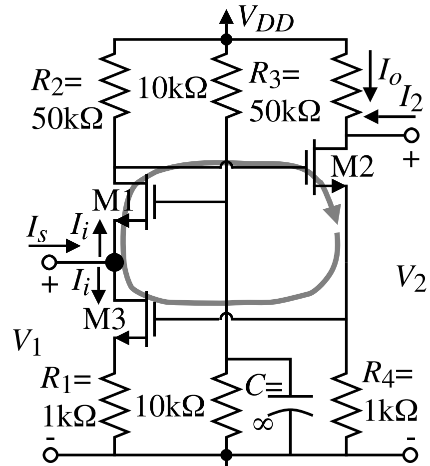
EXAMINATION NO. 3 - SOLUTIONS

(Average = 47, High = 65, Low = 27)

Problem 1 - (25 points)

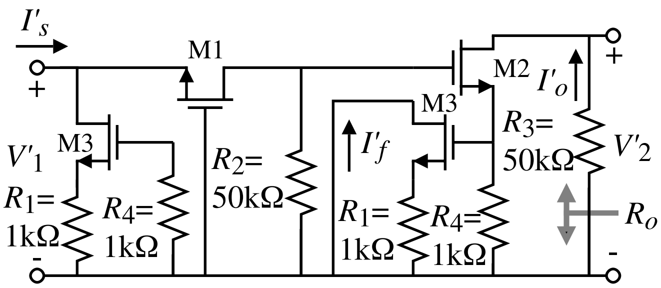
The simplified schematic of a feedback amplifier is shown. Use the method of feedback analysis to find V_2/V_1, $R_{in} = V_1/I_1$, and $R_{out} = V_2/I_2$. Assume that all transistors are matched and that g_m = 1mA/V and $r_{ds} = \infty$.



S02E3S1A

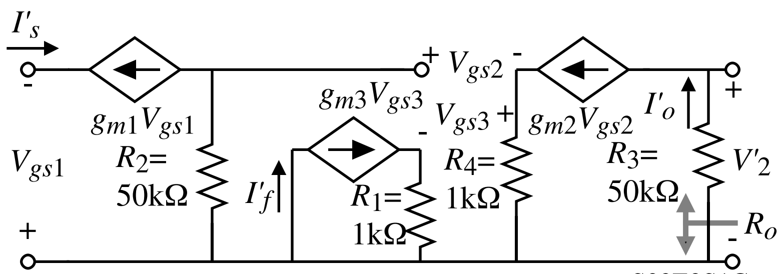
Solution

This feedback circuit is shunt-series. We begin with the open-loop, quasi-ac model shown below.



S02E3S1B

The small-signal, open-loop model is:



S02E3S1C

$$\frac{I'_o}{I'_s} = \left(\frac{I'_o}{V_{gs2}}\right)\left(\frac{V_{gs2}}{V_{gs1}}\right)\left(\frac{V_{gs1}}{I'_s}\right)$$

$$V_{gs2} = -g_{m1}V_{gs1}R_2 - g_{m2}V_{gs2}R_4$$

or

$$\frac{V_{gs2}}{V_{gs1}} = \frac{-g_{m1}R_2}{1+g_{m2}R_4} = -\frac{50}{2} = -25 \qquad \therefore\ a = \frac{I'_o}{I'_s} = (g_{m2})(-25)\left(\frac{-1}{g_{m1}}\right) = 25\text{A/A}$$

$$f = \frac{I'_f}{I'_o} = \left(\frac{I'_f}{V_{gs3}}\right)\left(\frac{V_{gs3}}{I'_o}\right) = (g_{m3})\left(\frac{R_4}{1+g_{m3}R_1}\right) = (1\text{mA/V})(0.5\text{k}\Omega) = 0.5$$

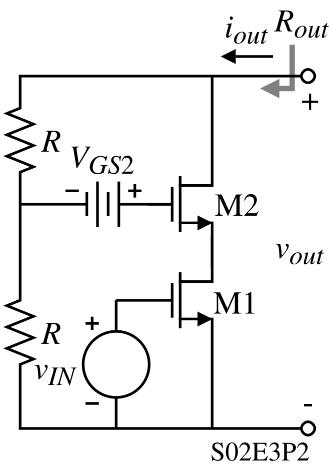
$$\therefore\ af = 25\cdot 0.5 = 12.5$$

$$R_i = \frac{v'_1}{I'_s} = \frac{1}{g_{m1}} = 1\text{k}\Omega \ \rightarrow\ \boxed{R_{in} = R_{if} = \frac{R_i}{1+af} = \frac{1000}{13.5} = 74.07\Omega}$$

$$\boxed{R_{out} = 50\text{k}\Omega \ \ (R_3 \text{ is outside the feedback loop})}$$

$$\frac{I_o}{I_s} = \frac{a}{1+af} = \frac{25}{1+12.5} = 1.852\text{ A/A} \ \rightarrow\ \boxed{\frac{v_2}{v_1} = \frac{I_o(-50\text{k}\Omega)}{I_s(74.07\Omega)} = -1240.1\text{ V/V}}$$

Problem 2 - (25 points)



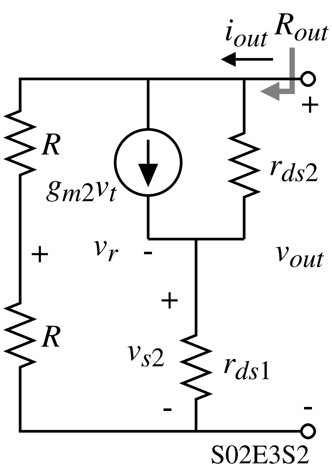
Use the Blackman's formula (see below) to calculate the small-signal output resistance of the stacked MOSFET configuration having identical drain-source drops for both transistors. Express your answer in terms of all the pertinent small-signal parameters and then simplify your answer if $g_m > g_{ds} > (1/R)$. Assume the MOSFETs are identical.

$$R_{out} = R_{out}\,(g_m{=}0)\left[\frac{1 + RR(\text{output port shorted})}{1 + RR(\text{output port open})}\right]$$

(You may use small-signal analysis if you wish but this circuit seems to be one of the rare cases where feedback analysis is more efficient.)

Solution

$$R_{out}\,(g_{m2}{=}0) = 2R \| (r_{ds1} + r_{ds2}) = \frac{2R(r_{ds1}+r_{ds2})}{2R+r_{ds1}+r_{ds2}}$$



RR(port shorted) = ?

$$v_r = 0 - v_{s2} = -\,g_{m2}v_t\left(\frac{r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}}\right)$$

$$\Rightarrow \quad RR(\text{port shorted}) = \frac{g_{m2}r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}}$$

RR(port open) = ?

$$v_r = -\,g_{m2}v_t\left(\frac{r_{ds2}(r_{ds1}+R)}{r_{ds1}+r_{ds2}+2R}\right)$$

$$\Rightarrow \quad RR(\text{port open}) = \frac{g_{m2}r_{ds2}(r_{ds1}+R)}{r_{ds1}+r_{ds2}+2R}$$

$$\therefore \quad R_{out} = \frac{2R(r_{ds1}+r_{ds2})}{2R+r_{ds1}+r_{ds2}}\left[\frac{1+\dfrac{g_{m2}r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}}}{1+\dfrac{g_{m2}r_{ds2}(r_{ds1}+R)}{r_{ds1}+r_{ds2}+2R}}\right] = 2R\left(\frac{r_{ds1}+r_{ds2}+g_{m2}r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}+2R+g_{m2}r_{ds2}(r_{ds1}+R)}\right)$$

Using the assumptions of $g_m > g_{ds} > (1/R)$ we can simplify R_{out} as

$$R_{out} \approx 2R\left(\frac{g_{m2}r_{ds1}r_{ds2}}{g_{m2}r_{ds2}R}\right) = \underline{\underline{2r_{ds1}}}$$

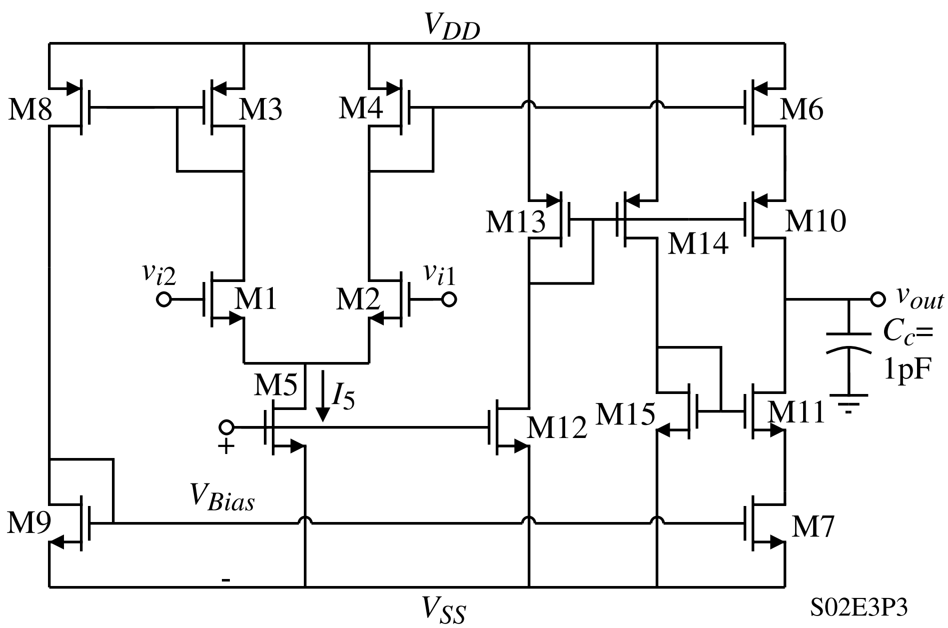
What is the insight? Well there are two feedback loops, one a series (the normal cascode) and one a shunt. Apparently they are working against each other and the effective output resistance is pretty much what it would be if there were two transistors in series without any feedback.

Problem 3 - (25 points)

Calculate the small-signal voltage gain, the SR (C_L = 1pF), and the P_{diss} for the op amp shown where I_5 = 100nA and all transistors M1-M11 have a W/L of 10µm/1µm and V_{DD} = -V_{SS} = 1.5V. If the minimum voltage across the drain-source of M6 and M7 are to be 0.1V, design the W/L ratios of M12-M15 that give the maximum plus and minus output voltage swing assuming that transistors M12 and M15 have a current of 50nA. The transistors are working in weak inversion and are modeled by the large signal model of

$$i_D = \frac{W}{L} I_{DO} \exp\left(\frac{v_{GS}}{nV_t}\right)$$

where I_{DO} = 2nA for PMOS and NMOS and n_P = 2.5 and n_N = 1.5. Assume V_t = 26mV and λ_N=0.4V^{-1} and λ_P=0.5V^{-1}.



Soluton

The small-signal voltage gain, A_v, is $g_{m1}R_{out}$ (see end of solution) where,

$$R_{out} = (r_{ds6}g_{m10}r_{ds10})||(r_{ds7}g_{m11}r_{ds11})$$

With the currents and W/L ratios of transistors M1 through M11 known, we get

$$g_{m1} = g_{m11} = \frac{50\text{nA}}{1.5\cdot 26\text{mV}} = 1.282\mu\text{S} \quad \text{and} \quad r_{ds7} = r_{ds11} = \frac{10^9}{0.04\cdot 50} = 0.5\text{x}10^9\Omega$$

$$g_{m10} = \frac{50\text{nA}}{2.5\cdot 26\text{mV}} = 0.769\mu\text{S} \quad \text{and} \quad r_{ds6} = r_{ds10} = \frac{10^9}{0.05\cdot 50} = 0.4\text{x}10^9\Omega$$

$$R_{out} = (0.4\text{x}10^9\cdot 1.282\text{x}10^{-6}\cdot 0.4\text{x}10^9)||(0.5\text{x}10^9\cdot 0.769\text{x}10^{-6}\cdot 0.5\text{x}10^9) = 9.924\text{x}10^{10}\Omega$$

$$\therefore \quad A_v = 1.282\text{x}10^{-6}\cdot 9.924\text{x}10^{10} = \underline{\underline{127{,}726 \text{ V/V}}}$$

$$SR = \frac{100\text{nA}}{1\text{pF}} = \underline{\underline{0.1\text{V}/\mu\text{s}}}$$

$$P_{diss} = 3\text{V}(50\text{nA}\cdot 6) = \underline{\underline{0.9\mu\text{W}}}$$

Problem 3 - Continued

Design of the *W/L*'s of M12 through M15:

To get 50nA in M12 means the $W_{12}/L_{12} = 0.5(W_5/L_5) = \underline{\underline{5\mu m/1\mu m}}$

M15:

$$V_{GS11} = n_N V_t \ln\left(\frac{I_D}{I_{DO}(W/L)}\right) = 1.5\cdot 0.026 \ln\left(\frac{50}{2\cdot 10}\right) = 0.0357\text{V}$$

$$\therefore \quad V_{GS15} = 0.1 + 0.0357 = 0.1357\text{V} \rightarrow \frac{W_{15}}{L_{15}} = \frac{50\text{nA}}{2\text{nA}\cdot\exp\left(\frac{135.7}{1.5\cdot 26}\right)} = \underline{\underline{0.77\mu m/1\mu m}}$$

M13 and M14:

$$V_{SG10} = n_P V_t \ln\left(\frac{I_D}{I_{DO}(W/L)}\right) = 2.5\cdot 0.026 \ln\left(\frac{50}{2\cdot 10}\right) = 0.0596\text{V}$$

$$\therefore \quad V_{GS13} = 0.1 + 0.0596 = 0.1596\text{V} \rightarrow \frac{W_{13}}{L_{13}} = \frac{50\text{nA}}{2\text{nA}\cdot\exp\left(\frac{159.6}{1.5\cdot 26}\right)} = 2.146\mu m/1\mu m$$

Thus

$$\frac{W_{13}}{L_{13}} = \frac{W_{14}}{L_{14}} = \underline{\underline{2.146\mu m/1\mu m}}$$

Comments on the small-signal gain:

It is much easier to use the expression $g_{m1}R_{out}$ for the small-signal voltage gain. However, some prefer the following expression,

$$v_{out} = \left(\frac{g_{m1}\cdot g_{m8}\cdot g_{m7}}{2g_{m3}\cdot g_{m9}} + \frac{g_{m2}\cdot g_{m6}}{2g_{m4}}\right) R_{out}$$

which is equivalent since $g_{m3}=g_{m8}$, $g_{m7}=g_{m9}$, $g_{m4}=g_{m6}$, and $g_{m1}=g_{m2}$.