EXAMINATION NO. 2 - SOLUTIONS

(Average Score = 70/100)

Problem 1 - (25 points)

A CMOS op amp is shown. All W/L values of all transistors are 10μm/1μm. Assume that K_N' = 110μA/V^2, K_P' = 50μA/V^2, V_{TN} = 0.7V, V_{TP} = -0.7V, λ_N = 0.04V^{-1}, and λ_P = 0.05V^{-1}. Find the low frequency differential voltage gain, v_{out}/v_{in}, the gainbandwidth, *GB*, the slew rate, *SR*, and the power dissipation, P_{diss} if V_{DD} = 2V.

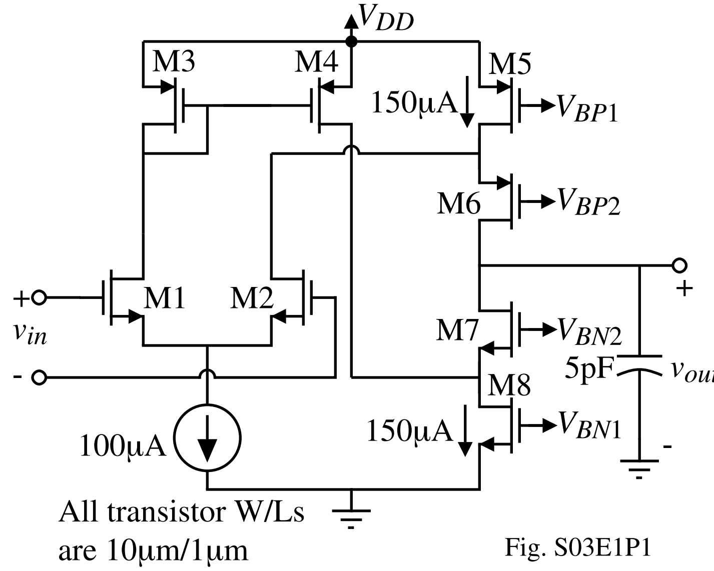


Fig. S03E1P1

Solution

The small-signal voltage gain can be expressed as,

$$\frac{v_{out}}{v_{in}} = g_{m1}R_{out} = g_{m2}R_{out}{}^*$$

where $R_{out} \approx [g_{m7}r_{ds7}(r_{ds4} \| r_{ds8})] \| [g_{m6}r_{ds6}(r_{ds2} \| r_{ds5})]$

Evaluating the small signal parameters,

$$g_{m1} = g_{m2} = \sqrt{2\cdot110\cdot10\cdot50} = 331.7\mu S,\ r_{ds1} = r_{ds2} = (25/50)M\Omega = 0.5M\Omega$$

$$g_{m6} = \sqrt{2\cdot50\cdot10\cdot100} = 316.2\mu S,\ r_{ds6} = (20/100)M\Omega = 0.2M\Omega$$

$$r_{ds5} = (20/150)M\Omega = 0.133M\Omega,\ r_{ds4} = (20/50)M\Omega = 0.4M\Omega$$

$$g_{m7} = \sqrt{2\cdot110\cdot10\cdot100} = 469\mu S,\ r_{ds7} = (25/100)M\Omega = 0.25M\Omega$$

$$r_{ds8} = (25/150)M\Omega = 0.167M\Omega$$

$$\therefore \quad R_{out} \approx [469\cdot0.25(0.4\|0.167)]\|[316.2\cdot0.2(0.5\|0.133)]M\Omega$$

$$= (13.796\|6.644)M\Omega = 4.484M\Omega$$

$$\frac{v_{out}}{v_{in}} = 331.7\cdot4.484 = \underline{\underline{1487\ V/V}}$$

$$GB = \frac{g_{m1}}{C_L} = \frac{331.7x10^{-6}}{5x10^{-12}} = 66.33x10^{+6} \rightarrow \underline{\underline{10.56MHz}}$$

$$SR = \frac{100\mu A}{C_L} = \frac{100x10^{-6}}{5x10^{-12}} = \underline{\underline{20V/\mu s}}$$

$$P_{diss} = 2(50\mu A+50\mu A+150\mu A) = \underline{\underline{500\mu W}}$$

* This expression ignores the fact that about half the signal is lost due to the input resistances at the sources of M6 and M7 are at an r_{ds} level.

Problem 2 - (25 points)

A two-stage, Miller compensated op amp has the following values: g_{mI} = 100µS, g_{mII} = 1000µS, C_c = 2pF, and C_L = 10pF.

a.) What value of nulling resistor, R_z, will cancel the output pole?

b.) If the output capacitance of the first stage is C_I = 1pF, what is the phase margin in part a.) if R_z is 5kΩ.

c.) If C_L is increased to 20pF and R_z = 5kΩ, what is the new phase margin?

Solution

a.) The zero is given as $z = \dfrac{1}{C_c\left(\dfrac{1}{g_{mII}} - R_z\right)}$ and the output pole is $p_2 = -\dfrac{g_{mII}}{C_c}$. Equating these two roots gives,

$$R_z = \frac{1}{g_{mII}}\left(\frac{C_L+C_c}{C_c}\right) = \frac{1}{1000\mu S}\left(\frac{12}{2}\right) = \underline{\underline{6k\Omega}}$$

b.) The pole due to R_z is

$$p_4 = -\frac{1}{R_zC_I} = -\frac{1}{5k\Omega\cdot 1pF} = -2x10^8 \text{ rads/sec.}$$

Also, the GB is

$$GB = \frac{g_{mI}}{C_c} = \frac{100\mu S}{2pF} = 50x10^6 \text{ rads/sec.}$$

The phase margin is,

$$PM = 180° - 90° - \tan^{-1}\left(\frac{GB}{|p_4|}\right) = 90° - \tan^{-1}\left(\frac{50}{200}\right) = 90° - 14° = \underline{\underline{76°}}$$

(You were intended to assume that z_1 still cancels p_2. If you did assume this, the answer is 71.2° and you were given full credit.)

c.) The new phase margin is,

$$PM = 180° - 90° + \tan^{-1}\left(\frac{GB}{|p_2|}\right) - \tan^{-1}\left(\frac{2GB}{|p_2|}\right) - \tan^{-1}\left(\frac{GB}{|p_4|}\right)$$

$$z_1 = -\frac{g_{mII}}{C_L} = -\frac{1000\mu S}{10pF} = -100x10^6 \text{ rads/sec.}$$

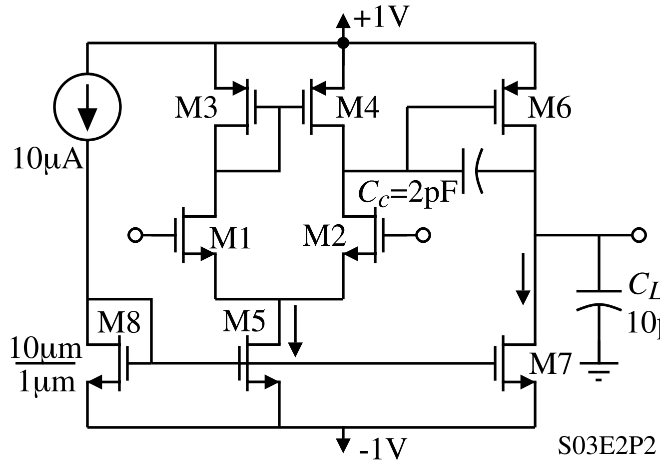
$$p_2 = -\frac{g_{mII}}{C_L} = -\frac{1000\mu S}{20pF} = -50x10^6 \text{ rads/sec.}$$

$$\therefore\ PM = 90° + \tan^{-1}\left(\frac{50}{50}\right) - \tan^{-1}\left(\frac{100}{50}\right) - \tan^{-1}\left(\frac{50}{200}\right) = 90° + 43° - 63.43° - 14° = \underline{\underline{57.52°}}$$

(Again, you were intended to assume that z_1 is at the old value of p_2. If you did assume this, the answer is 52.8° and you were given full credit.)

Problem 3 - (25 points)

For the CMOS op amp shown, assume the model parameters for the transistors are $K_N' = 110\mu A/V^2$, $K_P' = 50\mu A/V^2$, $V_{TN} = 0.7V$, $V_{TP} = -0.7V$, $\lambda_N = 0.04V^{-1}$, and $\lambda_P = 0.05V^{-1}$. Let all transistor lengths be 1µm and design the widths of every transistor and the dc currents I_5 and I_7 to satisfy the following specifications:



Slew rate = 10V/µs
+ICMR = 0.8V
-ICMR = 0V
GB = 10MHz
Phase margin = 60° ($g_{mII} = 10g_{mI}$ and $V_{SG4}=V_{SG6}$)

Solution

Slew rate $\rightarrow I_5 = SR{\cdot}C_L = 10V\mu s{\cdot}10pF = \underline{\underline{20\mu A}} \rightarrow W_5 = 20\mu m$ (check $-ICMR$ later)

$+ICMR \rightarrow 0.8 = 1 - V_{SG3} + 0.7 \rightarrow V_{SG3} = 0.9 \rightarrow V_{SD3}(\text{sat}) = 0.2V$

$$\frac{W_3}{L_3} = \frac{W_4}{L_4} = \frac{2I_3}{K_P(V_{SD3}(\text{sat}))^2} = \frac{2\cdot 10}{50(0.2)^2} = 10 \rightarrow W_3 = W_4 = \underline{\underline{10\mu m}}$$

$$GB \rightarrow \frac{g_{m1}}{C_c} = GB \rightarrow g_{m1} = GB{\cdot}C_c = 20\pi x10^6{\cdot}2x10^{-12} = 40\pi\ \mu S$$

$$g_{m1} = \sqrt{2K_N\frac{W_1}{L_1}I_1} \rightarrow g_{m1}{}^2 = 2K_N\frac{W_1}{L_1}I_1 \rightarrow \frac{W_1}{L_1} = \frac{g_{m1}{}^2}{2K_NI_1} = \frac{(40\pi)^2}{2\cdot 110\cdot 10} = 7.17$$

$W_1 = W_2 = \underline{\underline{7.17\mu m}}$

$$-ICMR \rightarrow 0 = V_{GS1} + V_{DS5}(\text{sat}) - 1 \rightarrow V_{DS5}(\text{sat}) = 1 - V_{GS1} = 1 - \sqrt{\frac{2I_1}{K_N(W_1/L_1)}} - V_{TN}$$

$$V_{DS5}(\text{sat}) = 1 - \sqrt{\frac{20}{110\cdot 7.17}} + 0.7 = 1 - 0.859 = 0.141V$$

$$\frac{W_5}{L_5} = \frac{2I_5}{K_N(V_{DS5}(\text{sat}))^2} = \frac{2\cdot 10}{110(0.141)^2} = 18.29 \rightarrow W_5 = 18.29\mu m \rightarrow W_5 = \underline{\underline{20\mu m}}$$

60° phase margin $\rightarrow g_{m6} = 10\ g_{m1} = 400\pi\ \mu S$

$$\text{Also, } g_{m4} = \sqrt{2\cdot 50\cdot 10\cdot 10} = 100\mu S \qquad \rightarrow W_6 = \frac{400\pi}{100}10 = 40\pi = \underline{\underline{125.66\ \mu m}}$$

$$I_6 = \frac{g_{m6}{}^2}{2K_P(W_6/L_6)} = \frac{(400\pi)^2}{100\cdot 125.66} = \underline{\underline{125.66\mu A}}$$

$$W_7 = W_5\frac{I_6}{I_5} = 20\frac{125.66}{20} = \underline{\underline{125.66\mu m}}$$

Problem 4 - (25 points)

a) $RI_{C9}=V_T ln(I_{C8}/I_{C9})$

$I_{C8}=(20V-1.4V)/50k\Omega=0.372mA$

$IC_9=20\mu A$ → $\underline{R=3.8k\Omega}$

b) $G_{mI} = \frac{g_{m4}}{1+g_{m4}r_{e2}} = \frac{g_{m4}}{2} = 0.1923\times 10^{-3}\ \frac{A}{V}$ $\quad$ *note that*: $r_{e2} = \frac{1}{g_{m2}} = \frac{1}{g_{m4}}$

$R_I = r_{07} \parallel r_{04}(1+g_{m4}\times r_{e2}) = r_{07}\parallel 2r_{04} = 5.65M\Omega$ $\quad$ *because*: $r_{07} = \frac{V_A}{I_7} = 13M\Omega$ *and* $r_{04} = 5l$

$G_{mII} = \frac{g_{m14}}{1+g_{m14}R_2} = 0.23\times 10^{-3}\ \frac{A}{V}$ $\quad$ *note that*: $g_{m14} = \sqrt{2\mu C_{ox}(\frac{W}{L})_{14} I_{M14}} = 0.424\times 10^{-3}\ \frac{A}{V}$

$R_{II} = r_{ds14}(1+g_{m14}\times R_2)\parallel r_{013}(1+g_{m13}\times R_3) = 1.848M\Omega \parallel 1.13M\Omega = 0.7M\Omega$

$A_v = G_{mI}R_I G_{mII}R_{II} \cong 175{,}000$

c) $GBW = \frac{G_{mI}}{C_C} \quad \Rightarrow \quad C_C = \frac{G_{mI}}{GBW} = 15.3pF$

d) $\Phi_M = 90^\circ - \tan^{-1}(\frac{2MHz}{p_2}) - \tan^{-1}(\frac{2MHz}{z})$

$p_2 = \frac{G_{mII}}{C_L} = \frac{0.23\times 10^{-3}}{5\times 10^{-12}} = 46\times 10^6\ \ rad/\sec$

$z = \frac{G_{mII}}{C_C} = \frac{0.23\times 10^{-3}}{15.3\times 10^{-12}} = 15.03\times 10^6\ \ rad/\sec$

$\Rightarrow \quad \Phi_M = 90^\circ - \tan^{-1}(\frac{2\times 2\pi}{46}) - \tan^{-1}(\frac{2\times 2\pi}{15.03}) = 90^\circ - 15.27^\circ - 39.89^\circ = 34.83^\circ$