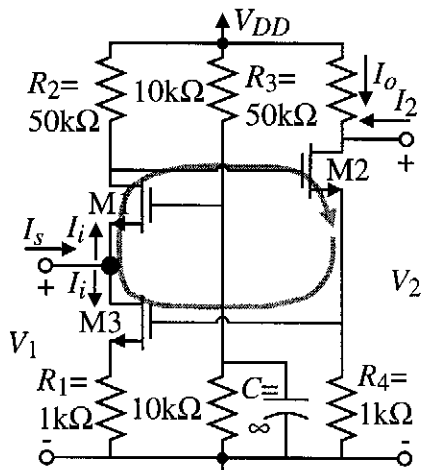
Homework Assignment No. 11 - Solutions

Problem 1 - (10 points)

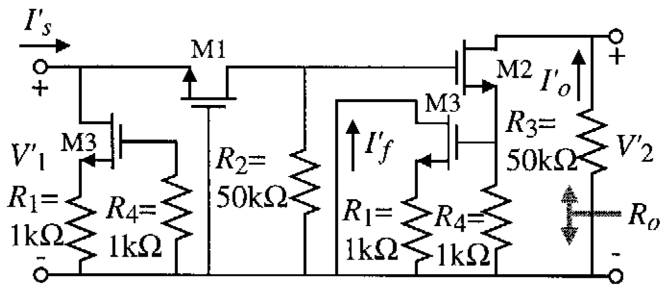
The simplified schematic of a feedback amplifier is shown. Use the method of feedback analysis to find V_2/V_1, $R_{in} = V_1/I_1$, and $R_{out} = V_2/I_2$. Assume that all transistors are matched and that g_m = 1mA/V and $r_{ds} = \infty$.



S02E3S1A

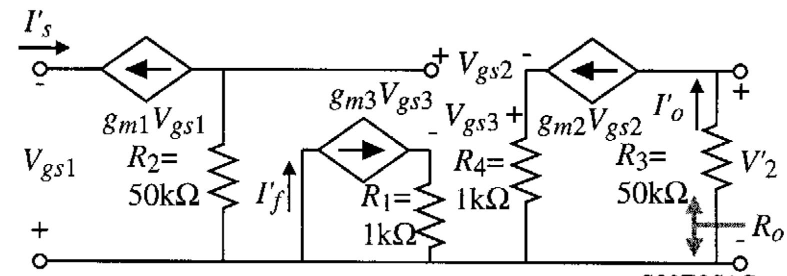
Solution

This feedback circuit is shunt-series. We begin with the open-loop, quasi-ac model shown below.



S02E3S1B

The small-signal, open-loop model is:



S02E3S1C

$$\frac{I'_o}{I'_s} = \left(\frac{I'_o}{V_{gs2}}\right)\left(\frac{V_{gs2}}{V_{gs1}}\right)\left(\frac{V_{gs1}}{I'_s}\right)$$

$V_{gs2} =$

$$-g_{m1}V_{gs1}R_2 - g_{m2}V_{gs2}R_4$$

or

$$\frac{V_{gs2}}{V_{gs1}} = \frac{-g_{m1}R_2}{1 + g_{m2}R_4} = -\frac{50}{2} = -25 \qquad \therefore\ a = \frac{I'_o}{I'_s} = (g_{m2})(-25)\left(\frac{-1}{g_{m1}}\right) = 25\text{A/A}$$

$$f = \frac{I'_f}{I'_o} = \left(\frac{I'_f}{V_{gs3}}\right)\left(\frac{V_{gs3}}{I'_o}\right) = (g_{m3})\left(\frac{R_4}{1 + g_{m3}R_1}\right) = (1\text{mA/V})(0.5\text{k}\Omega) = 0.5$$

$$\therefore\ af = 25\cdot 0.5 = 12.5$$

$$R_i = \frac{v'_1}{I'_s} = \frac{1}{g_{m1}} = 1\text{k}\Omega \ \rightarrow\ \boxed{R_{in} = R_{if} = \frac{R_i}{1 + af} = \frac{1000}{13.5} = 74.07\Omega}$$

$$\boxed{R_{out} = 50\text{k}\Omega\ \ (R_3 \text{ is outside the feedback loop})}$$

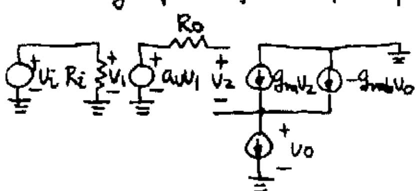
$$\frac{I_o}{I_s} = \frac{a}{1+af} = \frac{25}{1+12.5} = 1.852\text{ A/A} \ \rightarrow\ \boxed{\frac{v_2}{v_1} = \frac{I_o(-50\text{k}\Omega)}{I_s(74.07\Omega)} = -1240.1\text{ V/V}}$$

Problem 2 – (10 points)

Problem 8.30 of GHLM

(a)

The basic amplifier without the feedback signal inserted at the inverting input of the opamp



$g_m = \sqrt{2k'\frac{W}{L}I_D} = \sqrt{2\times 180\times 10^{-6}\times 100\times 0.5\times 10^{-3}}$
$= 4.2\times 10^{-3}$ A/V

$g_{mb} = \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} g_m = \frac{\gamma}{2\sqrt{2\phi_f}} g_m$
$= \frac{0.3}{2\sqrt{2\times 0.3}} 4.2\times 10^{-3} = 8.1\times 10^{-4}$ A/V

$v_o = g_m(a_v v_i - v_o)\frac{1}{g_{mb}}$

$a = \frac{v_o}{v_i} = a_v \frac{g_m}{g_m + g_{mb}}$

$f = 1$

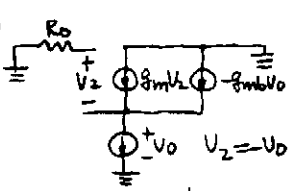
$af = a_v \frac{g_m}{g_m+g_{mb}} = 1000\frac{4.2}{4.2+0.81} = 838$

$A = \frac{a}{1+af} = \frac{838}{1+838} = 0.999$

$r_{ia} = R_i$

$R_{in} = r_{ia}(1+af) = R_i(1+af)$
$= 1M(1+838) = 839\ M\Omega$

$v_i = 0$

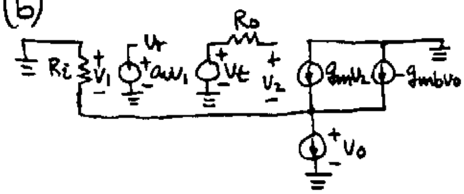


$r_{oa} = \frac{1}{g_m} \| \frac{1}{g_{mb}} = \frac{1}{g_m + g_{mb}}$

$R_{out} = \frac{r_{oa}}{1+af} = \frac{1}{g_m+g_{mb}}\frac{1}{1+af}$

$= \frac{1}{4.2\times 10^{-3} + 8.1\times 10^{-4}}\frac{1}{1+838}$
$= 0.238\ \Omega$

(b)



$v_o = g_m(v_t - v_o)(\frac{1}{g_{mb}} \| R_i)$

$v_o = \frac{g_m}{\frac{1}{R_i} + g_m + g_{mb}}$

$\mathcal{R} = a_v \frac{g_m}{\frac{1}{R_i} + g_m + g_{mb}}$

$\approx a_v \frac{g_m}{g_m + g_{mb}}$

$= 838$

$A_\infty = \frac{v_o}{v_i}\Big|_{a_v=\infty} = 1 \quad (v_1 = 0 \text{ and } v_o = v_i)$

$d = \frac{v_o}{v_i}\Big|_{a_v=0} = \frac{\frac{1}{g_m}\|\frac{1}{g_{mb}}}{R_i + \frac{1}{g_m}\|\frac{1}{g_{mb}}}$

$= \frac{\frac{1}{g_m+g_{mb}}}{R_i + \frac{1}{g_m+g_{mb}}}$

$\approx \frac{1}{(g_m+g_{mb})R_i}$

$= \frac{1}{(4.2\times 10^{-3} + 8.1\times 10^{-4})10^6}$

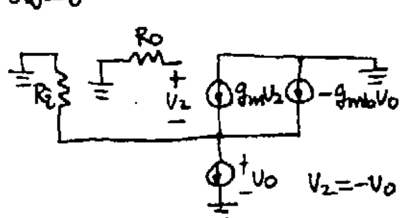
$= 2.00\times 10^{-4}$

$A = A_\infty \frac{\mathcal{R}}{1+\mathcal{R}} + \frac{d}{1+\mathcal{R}}$

$= 1\frac{838}{1+838} + \frac{2.00\times 10^{-4}}{1+838}$

$= 0.999$

$a_v = 0$



$R_{in}(a_v=0) = R_i + \frac{1}{g_m}\|\frac{1}{g_{mb}} \approx R_i = 1M\Omega$

$\mathcal{R}(\text{short}) = \mathcal{R} = 838$

$\mathcal{R}(\text{open}) = 0 \quad (v_1 = 0)$

$R_{in} = R_{in}(a_v=0)\frac{1+\mathcal{R}(\text{short})}{1+\mathcal{R}(\text{open})} \approx R_i\frac{1+\mathcal{R}}{1+0}$

$= R_i(1+\mathcal{R}) = 1M(1+838) = 839M\Omega$

$R_{out}(a_v=0) = R_i \| \frac{1}{g_m} \| \frac{1}{g_{mb}} \approx \frac{1}{g_m}\|\frac{1}{g_{mb}}$

$= \frac{1}{g_m+g_{mb}} = \frac{1}{4.2\times 10^{-3} + 8.1\times 10^{-4}} = 200\Omega$

$\mathcal{R}(\text{short}) = 0 \quad (v_o = 0)$

$\mathcal{R}(\text{open}) = \mathcal{R}$

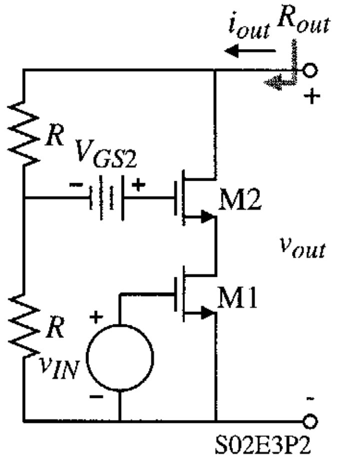
$R_{out} = R_{out}(a_v=0)\frac{1+\mathcal{R}(\text{short})}{1+\mathcal{R}(\text{open})}$

$\approx \frac{1}{g_m+g_{mb}}\frac{1+0}{1+\mathcal{R}} = \frac{1}{g_m+g_{mb}}\frac{1}{1+\mathcal{R}}$

$= 200\frac{1}{1+838} = 0.238\Omega$

Problem 3 - (10 points)

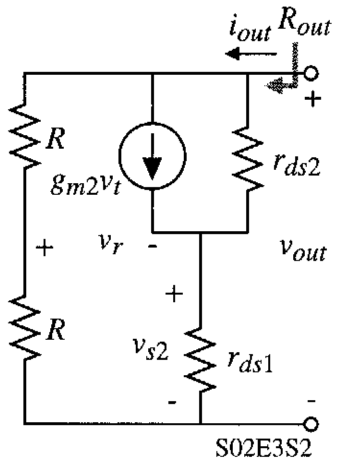
Use the Blackman's formula (see below) to calculate the small-signal output resistance of the stacked MOSFET configuration having identical drain-source drops for both transistors. Express your answer in terms of all the pertinent small-signal parameters and then simplify your answer if $g_m > g_{ds} > (1/R)$. Assume the MOSFETs are identical.



$$R_{out} = R_{out}\,(g_m{=}0)\left[\frac{1 + RR(\text{output port shorted})}{1 + RR(\text{output port open})}\right]$$

(You may use small-signal analysis if you wish but this circuit seems to be one of the rare cases where feedback analysis is more efficient.)

Solution



$$R_{out}\,(g_{m2}{=}0) = 2R \| (r_{ds1} + r_{ds2}) = \frac{2R(r_{ds1}+r_{ds2})}{2R+r_{ds1}+r_{ds2}}$$

RR(port shorted) = ?

$$v_r = 0 - v_{s2} = -\,g_{m2}v_t\left(\frac{r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}}\right)$$

$$\Rightarrow \qquad RR(\text{port shorted}) = \frac{g_{m2}r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}}$$

RR(port open) = ?

$$v_r = -\,g_{m2}v_t\left(\frac{r_{ds2}(r_{ds1}+R)}{r_{ds1}+r_{ds2}+2R}\right)$$

$$\Rightarrow \qquad RR(\text{port open}) = \frac{g_{m2}r_{ds2}(r_{ds1}+R)}{r_{ds1}+r_{ds2}+2R}$$

$$\therefore \qquad R_{out} = \frac{2R(r_{ds1}+r_{ds2})}{2R+r_{ds1}+r_{ds2}}\left[\frac{1+\dfrac{g_{m2}r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}}}{1+\dfrac{g_{m2}r_{ds2}(r_{ds1}+R)}{r_{ds1}+r_{ds2}+2R}}\right] = 2R\left(\frac{r_{ds1}+r_{ds2}+g_{m2}r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}+2R+g_{m2}r_{ds2}(r_{ds1}+R)}\right)$$

Using the assumptions of $g_m > g_{ds} > (1/R)$ we can simplify R_{out} as

$$R_{out} \approx 2R\left(\frac{g_{m2}r_{ds1}r_{ds2}}{g_{m2}r_{ds2}R}\right) = \underline{\underline{2r_{ds1}}}$$

What is the insight? Well there are two feedback loops, one a series (the normal cascode) and one a shunt. Apparently they are working against each other and the effective output resistance is pretty much what it would be if there were two transistors in series without any feedback.

Problem 4 – (10 points)

Find the dominant roots of the MOS follower and the BJT follower for the buffered, class-A op amp of Ex. 7.1-2. Use the capacitances of Table 3.2-1. Compare these root locations with the fact that GB = 5MHz? Assume the capacitances of the BJT are C_π = 10pf and C_μ = 1pF.

Solution

The model of just the output buffer of Ex. 7.1-2 is shown.

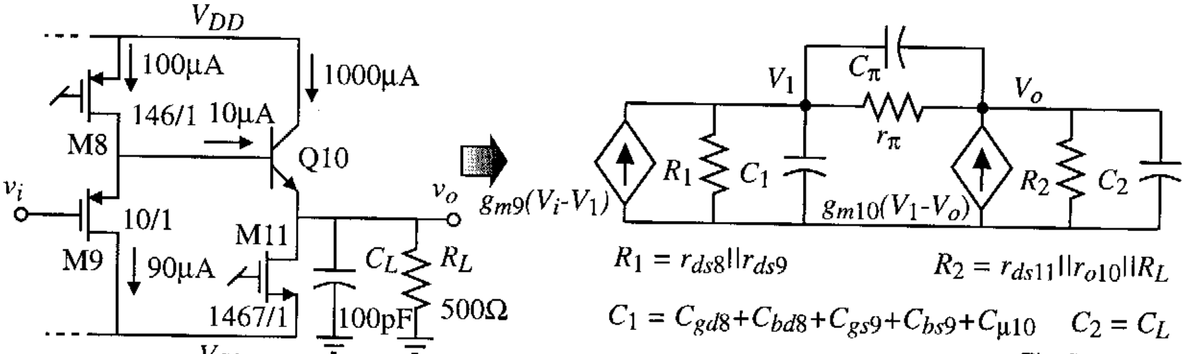


Fig. S7.1-10

The nodal equations can be written as,

$$g_{m9}V_i = (g_{m9} + G_1 + g_{\pi 10} + sC_{\pi 10} + sC_1)V_1 - (g_{\pi 10} + sC_{\pi 10})V_o$$

$$0 = -(g_{m10} + g_{\pi 10} + sC_{\pi 10})V_1 + (g_{m10} + G_2 + g_{\pi 10} + sC_{\pi 10} + sC_2)V_o$$

Solving for V_o/V_i gives,

$$\frac{V_o}{V_i} = \frac{g_{m9}(g_{m10}+ g_{\pi 10}+ sC_{\pi 10})}{(g_{\pi 10} + sC_{\pi 10})(g_{m9}+ G_1 + G_2 + sC_1 + sC_2) + (g_{m10}+ G_2+ sC_2)(g_{m9}+ G_1 + sC_1)}$$

$$\frac{V_o}{V_i} = \frac{g_{m9}(g_{m10}+ g_{\pi 10}+ sC_{\pi 10})}{a_0 + sa_1 + s^2a_2}$$

where

$$a_0 = g_{m9}g_{\pi 10} + g_{\pi 10}G_1 + \mathbf{g_{\pi 10}G_2} + \mathbf{g_{m9}g_{m10}} + \mathbf{g_{m10}G_1} + \mathbf{g_{m9}G_2} + G_1G_2$$

$$a_1 = g_{m9}C_{\pi 10}+G_1C_{\pi 10}+\mathbf{G_2C_{\pi 10}}+g_{\pi 10}C_1+\mathbf{g_{\pi 10}C_2}+\mathbf{g_{m10}C_1}+G_2C_1+\mathbf{g_{m9}C_2}+G_1C_2$$

$$a_2 = C_{\pi 10}C_1 + \mathbf{C_{\pi 10}C_2} + C_1C_2$$

The numerical value of the small signal parameters are:

$g_{m10} = \frac{1\text{mA}}{25.9\text{mV}}$ = 38.6mS, G_2 = 2mS, $g_{\pi 10}$ = 386μS, $g_{m9} = \sqrt{2\cdot 50\cdot 10\cdot 90}$ = 300μS, G_1 = $g_{ds8} + g_{ds9}$ = 0.05·100μA + 0.05·90μA = 9.5μS

$$C_2 = 100\text{pF},\ C_{\pi 10} = 10\text{pF},\ C_1 = C_{gs9} + C_{bs9} + C_{bd8} + C_{gd8} + C_{\mu 10}$$

$C_{gs9} = C_{ov} + 0.667C_{ox}W_9L_9 = (220\text{x}10^{-12})(10\text{x}10^{-6})$

$+0.667(24.7\text{x}10^{-4})(10\text{x}10^{-12})$ = 18.7fF

Problem 7.1-10 – Continued

$$C_{bs9} = 560\text{x}10^{-6}(30\text{x}10^{-12})+350\text{x}10^{-12}(26\text{x}10^{-6}) = 25.9\text{fF}$$

(Assumed area=3μmx10μm = 30μm and perimeter is 3μm+10μm+3μm+10μm = 26μm)

$$C_{bd8} = 560\text{x}10^{-6}(438\text{x}10^{-12})+350\text{x}10^{-12}(298\text{x}10^{-6}) = 349\text{fF}$$

$$C_{gd8} = C_{ov} = (220\text{x}10^{-12})(146\text{x}10^{-6}) = 32.1\text{fF}$$

$$\therefore \quad C_1 = 18.7\text{fF} + 25.9\text{fF} + 349\text{fF} + 32.1\text{fF} + 1000\text{fF} = 1.43\text{pF}$$

(We have ignored any reverse bias influence on pn junction capacitors.)

The dominant terms of a_0 , a_1, and a_2 based on these values are shown in boldface above.

$$\therefore \frac{V_o}{V_i} \approx \frac{g_{m9}(g_{m10}+ g_{\pi10}+ sC_{\pi10})}{g_{m9}g_{m10}+ g_{\pi10}G_2+s(G_2C_{\pi10}+g_{\pi10}C_2+g_{m10}C_1+g_{m9}C_2)+s^2C_2C_{\pi10}}$$

$$\frac{V_o}{V_i} = \frac{g_{m9}g_{m10}}{g_{m9}g_{m10}+ g_{\pi10}G_2}\left[\frac{1 + \frac{sC_{\pi10}}{g_{m10}}}{1 + s\left(\frac{G_2C_{\pi10}+g_{\pi10}C_2+g_{m10}C_1+g_{m9}C_2}{g_{m9}g_{m10}+ g_{\pi10}G_2}\right)+ s^2\frac{C_2C_{\pi10}}{g_{m9}g_{m10}+ g_{\pi10}G_2}}\right]$$

Assuming negative real axis roots widely spaced gives,

$$p_1 = -\frac{1}{\text{a}} = \frac{-(g_{m9}g_{m10}+ g_{\pi10}G_2)}{G_2C_{\pi10}+g_{\pi10}C_2+g_{m10}C_1+g_{m9}C_2} = -\frac{1.235\text{x}10^{-5}}{1.465\text{ x}10^{-13}} = \underline{\underline{-84.3\text{ x}10^6\text{ rads/sec.}}}$$

$= -13.4\text{MHz}$

$$p_2 = -\frac{\text{a}}{\text{b}} = \frac{-(G_2C_{\pi10}+g_{\pi10}C_2+g_{m10}C_1+g_{m9}C_2)}{C_2C_{\pi10}} = -\frac{1.465\text{ x}10^{-13}}{100\text{x}10^{-12}\cdot10\text{x}10^{-12}}$$

$= \underline{\underline{-146.5\text{ x}10^6\text{ rads/sec.}}} \rightarrow -23.32\text{MHz}$

$$z_1 = -\frac{g_{m10}}{C_{\pi10}} = -\frac{38.6\text{x}10^{-3}}{10\text{ x}10^{-12}} = \underline{\underline{-3.86\text{ x}10^9\text{ rads/sec.}}} \rightarrow -614\text{MHz}$$

We see that neither p_1 or p_2 is greater than $10GB$ if GB = 5MHz so they will deteriorate the phase margin of the amplifier of Ex. 7.1-2.

Problem 5 - (10 points)

Find the GB of a two-stage op amp using Miller compensation using a nulling resistor that has 60° phase margin where the second pole is -10x10^6 rads/sec and two higher poles both at -100x10^6 rads/sec. Assume that the RHP zero is used to cancel the second pole and that the load capacitance stays constant. If the input transconductance is 500μA/V, what is the value of C_c?

Solution

The resulting higher-order poles are two at -100x10^6 radians/sec. The resulting phase margin expression is,

$$\text{PM} = 180° - \tan^{-1}(A_v(0)) - 2\tan^{-1}\left(\frac{GB}{10^7}\right) = 90° - 2\tan^{-1}\left(\frac{GB}{10^7}\right) = 60°$$

$$\therefore\ 30° = 2\tan^{-1}\left(\frac{GB}{10^7}\right) \rightarrow \left(\frac{GB}{10^7}\right) = \tan(15°) = 0.2679$$

$$GB = 2.679\text{x}10^7 = \frac{g_{m1}}{C_c} \rightarrow C_c = \frac{500\text{x}10^{-6}}{26.79\text{x}10^7} = \underline{\underline{18.66\text{pF}}}$$