_1 \left(1 + \frac{R_F/R_1}{1 + A_0}\right) = R_1 \left(1 + \frac{1005}{1 + 2 \times 10^5}\right) = 1.005R_1$$

Example 6 *An op-amp has a dc gain $A_0 = 2 \times 10^5$ and a gain bandwidth product $f_x = 1\,\text{MHz}$. The op-amp is used in the inverting amplifier of Fig. 2.5(a). The circuit element values are $R_1 = 1\,\text{k}\Omega$ and $R_F = 100\,\text{k}\Omega$. Calculate the dc gain of the amplifier, the upper cutoff frequency, and the value of the elements in the equivalent circuit for the input impedance. In addition, calculate the zero and the pole frequencies in Hz for the impedance Bode plot of Fig. 2.7(b).*

Solution. The dc voltage gain is $-A_{0f}$. Eq. (2.14) can be used to calculate A_{0f} to obtain

$$A_{0f} = \frac{2 \times 10^5 \times (1\text{k}\|100\text{k})/1\text{k}}{1 + 2 \times 10^5 \times (1\text{k}\|100\text{k})/100\text{k}} = 99.95$$

By Eqs. (2.3) and (2.15), the upper cutoff frequency f_{0f} is given by

$$f_{of} = \frac{\omega_{0f}}{2\pi} = \frac{10^6}{2 \times 10^5}\left(1 + 2 \times 10^5 \frac{1\text{k}\|100\text{k}}{100\text{k}}\right) = 9.91\,\text{kHz}$$

The element values in the equivalent circuit of Fig. 2.7(a) for the input impedance are as follows:

$$R_1 = 1\ \text{k}\Omega,\ R_F = 100\ \text{k}\Omega,\ R_2 = 0.5\ \Omega,\ \text{and}\ \ L = 15.9\ \text{mH}$$

where Eqs. (2.18) and (2.19) have been used for R_2 and L. The pole and zero frequencies in the Bode impedance plot are

$$f_p = f_0\,(1 + A_0) = \frac{f_x}{A_0}\,(1 + A_0) = 1.00\ \text{MHz}$$

$$f_z = f_0 \left(1 + A_0 \frac{R_1}{R_1 + R_F}\right) = 9.91\ \text{kHz}$$

2.2 Effects of Finite Input Resistance

2.2.1 Differential Input Resistance

Because no signal currents flow in the input leads of the ideal op-amp, the input resistance to either lead is infinite. For a physical op-amp, the input resistance is not infinite. To a first approximation, the signal currents which flow in the input leads can be modeled by placing a resistor R_I between the two leads. Fig. 2.8 shows the op-amp symbol with such a resistor added as an external element. The resistor is called the differential input resistance. A typical value for R_I is 1 MΩ or greater. In the following, we calculate the effects of R_I on the non-inverting and the inverting amplifiers.

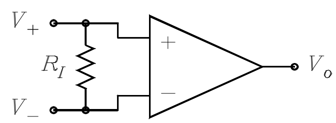

Figure 2.8: Op amp with its internal input resistance modeled by an external resistor.

2.2.2 Non-Inverting Amplifier

Figure 2.9(a) shows the circuit diagram of a non-inverting amplifier with the differential input resistance modeled by an external resistor. Fig. 2.9(b) shows an equivalent circuit which can be used to solve for V_-. By inspection, we can write

$$V_o = A(s)(V_i - V_-) \tag{2.24}$$

$$V_- = V_o \frac{R_1}{R_1 + R_F} \times \frac{R_I}{R_I + R_1 \| R_F} + V_i \frac{R_1 \| R_F}{R_I + R_1 \| R_F} \tag{2.25}$$

where $A(s)$ is the op-amp open-loop voltage-gain transfer function. Simultaneous solution of these equations for V_o/V_i yields

$$\frac{V_o}{V_i} = \frac{A'(s)}{1 + A'(s) R_1/(R_1 + R_F)} = \frac{1 + R_F/R_1}{1 + (1 + R_F/R_1)/A'(s)} \tag{2.26}$$

where $A'(s)$ is given by

$$A'(s) = \frac{R_I}{R_I + R_1 \| R_F} A(s) \tag{2.27}$$

Equation (2.6) gives the voltage-gain transfer function for $R_I = \infty$. When this equation is compared to Eq. (2.26), it can be concluded that the effect of R_I on the voltage gain is to reduce the open-loop transfer function $A(s)$ by the factor $R_I/(R_I + R_1 \| R_F)$. The effective transfer function is denoted by $A'(s)$. When Eq. (2.2) is used for $A(s)$, $A'(s)$ can be written

$$A'(s) = \frac{A'_0}{1 + s/\omega_0} \tag{2.28}$$

where A'_0 is given by

$$A'_0 = \frac{R_I}{R_I + R_1 \| R_F} A_0 \tag{2.29}$$

We see that the effect of R_I is to reduce the dc gain constant of $A(s)$ by the factor $R_I/(R_I + R_1 \| R_F)$. Because the pole frequency is unchanged, it follows that the gain bandwidth product is reduced by the factor $R_I/(R_I + R_1 \| R_F)$.

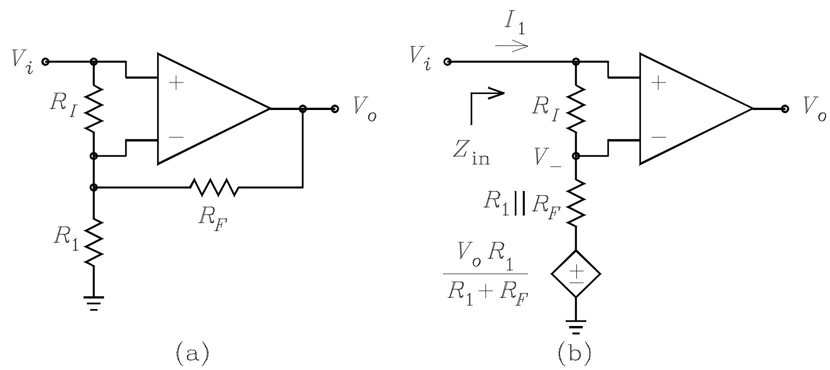

Figure 2.9: (a) Non-inverting amplifier. (b) Equivalent circuit for calculating Z_{in} and V_o/V_i.

It follows from Fig. 2.9(b) that the input impedance of the non-inverting amplifier is given by

$$Z_{in} = \frac{V_i}{I_1} = \frac{V_i}{\left(\frac{V_i - V_o R_1/(R_1+R_F)}{R_I + R_1\|R_F}\right)} = \frac{R_I + R_1\|R_F}{1 - (V_o/V_i)\, R_1/\,(R_1+R_F)} \tag{2.30}$$

When Eq. (2.26) is used for V_o/V_i, this expression reduces to

$$Z_{in} = \left(1 + \frac{R_1 A(s)}{R_1 + R_F}\right) R_I + R_1\|R_F \tag{2.31}$$

Fig. 2.10(a) shows the equivalent circuit for Z_{in}, where Eq. (2.2) is assumed for $A(s)$. The resistor R and the capacitor C in the figure are given by

$$R = \frac{R_1}{R_1 + R_F} A_0 R_I \tag{2.32}$$

$$C = \frac{1}{\omega_0 R} = \frac{1 + R_F/R_1}{A_0 \omega_0 R_I} \tag{2.33}$$

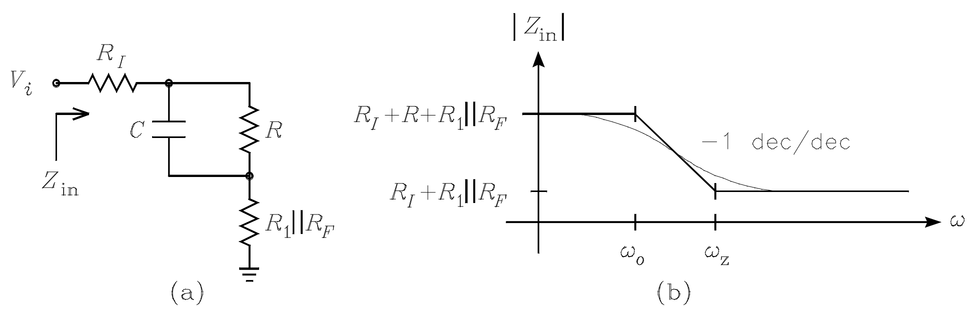

Figure 2.10: (a) Equivalent circuit for Z_{in}. (b) Bode plot for $|Z_{in}|$.

At low frequencies where the capacitor is an open circuit, Z_{in} is resistive and equal to $R_I + R + R_1\|R_F$. At high frequencies where the capacitor is a short circuit, Z_{in} is resistive and equal to $R_I + R_1\|R_F$. It follows that the input impedance function is a low-pass shelving function. The transfer function for Z_{in} can be written

$$Z_{in} = (R_I + R + R_1 \| R_F) \frac{1 + s/\omega_z}{1 + s/\omega_0} \tag{2.34}$$

where ω_z is given by

$$\omega_z = \left(1 + \frac{R}{R_I + R_1 \| R_F}\right) \omega_0 \tag{2.35}$$

The Bode magnitude plot for Z_{in} is shown in Fig. 2.10(b). Because $|Z_{in}| \geq R_I$ and R_I is usually very large, it follows that Z_{in} can be approximated by an open circuit in most applications.

2.2.3 Inverting Amplifier

Figure 2.11(a) shows the circuit diagram of an inverting amplifier with the differential input resistance of the op-amp modeled as an external resistor. Fig. 2.11(b) shows an equivalent circuit which can be used to solve for V_-. By inspection, we can write

$$V_o = -A(s) V_- \tag{2.36}$$

$$V_- = \left(\frac{V_i}{R_1} + \frac{V_o}{R_F}\right) (R_I \| R_1 \| R_F) = \frac{R_I}{R_I + R_1 \| R_F} \left(\frac{V_i}{R_1} + \frac{V_o}{R_F}\right) (R_1 \| R_F) \tag{2.37}$$

where $A(s)$ is the op-amp open-loop voltage-gain transfer function. These equations can be solved for the voltage gain of the circuit to obtain

$$\frac{V_o}{V_i} = -\frac{(1/R_1) A'(s) (R_1 \| R_F)}{1 + (1/R_F) A'(s) (R_1 \| R_F)} = -\frac{R_F/R_1}{1 + (1 + R_F/R_1)/A'(s)} \tag{2.38}$$

where $A'(s)$ is the effective open-loop gain given by Eq. (2.27).

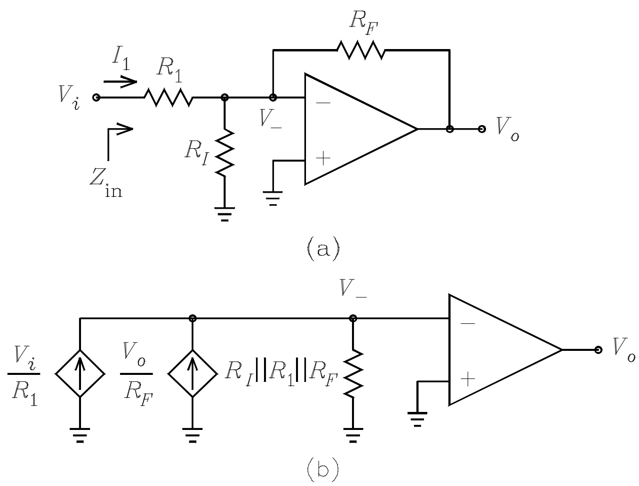

Figure 2.11: (a) Inverting amplifier. (b) Equivalent circuit for calculating V_o/V_i.

Equation (2.12) gives the voltage-gain transfer function for $R_I = \infty$. When this equation is compared to Eq. (2.38), it can be seen that the effect of R_I on the voltage gain is to reduce the open-loop transfer function $A(s)$ by the factor $R_I/(R_I + R_1 \| R_F)$. This is the same as the effect of R_I on the non-inverting amplifier.

It follows from Fig. 2.11(a) that the input impedance of the inverting amplifier is given by

$$Z_{in} = \frac{V_i}{I_1} = \frac{V_i}{(V_i - V_-)/R_1} = \frac{R_1}{1 - (V_-/V_o)(V_o/V_i)} \tag{2.39}$$

To put this in the desired form, we let $V_-/V_o = -1/A(s)$ and use Eq. (2.38) for V_o/V_i. The equation for Z_{in} reduces to

$$\begin{aligned} Z_{in} &= R_1 + R_I \| \left(\frac{R_F}{1 + A(s)} \right) \\ &= R_1 + R_I \| \left[\frac{1}{R_F} + \left(\frac{R_F}{A_0} + \frac{R_F}{A_0 \omega_0} s \right)^{-1} \right]^{-1} \end{aligned} \tag{2.40}$$

The analogous circuit for Z_{in} is shown in Fig. 2.12(a), where R_2 and L are given by Eqs. (2.18) and (2.19).

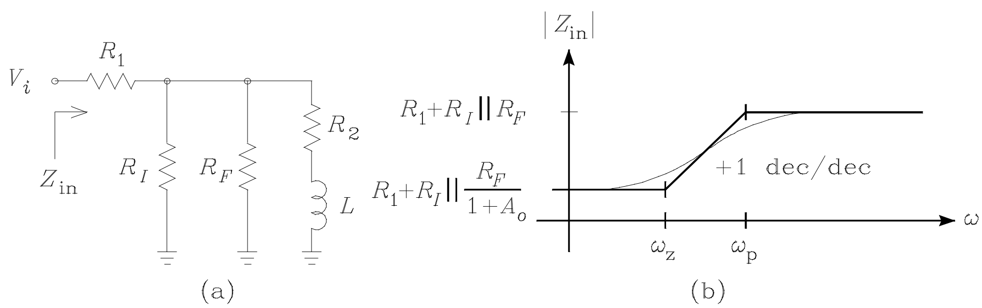

Figure 2.12: (a) Equivalent circuit for Z_{in}. (b) Bode plot for $|Z_{in}|$.

The impedance transfer function for Z_{in} is of the form of a high-pass shelving transfer function given by

$$Z_{in}(s) = R_{DC} \frac{1 + s/\omega_z}{1 + s/\omega_p} \tag{2.41}$$

where R_{DC} is the dc resistance, ω_p is the pole frequency, and ω_z is the zero frequency. These are given by

$$R_{DC} = R_1 + R_I \| \left(\frac{R_F}{1 + A_0} \right) \tag{2.42}$$

$$\omega_p = \frac{R_2 + R_I \| R_F \| R_1}{L} = \omega_0 \left(1 + \frac{A_0 R_1 \| R_I}{R_1 \| R_I + R_F} \right) \tag{2.43}$$

$$\omega_z = \frac{R_2 + R_I \| R_F}{L} = \omega_0 \left(1 + \frac{A_0 R_I}{R_I + R_F} \right) \tag{2.44}$$

The Bode magnitude plot for Z_{in} is shown in Fig. 2.12(b).

2.3 Effects of Non-Zero Output Resistance

2.3.1 Open-Loop Output Resistance

The output impedance of the ideal op-amp is zero. For a physical op-amp, the output impedance is not zero. To model it, we place a resistor R_O in series with the op-amp output. Fig. 2.13 shows the op-amp symbol with such a resistor added as an external element. The resistor is called the open-loop output resistance. A typical value for R_O is $100\,\Omega$. We use the model in Fig. 2.13 to calculate the effects of R_O on the inverting and the non-inverting amplifiers in the following. In the analyses, we assume that the differential input resistance can be neglected, i.e. replaced by an open circuit.

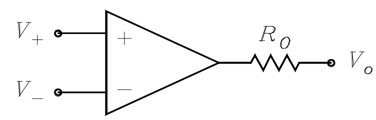

Figure 2.13: Op-amp symbol with the output resistance modeled by an external resistor.

2.3.2 Non-Inverting Amplifier

Figure 2.14 shows the circuit diagram of a non-inverting amplifier with the op-amp output resistance modeled as an external resistor. To solve for the voltage gain of the circuit, we can write by inspection

$$V_o = A(s)(V_i - V_-)\frac{R_F + R_1}{R_O + R_F + R_1} \tag{2.45}$$

$$V_- = V_o \frac{R_1}{R_F + R_1} \tag{2.46}$$

where a voltage divider relation is used in the former equation. These equations can be solved for the voltage gain to obtain

$$\frac{V_o}{V_i} = \frac{A'(s)}{1 + A'(s) R_1 / (R_1 + R_F)} = \frac{1 + R_F/R_1}{1 + (1 + R_F/R_1)/A'(s)} \tag{2.47}$$

where $A'(s)$ is given by

$$A'(s) = A(s)\frac{R_F + R_1}{R_O + R_F + R_1} \tag{2.48}$$

(The $A'(s)$ here is not the same as that defined in Sec. 2.2.)

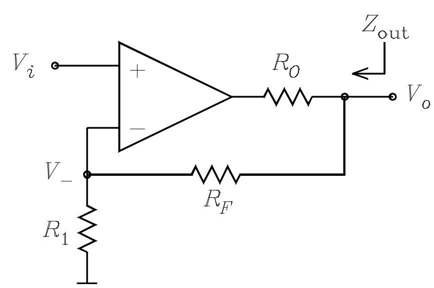

Figure 2.14: Non-inverting amplifier.

It can be concluded that the effect of R_O is to reduce the effective open-loop voltage-gain transfer function by the factor $(R_F + R_1)/(R_O + R_F + R_1)$. When $A(s)$ is modeled by the transfer function in Eq. (2.2), it follows that the dc gain constant A_0 is reduced by the same factor, the pole frequency is not changed, and the gain-bandwidth product is reduced by the factor $(R_F + R_1)/(R_O + R_F + R_1)$.

The output impedance of the non-inverting amplifier is given by the ratio of the open-circuit output voltage $V_{o(oc)}$ to the short-circuit output current $I_{o(sc)}$, or equivalently the ratio of $V_{o(oc)}/V_i$ to $I_{o(sc)}/V_i$. Eq. (2.47) gives $V_{o(oc)}/V_i$. To solve for the short-circuit output current, we connect the V_o node in Fig. 2.14 to ground. The current which flows in the ground connection is the short-circuit output current. When the V_o node is grounded, there is no feedback voltage, i.e. $V_- = 0$. It follows that the current which flows in the ground connection is $I_{o(sc)} = A(s)V_i/R_O$ so that $I_{o(sc)}/V_i = A(s)/R_O$. Thus the output impedance of the amplifier is obtained by dividing Eq. (2.47) by $A(s)/R_O$. It is given by

$$Z_{out} = \frac{V_{o(oc)}}{I_{o(sc)}} = \frac{R_O \| (R_1 + R_F)}{1 + A'(s) R_1/(R_1 + R_F)} \tag{2.49}$$

When the transfer function in Eq. (2.2) is used for $A(s)$, the expression for Z_{out} reduces to

$$Z_{out} = R_{DC} \frac{1 + s/\omega_0}{1 + s/\omega_{\mathrm{p}}} \tag{2.50}$$

where R_{DC} and ω_{p} are given by

$$R_{DC} = \frac{R_O \| (R_1 + R_F)}{1 + A_0 R_1/(R_O + R_F + R_1)} \tag{2.51}$$

$$\omega_p = \left(1 + A_0 \frac{R_1}{R_O + R_F + R_1}\right) \omega_0 \tag{2.52}$$

The equivalent circuit for Z_{out} is given in Fig. 2.15(a), where R_2 and L are given by

$$R_2 = [R_O \| (R_F + R_1)] \frac{A_0 R_1/(R_O + R_F + R_1)}{1 + A_0 R_1/(R_O + R_F + R_1)} \tag{2.53}$$

$$L = \frac{R_2}{\omega_p} \tag{2.54}$$

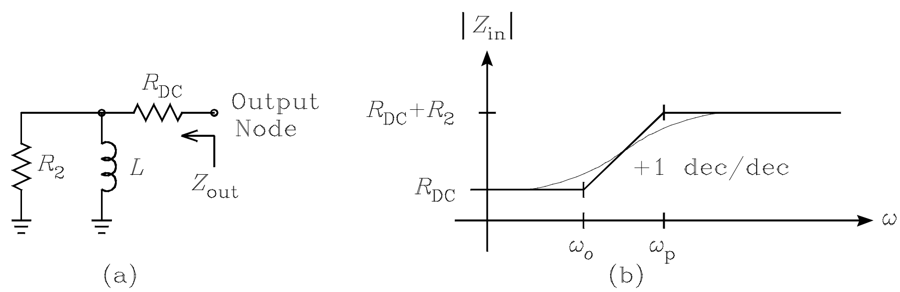

Figure 2.15: (a) Equivalent circuit for Z_{out}. (b) Bode plot for $|Z_{out}|$.

Example 7 *At very low frequencies, an op-amp has the frequency independent open-loop gain* $A(s) = A_0 = 2 \times 10^5$ *and an open-loop output resistance* $R_O = 100\,\Omega$. *The op-amp is to be used in a non-inverting amplifier having a voltage gain of* 100. *If the amplifier is designed with the assumption that the op-amp is ideal, calculate the actual gain of the circuit and its output resistance.*

Solution. To obtain a voltage gain of 100 with an ideal op-amp, we require $1 + R_F/R_1 = 100$. To satisfy this, we can choose $R_1 = 100\,\Omega$ and $R_F = 9.9\,\mathrm{k}\Omega$. The voltage gain is calculated from Eqs. (2.48) and (2.47) as follows:

$$A' = 2 \times 10^5 \frac{9.9\mathrm{k} + 100}{100 + 9.9\mathrm{k} + 100} = 1.98 \times 10^5$$

$$\frac{V_o}{V_i} = \frac{1.98 \times 10^5}{1 + 1.98 \times 10^5 \times 100/(100 + 9.9\mathrm{k})} = 99.95$$

The output resistance is calculate from Eq. (2.51) to obtain

$$R_{DC} = \frac{100 \| (100 + 9.9\mathrm{k})}{1 + 1.98 \times 10^5 \times 100/(100 + 9.9\mathrm{k})} = 0.05\,\Omega$$

2.3.3 Inverting Amplifier

Figure 2.16(a) shows the circuit diagram of an inverting amplifier with R_O shown as an external resistor. Fig. 2.16(b) shows an equivalent circuit which can be used to calculate V_- and V_o. By inspection, we can write

$$V_- = \left(\frac{V_i}{R_1} + \frac{V_o}{R_F}\right)(R_1 \| R_F) \tag{2.55}$$

$$V_o = -A(s)\,V_- \frac{R_F}{R_O + R_F} + V_- \frac{R_O}{R_O + R_F} \tag{2.56}$$

where two voltage divider relations are used in the latter equation. These equations can be solved for the voltage gain to obtain

$$\frac{V_o}{V_i} = -\frac{(1/R_1)\,A''(s)\,(R_1 \| R_F)}{1 + (1/R_F)\,A''(s)\,(R_1 \| R_F)} = -\frac{R_F/R_1}{1 + (1 + R_F/R_1)\,/A''(s)} \tag{2.57}$$

where $A''(s)$ is given by

$$A''(s) = A(s)\,\frac{R_F}{R_O + R_F} - \frac{R_O}{R_O + R_F} \tag{2.58}$$

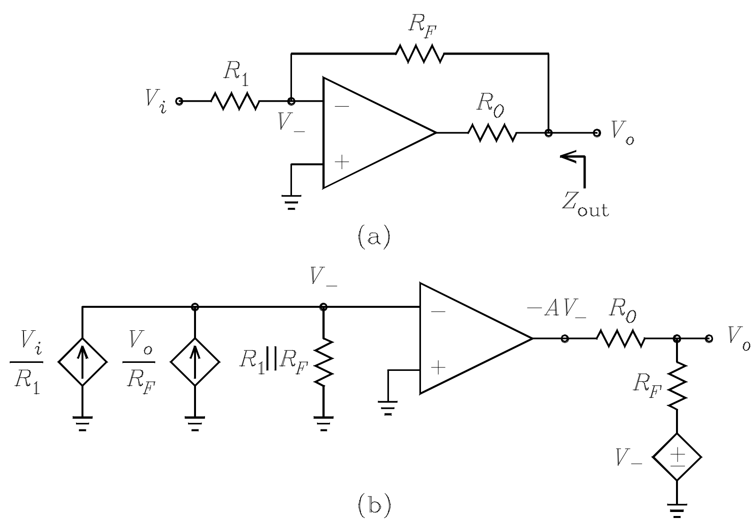

Figure 2.16: (a) Inverting amplifier. (b) Equivalent circuit for calculating V_o/V_i.

Equation (2.12) gives the voltage-gain transfer function for the inverting amplifier for the case $R_O = 0$. When this equation is compared to Eq. (2.57), it can be seen that the effect of R_O is to cause the open loop transfer function $A(s)$ to be changed to $A''(s)$ given by Eq. (2.58). When Eq. (2.2) is used for $A(s)$, $A''(s)$ can be written

$$A''(s) = A_0'' \frac{1 - s/\omega_z}{1 + s/\omega_0} \tag{2.59}$$

where A_0 and ω_z are given by

$$A_0'' = \frac{R_F}{R_O + R_F}\left(A_0 - \frac{R_O}{R_F}\right) \tag{2.60}$$

$$\omega_z = \left(A_0 \frac{R_F}{R_O} - 1\right)\omega_0 \tag{2.61}$$

Thus the effect of R_O is to reduce the gain constant from A_0 to A_0'' and to introduce a right-half-plane zero into the transfer function.

When Eq. (2.59) is used in Eq. (2.57), it follows that the voltage-gain transfer function for the inverting amplifier reduces to

$$\frac{V_o}{V_i} = A_{0f}\frac{1 - s/\omega_z}{1 + s/\omega_{0f}} \tag{2.62}$$

where A_{0f} and ω_{0f} are given by

$$A_{0f} = \frac{A_0'' R_F / (R_1 + R_F)}{1 + A_0'' R_1 / (R_1 + R_F)} = \frac{R_F / R_1}{1 + (1 + R_F/R_1)/A_0''} \tag{2.63}$$

$$\omega_{0f} = \frac{1 + A_0'' R_1 / (R_1 + R_F)}{1 - (\omega_0/\omega_1) R_1 / (R_1 + R_F)}\omega_0 \tag{2.64}$$

The output impedance of the inverting amplifier is the same as that for the non-inverting amplifier given by Eq. (2.50). This follows because the circuit seen looking into the output terminal with the source zeroed is the same for both configurations.

2.4 Output Waveform Distortion

2.4.1 Types of Distortion

The output voltage waveform from a physical op-amp is said to be distorted when it does not correspond to what would be expected if the op-amp were ideal. Distortion can be divided into two categories, linear distortion and non-linear distortion. The simplest way to differentiate between the two is to compare their effects when the op-amp input signal is a sine wave. If the output signal is a pure sine wave having the same frequency as the input sine wave, the distortion is said to be linear. For example, the gain of all physical op-amps decreases as frequency is increased. This is a linear distortion mechanism. Another example of linear distortion is a phase shift in the output sine wave. In contrast, if the output signal contains sine-wave components at frequencies different from the frequency of the input sine wave, the distortion is said to be non-linear. The three principle mechanisms of non-linear distortion in op-amps are peak clipping, current limiting, and slew rate limiting. These are discussed in this section.

2.4.2 Peak Clipping

Physical op-amps have two external leads to which dc power supply voltages must be applied in order for the op-amps to operate. Fig. 2.17(a) shows the op-amp symbol with the power supply leads shown explicitly. The diagram shows the dc voltages V^+ and V^- applied to the leads. In the majority of applications, the power supply voltages are bipolar, i.e. $V^+ = -V^-$. In the following, it is assumed that this condition on the two power supply voltages holds unless stated otherwise.

In general, the output voltage v_O from an op-amp must satisfy the inequality $V^- < v_O < V^+$. This relation says that v_O can never be equal to either power supply voltage. We denote the maximum positive peak value of v_O by V_{SAT}^+ and the maximum negative peak value by V_{SAT}^-. These two voltages are called the op-amp saturation voltages. Typically, V_{SAT}^+ is two to three volts less than V^+ and V_{SAT}^- is two to three volts greater than V^-. For example, if $V^+ = -V^- = 15$ V, typical values for the saturation voltages might be $V_{SAT}^+ = -V_{SAT}^- = 12$ V. This example illustrates the case where the saturation voltages are symmetrical. It is common to assume symmetrical saturation voltages when the op-amp is powered by bipolar power supply voltages. In this case, we denote the saturation voltage by V_{SAT}.

Figure 2.17(b) shows a plot of the op-amp output voltage v_O versus the differential input voltage $v_+ - v_-$, where symmetrical saturation voltages are assumed. For $-V_{SAT} < v_O < +V_{SAT}$, the output voltage is given by $v_O = A(v_+ - v_-)$, where A is the open-loop gain. The slope of the curve in this region is equal to A. For $|v_+ - v_-| \geq V_{SAT}/A$, the slope of the curve is zero. In the two regions where the slope is zero, the output

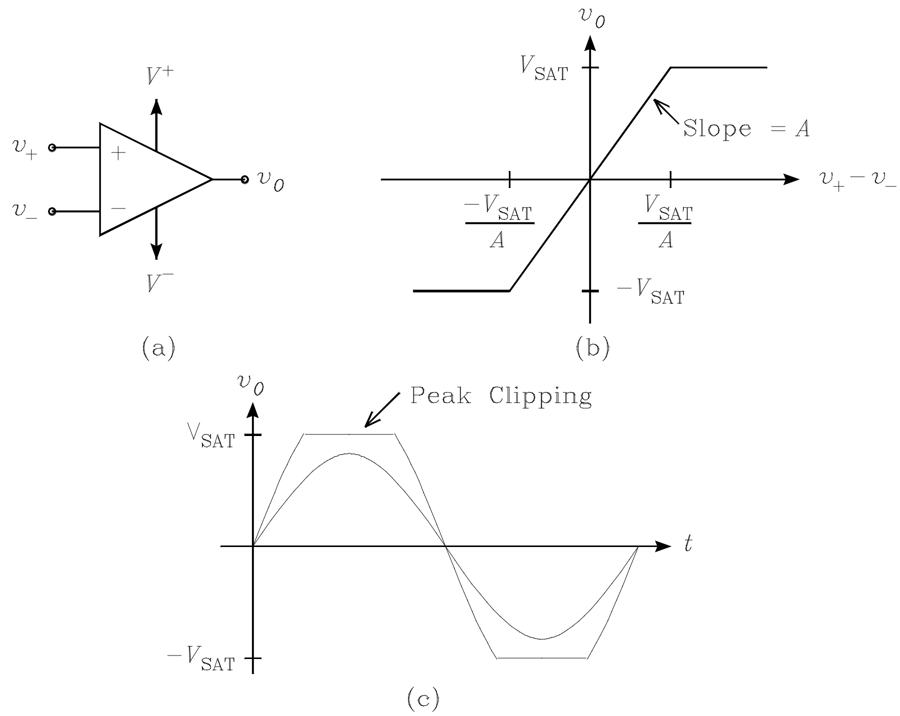

Figure 2.17: (a) Op amp with power supply connections shown. (b) v_O versus $v_+ - v_-$. (c) Output waveforms.

voltage does not change when the input voltage is changed. Fig. 2.17(c) shows the effect of clipping on a sine wave output signal. For the larger amplitude waveform, it can be seen that the maximum value of $|v_O|$ is limited to V_{SAT}. The output is said to be peak clipped at this value. The smaller amplitude waveform is not clipped.

Peak clipping is a non-linear distortion mechanism. For a sine-wave input signal, a Fourier series analysis can be used to show that a peak clipped output signal contains frequency components that are not at the frequency of the input signal. If the clipping voltages are symmetrical, it can be shown that the distortion components in the output signal are at odd harmonics of the input signal. For example, a 1 kHz input signal would generate a 1 kHz output signal plus distortion components at 3 kHz, 5 kHz, 7 kHz, etc. When the clipping voltages are not symmetrical, it can be shown that both even and odd order harmonics are generated by the clipping. We have illustrated peak clipping here for an op-amp with no feedback. If feedback is added, the peak clipping voltages are not changed. Thus the graph of the output voltage versus input voltage is the same as that shown in Fig. 2.17(b) except that the slope of the curve in the center region is reduced by the feedback. To prevent peak clipping from occurring, either the peak value of the input signal or the gain of the op-amp must be reduced.

2.4.3 Current Limiting

All physical op-amps have internal current limiting circuits which limit the maximum output current to prevent failure of the internal transistors that supply the current. Fig. 2.18(a) shows an op-amp with a load resistor connected to its output. The output current is given by $i_O = v_O/R_L$. For a given output voltage, the output current varies inversely with the load resistance. If the resistance is decreased, the output current will increase until the internal protection circuits are activated to limit the current. When this occurs, the op-amp exhibits peak clipping at its output. Current limiting causes the op-amp to clip at an output voltage that is less than its saturation voltage.

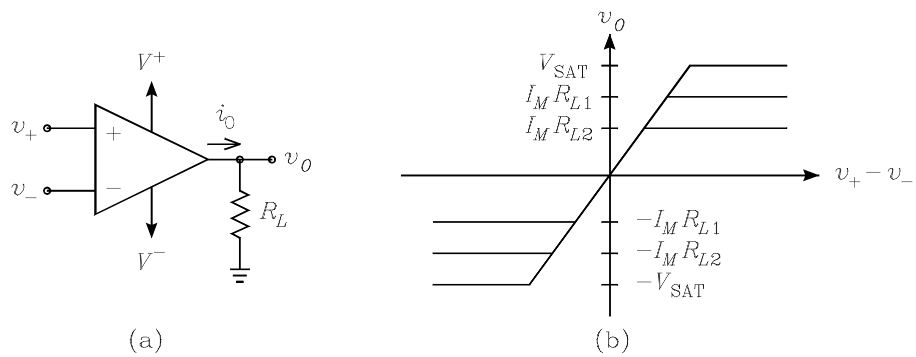

Figure 2.18: (a) Op amp with a load resistor. (b) Plots of v_O versus $v_+ - v_-$ showing effects of current limiting.

Let the maximum value of the op-amp output current be denoted by I_M. For a given load resistance R_L, the magnitude of the peak output voltage is limited to $I_M R_L$. If $R_L > V_{SAT}/I_M$, the op-amp clips before current limiting occurs. If $R_L < V_{SAT}/I_M$, the op-amp exhibits current limiting before clipping occurs. Fig. 2.18(b) shows the graph of the op-amp output voltage versus differential input voltage for three cases. One case corresponds to no load resistor so that the graph is identical to that shown in Fig. 2.17(b). The other cases illustrate the effects of current limiting for two values of load resistance. The graph assumes that $R_{L1} > R_{L2}$.

Example 8 *An op-amp has the saturation voltages $V_{SAT}^+ = V^+ - 2.5$ V and $V_{SAT}^- = V^- + 2.5$ V. The current limited output current is $I_M = 25$ mA. The op-amp is powered by bipolar supply voltages of $+15$ V and -15 V. Determine the lowest load resistance that can be driven without current limiting. Determine the peak output voltage for a load resistance of $100\,\Omega$.*

Solution. The magnitude of the peak output voltage is $15 - 2.5 = 12.5$ V. For a current limit of 25 mA, the minimum load resistance that can be driven to a peak voltage of 12.5 V is $R_{L(\min)} = 12.5/0.025 = 500\,\Omega$. For $R_L = 100\,\Omega$, the magnitude of the peak output voltage is $100 \times 0.025 = 2.5$ V.

Example 9 *Figure 2.19(a) shows a non-inverting amplifier with a capacitive load. The input signal to the amplifier is a square wave. The magnitude of the output current is limited to the value I_M. Determine the waveform of the amplifier output voltage.*

Solution. A voltage step applied to a capacitor causes an impulse of current to flow. Because the op-amp is current limited, it cannot supply an impulse of current to the load capacitor. Each time the input square wave switches states, the op-amp is driven into current limiting. For an output current $i_O = \pm I_{MAX}$, the capacitor voltage has a time derivative given by $dv_O/dt = \pm I_M/C_L$, where we assume that the current in R_F can be neglected. Thus current limiting has the effect of limiting the maximum time derivative of v_O with the capacitive load. Fig. 2.19(b) shows plot of the output voltage waveform, where it is assumed that the frequency of the square wave is low enough so that the output voltage reaches its final peak value each half cycle of the signal. The dotted lines in the figure represent the waveform for the case of no current limiting.

2.4.4 Slew Rate Limiting

An op-amp amplifier is said to be unstable if it puts out an ac signal with no input signal. To prevent instability problems, the internal circuits of most op-amps contain a capacitor that is called the compensation

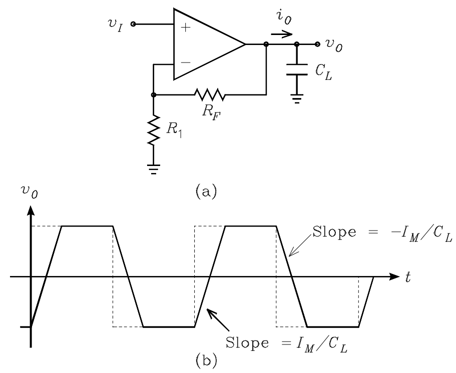

Figure 2.19: (a) Non-inverting amplifier with capacitive load. (b) Square wave response showing the effects of current limiting.

capacitor. The op-amp output voltage is proportional to the voltage across this capacitor. The circuits which charge and discharge the capacitor are current limited. As is illustrated by Example 9, the time derivative of the voltage across a capacitor is limited by the current available to charge it. Thus the compensation capacitor and the current available to charge it set the maximum time derivative of the op-amp output voltage. This maximum time derivative is called the op-amp slew rate.

The basic units of slew rate are volts per second (V/s). In op-amp specifications, it is usually specified in volts per microsecond (V/μs). The slew rate is related to the compensation capacitor and the maximum current available to charge it by the equation

$$SR = \frac{I_1}{C_c} \tag{2.65}$$

where C_c is the capacitor and I_1 is the peak value of the charging current. A typical general purpose op-amp might have a compensation capacitor with a value of 30 pF and a slew rate of 1 V/μs. Eq. (2.65) can be used to calculate $I_1 = 10^6 \times 30\text{p} = 30\ \mu\text{A}$. This calculation illustrates how small the internal currents in op-amps can be. Most op-amps have symmetrical slew rates. That is the slew rate is the same for an increasing or a decreasing output voltage. The slew rates are not symmetrical if the peak positive current available to charge the compensation capacitor is not the same as the peak negative current.

If an op-amp does not exhibit slewing, the time derivative of the output voltage is proportional to the time derivative of the input voltage. This is not true when the op-amp slews. Fig. 2.20(a) illustrates the effect of slewing on the sine-wave response of an op-amp. The figure shows the waveforms of the output voltage when the op-amp is slewing and when it is not slewing. The waveform with slewing is a triangle wave having a peak voltage obtained by multiplying the slew rate by one-fourth the period. It is given by

$$V_P = SR \times \frac{T}{4} = \frac{I_1}{4fC_c} \tag{2.66}$$

where the period T is related to the frequency f by $T = 1/f$. It can be seen from this expression that the peak output voltage is not a function of the amplitude the input voltage. Thus the output voltage does not increase if the input voltage is increased. This means that slewing is a non-linear phenomenon.

The non-linear distortion generated when an op-amp slews is referred to as slewing induced distortion. With a sine wave input signal, a Fourier series analysis can be used to show that the distortion components in the output signal are at odd harmonics of the signal frequency. This assumes symmetrical slew rates. If the op-amp slew rates are not symmetrical, both even and odd harmonics are generated.

If an op-amp does not slew, the maximum peak output voltage is V_{SAT}. For a sine-wave input signal, the corresponding output voltage is given by

$$v_O(t) = V_{SAT} \sin(2\pi f t) \tag{2.67}$$

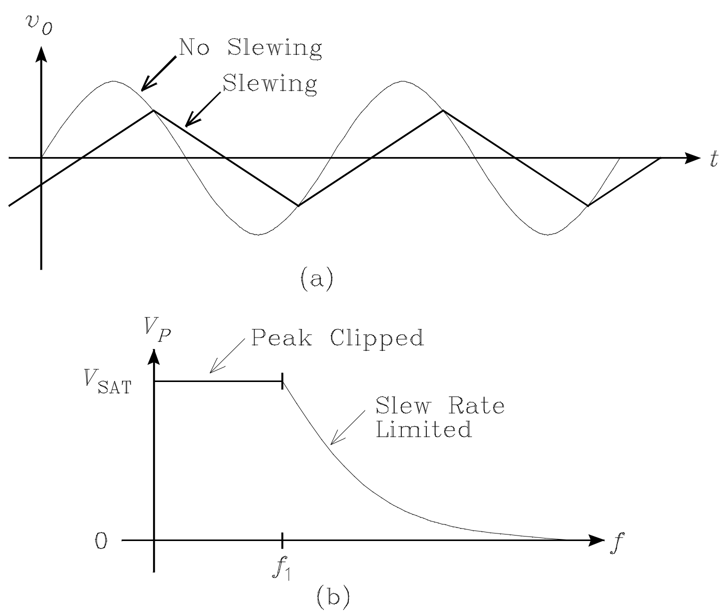

Figure 2.20: (a) Sine-wave output without slewing and with full slewing. (b) Peak sine-wave output voltage versus frequency.

The time derivative of this expression is

$$\frac{d}{dt}v_O(t) = 2\pi f V_{SAT}\cos(2\pi f t) \tag{2.68}$$

The maximum value of the magnitude of the derivative is $2\pi f V_{SAT}$. This must be less than the slew rate of the op-amp if it is not to exhibit slewing. The frequency above which slewing occurs is given by

$$f_1 = \frac{SR}{2\pi V_{SAT}} \tag{2.69}$$

For $f < f_1$, the peak output voltage is limited by clipping. For $f \geq f_1$, the peak output voltage is limited by slewing. The maximum undistorted peak sine-wave output voltage is given by

$$\begin{aligned} V_P &= V_{SAT} \text{ for } f \leq f_1 \\ &= \frac{SR}{2\pi f} \text{ for } f > f_1 \end{aligned} \tag{2.70}$$

Figure 2.20(b) shows a plot of V_P as a function of frequency. Because the voltage decreases with increasing frequency for $f \geq f_1$, f_1 is called the large-signal bandwidth of the op-amp.

Example 10 *The op-amp of Example 8 has a slew rate of* 1 *V/μs. Calculate the large-signal bandwidth of the op-amp. Calculate the peak value of the largest amplitude sine-wave that the op-amp can put out at a frequency of* 20 kHz*.*

Solution. By Eq. (2.69), the large signal bandwidth is $f_1 = 10^6/(2\pi \times 12.5) = 12.7\,\text{kHz}$. By Eq. (2.70), the peak value of the largest amplitude output sine wave is $V_P = 10^6/(2\pi \times 20\text{k}) = 7.96$ V.

2.5 DC Offsets

2.5.1 Offset Voltage

Although integrated-circuit op-amps are fabricated with precision, it is impossible to achieve circuits which have a zero dc output voltage when both input voltages are zero. The output offset voltage of a physical op-amp is the dc voltage that is present at its output when both of its inputs are grounded. The expression for the op-amp output voltage when an offset voltage is present is written

$$v_O = A\left(v_+ - v_- + V_{OS}\right) \tag{2.71}$$

With $v_+ = v_- = 0$, the dc offset voltage at the output is AV_{OS}. The voltage V_{OS} is defined as the input offset voltage. It is equivalent to the dc voltage at the input of an ideal op-amp that produces the same dc offset voltage at its output. A typical value for V_{OS} is 5 mV or less.

It can be seen from Eq. (2.71) that $v_O = 0$ if $v_- - v_+ = V_{OS}$. Thus an alternate definition of the input offset voltage is the differential dc voltage which must be applied across the op-amp inputs in order to achieve a zero dc output voltage, where the positive reference node is the v_- input. In op-amp specifications, V_{OS} is commonly specified without regard to its algebraic sign. The specified value represents the maximum value of the magnitude of the offset voltage for that particular op-amp.

Example 11 *Figure 2.21 shows the circuit diagram of a voltage follower with its input grounded. If the op-amp has the input offset voltage V_{OS}, solve for the output voltage.*

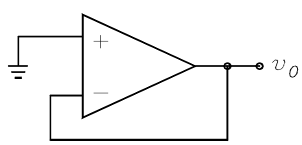

Figure 2.21: Circuit for measuring V_{OS}.

Solution. The output voltage is obtained by setting $v_+ = 0$ and $v_- = v_{\mathrm{O}}$ in Eq. (2.71) to obtain

$$v_O = \frac{A}{1+A} V_{OS}$$

For $A >> 1$, it follows that $v_{\mathrm{O}} \cong V_{OS}$. This result suggests a very convenient method for measuring V_{OS} for an op-amp.

2.5.2 Input Currents

The input stage of a physical op-amp is commonly a transistor differential amplifier. The dc bias currents which flow in the differential amplifier cause dc currents to flow in the two op-amp input leads. In many applications, these currents are small enough that they can be neglected. However, they may cause dc offset voltages at the op-amp output that are unacceptable. Fig. 2.22(a) shows the op-amp symbol with the dc input currents I_+ and I_- labeled. The reference directions show the currents flowing into the op-amp. Depending on the particular op-amp, the actual direction of the dc input currents can be either into or out of the op-amp.

The dc input currents are commonly specified by giving the common-mode and differential components. The common-mode component is called the input bias current and is denoted by I_{B}. The differential component is called the input offset current and is denoted by I_{OS}. These are related to I_+ and I_- by

$$I_{\mathrm{B}} = \frac{I_+ + I_-}{2} \tag{2.72}$$

$$I_{OS} = I_+ - I_- \tag{2.73}$$

If $I_+ = I_-$, we note that $I_\mathrm{B} = I_+ = I_-$ and $I_{OS} = 0$. Typical values are 100 nA or less for I_B and 20 nA or less for I_{OS}. In op-amp specifications, I_B and I_{OS} are commonly specified without regard to algebraic sign. The specified values represents the maximum value of the magnitude of the currents for that particular op-amp. Fig. 2.22(b) shows an equivalent circuit of the op-amp with the input currents represented by external common-mode and differential current sources.

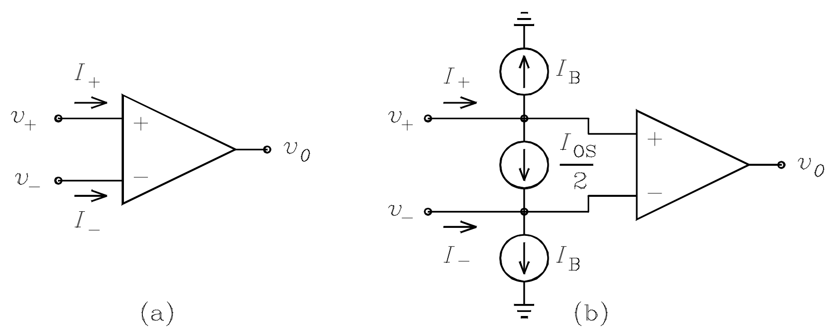

Figure 2.22: (a) Op amp with input currents labeled. (b) Equivalent circuit for input currents.

Example 12 *Figure 2.23(a) shows an inverting op-amp amplifier with a resistor connected in series with its non-inverting input. The op-amp has the input bias current I_B. Solve for the output voltage v_O. Assume that $A \to \infty$, $I_{OS} = 0$, and $V_{OS} = 0$.*

Solution. Because $I_{OS} = 0$, the dc input currents are $I_+ = I_- = I_\mathrm{B}$. Fig. 2.23(b) shows the equivalent circuit which can be used to calculate V_+ and V_-. These are given by

$$V_+ = -I_\mathrm{B} R_2$$

$$V_- = \left(\frac{V_I}{R_1} + \frac{V_O}{R_F} - I_\mathrm{B}\right)(R_1 \| R_F)$$

Because $A \to \infty$, the output voltage can be solved for by setting $V_+ = V_-$ to obtain

$$V_O = -\frac{R_F}{R_1}\left[V_I + \left(1 + \frac{R_1}{R_F}\right)(R_2 - R_1 \| R_F)\, I_\mathrm{B}\right]$$

It can be seen from this equation that the dc offset at the output caused by I_B is zero if $R_2 = R_1 \| R_F$. This is equivalent to the condition that the resistance seen looking out of the V_+ and V_- inputs be equal with V_I and V_O zeroed.

Example 13 *Figure 2.24 shows the circuit diagram of a non-inverting amplifier with a resistor connected in series with its input. The op-amp has the input bias current I_{IB}. Solve for the output voltage V_O. Assume that $A \to \infty$, $I_{OS} = 0$, and $V_{OS} = 0$.*

Solution. Figure 2.24(b) shows an equivalent circuit for calculating V_+ and V_-. By inspection, these are given by

$$V_+ = V_I - I_\mathrm{B} R_2$$

$$V_- = V_O \frac{R_1}{R_1 + R_F} - I_\mathrm{B}(R_1 \| R_F)$$

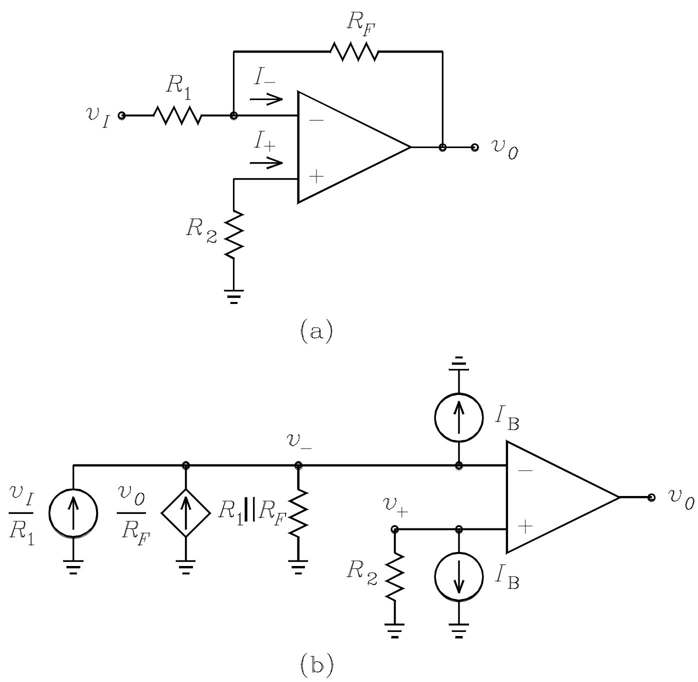

Figure 2.23: (a) Inverting amplifier. (b) Equivalent circuit for calculating v_+ and v_-.

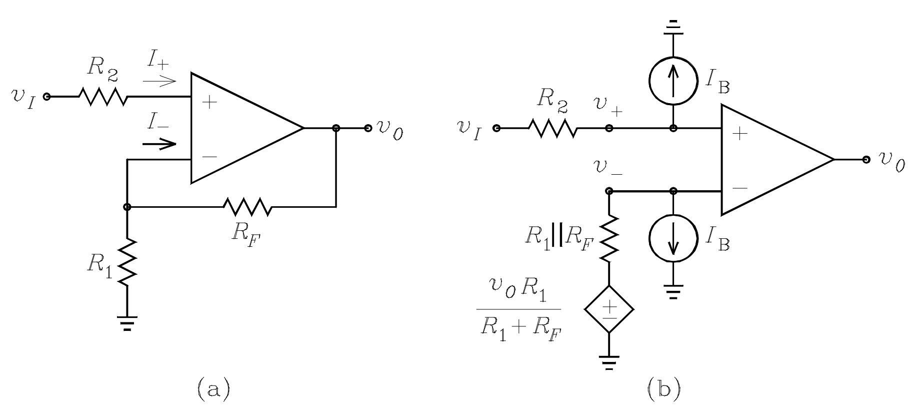

Figure 2.24: (a) Non-inverting amplifier. (b) Equivalent circuit for calculating v_+ and v_-.

The output voltage can be solved for by setting $v_+ = v_-$ to obtain

$$V_O = \left(1 + \frac{R_F}{R_1}\right)[V_I + (R_1 \| R_F - R_2) I_B]$$

It can be seen from this equation that the dc offset at the output caused by I_B is zero if $R_2 = R_1 \| R_F$. This is the same condition as for the inverting amplifier of Example 12.

2.5.3 Condition for Zero Offset Due to I_B

Examples 12 and 13 show that the condition for zero offset voltage at the op-amp output due to I_B are the same for the inverting amplifier and the non-inverting amplifier. The condition is that the resistance seen looking out of the v_+ and v_- inputs be equal, where the two resistances are calculated with v_I and v_O zeroed. Without resistor R_2 in the circuits, the condition cannot be met. In circuits containing capacitors, each capacitor must be replaced with an open-circuit when calculating the resistance seen looking out of the v_+ and v_- inputs.

2.6 Miscellaneous Specifications

This section covers several specifications on physical op-amps that have not been covered in the preceding sections.

2.6.1 Common-Mode Rejection Ratio

The output voltage from an ideal op-amp is a function only of the difference or differential voltage across its two inputs. In contrast, physical op-amps have an output voltage that can be written

$$V_o = A(s)(V_+ - V_-) + \frac{A(s)}{\rho} \times \frac{V_+ + V_-}{2} \tag{2.74}$$

where $A(s)$ is the differential voltage gain and $A(s)/\rho$ is the common-mode voltage gain. The constant ρ is the op-amp common-mode rejection ratio. It represents the ratio of the differential gain to the common-mode gain. It is commonly expressed in dB by the equation $20 \log \rho$. A typical value for ρ is 10^5 (100 dB). In most applications, the common-mode rejection ratio is large enough so that the common-mode term in Eq. (2.74) can be neglected.

2.6.2 Input Common-Mode Range

In some applications of op-amps, the external circuits cause a common-mode voltage, e.g. a dc voltage, to be present at both inputs. If this common-mode voltage is out of range, the differential amplifier input stage to the op-amp can cease to operate. The input common-mode range is the range on the common-mode input voltage over which the differential amplifier remains linear. For example, with the bipolar power supply voltages $V^+ = -V^- = 15$ V, the input common-mode range might be from -10 V to $+10$ V. A dc common-mode voltage applied to the two op-amp input terminals that is outside this range can cause the op-amp to cease to operate.

2.6.3 Input Differential Range

The input differential range is the maximum difference voltage that can be safely applied between the two op-amp inputs. When an op-amp is operated with negative feedback, the difference voltage between the inputs is very small. (It is zero for the ideal op-amp.) However, if the op-amp clips, current limits, or slews, it loses feedback and the differential input voltage can become large. Another example of a case where the differential input voltage might be large is when the op-amp is used as a comparator.

2.6.4 Power Supply Rejection Ratio

When either power supply voltage changes, the output offset voltage of a physical op-amp can change. This change in output offset voltage can be converted to a change in the input offset voltage by dividing by the open-loop gain of the op-amp. The power supply rejection ratio (PSRR) is defined as the ratio of the change in input offset voltage to the change in the power supply voltage, where one power supply voltage is varied and the other is held constant. In general, a different value is obtained when each power supply voltage is changed. A typical maximum value is 20 μV/V.

2.7 Linear Op-Amp Macromodels

2.7.1 Macromodels

A *macromodel* is a circuit model which is simpler than the original circuit but retains an accurate representation of the performance of that circuit. In this section, we develop a linear macromodel of the op-amp. The macromodel can be used with computer simulation programs such as SPICE to predict the voltage gain, input impedance, and output impedance of op-amp circuits. More accurate macromodels which model non-linear effects such as clipping, current limiting, and slewing require the addition of diodes and transistors to the model.

2.7.2 Modeling Input and Output Resistance

Figure 2.1(b) gives the simplest controlled-source model of the op-amp. The input resistance between the v_+ and the v_- terminals is infinite. The output resistance seen looking into the v_O node is zero. We can model the differential input resistance by adding a resistor R_I between the v_+ and the v_- nodes. In addition, we can model the open-loop output resistance by adding a resistor R_O in series with the output lead. The modified circuit is shown in Fig. 2.25. The next step in developing the macromodel circuit is to model the finite bandwidth of the op-amp.

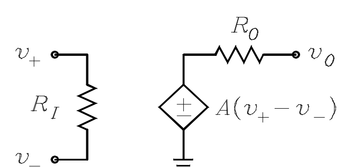

Figure 2.25: Simple macromodel with input and output resistances added.

2.7.3 Modeling the Open-Loop Transfer Function

The input stage of a typical op-amp operates as a voltage controlled current source with a load impedance that consists of a parallel RC circuit. The current source is controlled by the differential voltage between the op-amp input terminals. Such a circuit is shown in Fig. 2.26(a). The transconductance of the controlled source in this figure is denoted by g_{m1}. The voltage V_1 is given by

$$V_1 = -g_{m1}\left(V_+ - V_-\right)\left(R_1 \| \frac{1}{C_1 s}\right) = -\frac{g_{m1} R_1}{1 + R_1 C_1 s}\left(V_+ - V_-\right) \qquad (2.75)$$

The second stage of the typical op-amp operates as a voltage-controlled voltage source having the input voltage V_1 and an open-circuit output voltage equal to the op-amp open-circuit output voltage. Such a circuit is shown in Fig. 2.26(b). The voltage gain of the controlled source is denoted by $-A_V$, where A_V is a positive constant. The output resistance of the source is R_O. The open-circuit output voltage is given by $V_{o(oc)} = -A_V V_1$.

The op-amp macromodel consists of the two circuits of Fig. 2.26 in combination. Let the open-circuit voltage gain of this circuit be denoted by $A(s)$. It is given by

$$A(s) = \frac{V_{o(oc)}}{V_+ - V_-} = \frac{g_{m1} R_1 A_V}{1 + R_1 C_1 s} \tag{2.76}$$

This is a single-pole low-pass transfer function having a dc gain constant A_0 and a radian pole frequency ω_0 given by

$$A_0 = g_{m1} R_1 A_V \tag{2.77}$$

$$\omega_0 = \frac{1}{R_1 C_1} \tag{2.78}$$

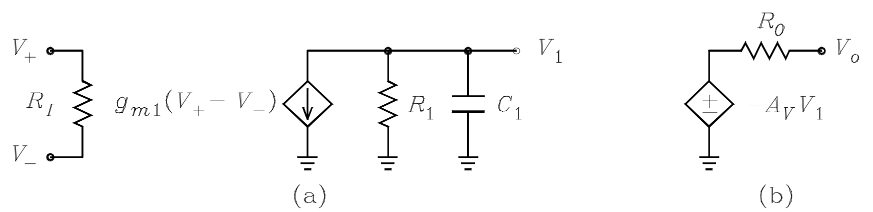

Figure 2.26: (a) Input stage model. (b) Gain stage model.

Figure 2.27 shows a modification to the circuit of Fig. 2.26 which makes the circuit more closely agree with the internal architecture of physical op-amps. The capacitor C_1 from the V_1 node to ground in the original circuit is replaced by the capacitor C_c from the V_1 node to the top of the voltage-controlled voltage source. By the Miller theorem, the load capacitance on the voltage-controlled current source input stage is the same if C_1 and C_c satisfy the relation

$$C_1 = (1 + A_V)\, C_c \tag{2.79}$$

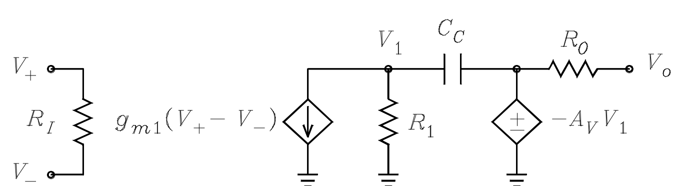

Figure 2.27: Modified macromodel circuit.

2.7.4 Completed Model

The circuit of Fig. 2.27 is a better model of physical op-amps if the output resistor R_O is broken into two parts R_{O1} and R_{O2} as shown in Fig. 2.28, where $R_O = R_{O1} + R_{O2}$. This circuit more accurately models the output impedance of a physical op-amp. In addition, it better models the variation in the op-amp gain with load impedance. This is the completed linear macromodel of the op-amp. A modification to this circuit that is often used in computer simulations is to make a Norton equivalent circuit of the resistor R_{O2} in series with the voltage source $A_V V_1$. The circuit is shown in Fig. 2.29, where $g_{m2} = A_V / R_{O2}$.

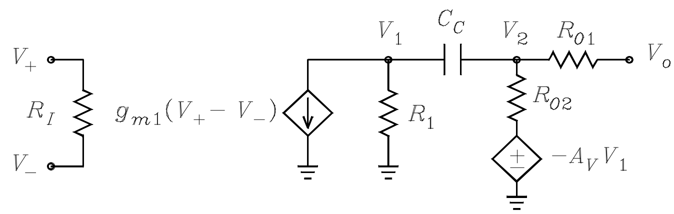

Figure 2.28: Further modification to the macromodel.

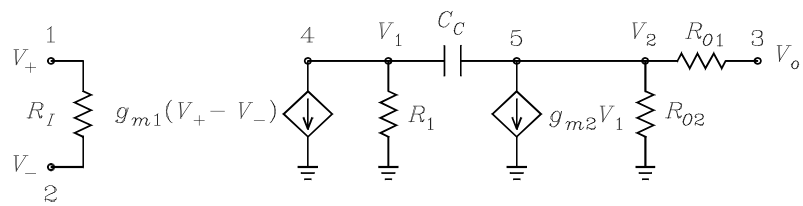

Figure 2.29: Final macromodel circuit.

Example 14 *Solve for the open-circuit voltage-gain transfer function of the op-amp macromodel in Fig. 2.29. Compare it to the transfer function of Eq. (2.76).*

Solution. It follows from Fig. 2.29 that

$$V_1 = \left[-g_{m1}\left(V_+ - V_-\right) + V_2 C_c s\right]\left(R_1 \| \frac{1}{C_c s}\right)$$

$$V_{o(oc)} = V_2 = \left[-g_{m2} V_1 + V_1 C_c s\right]\left(R_{O2} \| \frac{1}{C_c s}\right)$$

These equations can be solved for the voltage gain of the circuit to obtain

$$\frac{V_{o(oc)}}{V_+ - V_-} = g_{m1} R_1 g_{m2} R_{O2} \frac{1 - \left(C_c/g_{m2}\right) s}{1 + \left[R_1\left(1 + g_{m2} R_{O2}\right) + R_{O2}\right] C_c s}$$

The above equation is of the form of a dc gain constant multiplied by a low-pass shelving transfer function, where the zero in the transfer function is in the right-half complex plane. In contrast, the transfer function of Eq. (2.76) is of the form of a dc gain constant multiplied by a low-pass transfer function.

Example 15 *Solve for the output impedance transfer function for the op-amp macromodel in Fig. 2.29. Form the equivalent circuit which has this impedance transfer function.*

Solution. To solve for the output impedance, we zero the differential input voltage and drive the output node with a test current source I_t. The circuit is shown in Fig. 2.30. For this circuit, we can write

$$V_o = I_t R_{O1} + V_2$$

$$V_2 = \left(I_t - g_{m2} V_1\right) \times R_{O2} \| \left(R_{O1} + \frac{1}{C_c s}\right)$$

$$V_1 = V_2 \frac{R_{O1}}{R_{O1} + 1/C_c s}$$

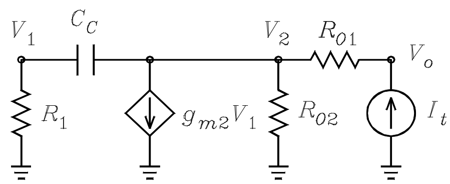

Figure 2.30: Circuit for solving for Z_{out}.

These equations can be solved for the output impedance to obtain

$$Z_{out} = \frac{V_o}{I_t} = R_{O1} + R_{O2} \frac{1 + R_1 C_c s}{1 + [R_1 (1 + g_{m2} R_{O2}) + R_{O2}] C_c s}$$

The above equation is of the form of a resistance (R_{O1}) plus a resistance (R_{O2}) multiplied by a low-pass shelving transfer function. At low frequencies, the impedance has the value $Z_{out} = R_{O1} + R_{O2}$. At high frequencies, it has the value $Z_{out} = R_{O1} + R_{O2} R_1 / [R_1 (1 + g_{m2} R_{O2}) + R_{O2}]$. The equivalent circuit which has this same impedance is given in Fig. 2.31. The elements R and C in this circuit are given by

$$R = \frac{R_1}{1 + g_{m2} R_1}$$

$$C = C_c (1 + g_{m2} R_1)$$

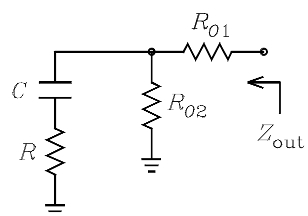

Figure 2.31: Equivalent circuit for Z_{out}.

Example 16 *The macromodel circuit of Fig. 2.29 can be used to model the 741 op-amp with the following element values:* $R_I = 2$ *M*Ω, $g_{m1} = 1.38 \times 10^{-4}$ *S,* $R_1 = 100\,\text{k}\Omega$, $C_c = 20$ *pF,* $g_{m2} = 106\,\text{S}$, $R_{O1} = 150\,\Omega$, *and* $R_{O2} = 150\,\Omega$. *Calculate the dc gain* A_0*, the pole frequency* f_0*, the gain-bandwidth product* f_x*, and the zero frequency* f_z *in the voltage-gain transfer function for the 741 macromodel. Calculate the element values for* R *and* C *in the equivalent circuit for the output impedance in Fig. 2.31.*

Solution. It follows from Example 14 that the dc gain A_0, the pole frequency f_0, the gain bandwidth product f_x, and the zero frequency f_z for the 741 macromodel are given by

$$A_0 = g_{m1} R_1 g_{m2} R_{O2} = 2.2 \times 10^5$$

$$f_0 = \frac{1}{2\pi [R_1 (1 + g_{m2} R_{O2}) + R_{O2}]} C_c = 5\,\text{Hz}$$

$$f_x = A_0 f_0 = 1.1\,\text{MHz}$$

$$f_z = \frac{g_{m2}}{2\pi C_c} = 8.4 \times 10^5 \,\text{MHz}$$

Because the zero frequency f_z is so much higher than the gain-bandwidth product f_x, the zero term in the transfer function can be neglected for all practical purposes. The values for the elements R and C in the output impedance equivalent circuit are $R = 9.43$ mΩ and $C = 212$ μF.

2.7.5 Example SPICE Macromodel Subcircuits

The 741 and the LF351 are two general purpose integrated-circuit op-amps that are commonly used in analog design. The 741 is a bipolar op-amp, i.e. it is fabricated entirely with bipolar-junction transistors (BJTs). The LF351 is a bi-fet op-amp. It is fabricated with junction field-effect transistors (JFETs) in the input diff-amp stage and BJT's in the following stages. The macromodels for these op-amps can be simulated in SPICE with subcircuits. A subcircuit in SPICE is a group of statements that is referenced as a single entity. It is defined by a block of statements starting with a .SUBCKT statement and ending with a .ENDS statement. In between are one or more statements. Once a subcircuit is defined, it can be called as a device having a name that starts with an "X". The codes for the 741 and LF351 subcircuits are given below. The SPICE node numbers for each code are labeled in Fig. 2.29.

```
*741 OP-AMP SUBCIRCUIT   *LF351 OP-AMP SUBCIRCUIT
SUBCKT 741 1 2 3         SUBCKT LF351 1 2 3
RI 1 2 2E6               RI 1 2 2E12
GM1 4 0 1 2 1.38E-4      GM1 4 0 1 2 2.83E-4
R1 4 0 1E5               R1 4 0 1E5
CC 4 5 20E-12            CC 4 5 15E-12
GM2 5 0 4 0 106          GM2 5 0 4 0 283
R01 3 5 150              R01 3 5 50
R02 5 0 150              R02 5 0 25
.ENDS 741                .ENDS LF351
```