(READING: GHLM - 472-480, AH - 269-286)

## INTRODUCTION

## Objective

The objective of this presentation is:
1.) Develop the design equations for a two-stage CMOS op amp
2.) Illustrate the design of a two-stage CMOS op amp

## Outline

- Design relationships
- Design of Two Stage CMOS Op Amp
- Summary

OP AMP DESIGN

## Unbuffered, Two-Stage CMOS Op Amp



Notation:

$$
S_{i}=\frac{W_{i}}{L_{i}}=W / L \text { of the ith transistor }
$$

## Design Relationships for the Two-Stage Op Amp

Slew rate $S R=\frac{I_{5}}{C_{c}}\left(\right.$ Assuming $I_{7} \gg I_{5}$ and $\left.C_{L}>C_{c}\right)$
First-stage gain $A_{v 1}=\frac{g_{m 1}}{g_{d s 2}+g_{d s 4}}=\frac{2 g_{m 1}}{I_{5}\left(\lambda_{2}+\lambda_{4}\right)}$
Second-stage gain $A_{v 2}=\frac{g_{m 6}}{g_{d s 6}+g_{d s 7}}=\frac{g_{m 6}}{I_{6}\left(\lambda_{6}+\lambda_{7}\right)}$
Gain-bandwidth $G B=\frac{g_{m 1}}{C_{c}}$
Output pole $p_{2}=\frac{-g_{m 6}}{C_{L}}$
RHP zero $z_{1}=\frac{g_{m 6}}{C_{c}}$
$60^{\circ}$ phase margin requires that $g_{m 6}=2.2 g_{m 2}\left(C_{L} / C_{c}\right)$ if all other roots are $\geq 10 G B$.
Positive ICMR $\left.V_{\mathrm{in}(\max )}=V_{D D}-\sqrt{\frac{I_{5}}{\beta 3}}-\left|V_{T 03}\right|(\max )+V_{T 1(\min )}\right)$
Negative ICMR $V_{\mathrm{in}(\min )}=V_{S S}+\sqrt{\frac{I_{5}}{\beta_{1}}}+V_{T 1(\max )}+V_{D S 5}($ sat $)$
Saturation voltage $V_{D S}($ sat $)=\sqrt{\frac{2 I_{D S}}{\beta}} \quad$ (all transistors are saturated)

## Op Amp Specifications

The following design procedure assumes that specifications for the following parameters are given.

1. Gain at dc, $A_{v}(0)$
2. Gain-bandwidth, $G B$
3. Phase margin (or settling time)
4. Input common-mode range, ICMR
5. Load Capacitance, $C_{L}$
6. Slew-rate, $S R$
7. Output voltage swing
8. Power dissipation, $P_{\text {diss }}$


## Unbuffered Op Amp Design Procedure

This design procedure assumes that the gain at dc $\left(A_{\nu}\right)$, unity gain bandwidth $(G B)$, input common mode range $\left(V_{i n}(\min )\right.$ and $\left.V_{i n}(\max )\right)$, load capacitance $\left(C_{L}\right)$, slew rate $(S R)$, settling time $\left(T_{s}\right)$, output voltage swing ( $V_{\text {out }}(\max )$ and $V_{\text {out }}(\min )$ ), and power dissipation $\left(P_{\text {diss }}\right)$ are given. Choose the smallest device length which will keep the channel modulation parameter constant and give good matching for current mirrors.

1. From the desired phase margin, choose the minimum value for $C_{c}$, i.e. for a $60^{\circ}$ phase margin we use the following relationship. This assumes that $z \geq 10 \mathrm{~GB}$.

$$
C_{C}>0.22 C_{L}
$$

2. Determine the minimum value for the "tail current" $\left(I_{5}\right)$ from the largest of the two values.

$$
I_{5}=S R \cdot C_{c} \quad \text { or } \quad I_{5} \cong 10\left(\frac{V_{D D}+\left|V_{S S}\right|}{2 \cdot T_{s}}\right)
$$

3. Design for $S_{3}$ from the maximum input voltage specification.

$$
S_{3}=\frac{I_{5}}{K_{3}\left[V_{D D}-V_{i n}(\max )-\left|V_{T 03}\right|(\max )+V_{T 1}(\min )\right]^{2}}-1
$$

4. Verify that the pole of M 3 due to $C_{g s 3}$ and $C_{g s 4}\left(=0.67 \mathrm{~W}_{3} \mathrm{~L}_{3} C_{o x}\right)$ will not be dominant by assuming it to be greater than $10 G B$

$$
\frac{g_{m 3}}{2 C_{g s 3}}>10 G B .
$$

Unbuffered Op Amp Design Procedure - Continued
5. Design for $S_{1}\left(S_{2}\right)$ to achieve the desired $G B$.

$$
g_{m 1}=G B \cdot C_{c} \rightarrow S_{2}=\frac{g_{m 2^{2}}}{K_{2}^{\prime} I_{5}}
$$

6. Design for $S_{5}$ from the minimum input voltage. First calculate $V_{D S 5}(\mathrm{sat})$ then find $S_{5}$.

$$
V_{D S 5}(\mathrm{sat})=V_{i n}(\mathrm{~min})-V_{S S^{-}} \sqrt{\frac{I_{5}}{\beta_{1}}}-V_{T 1}(\max ) \geq 100 \mathrm{mV} \rightarrow S_{5}=\frac{2 I_{5}}{K_{5}^{\prime}\left[V_{D S 5}(\mathrm{sat})\right]^{2}}
$$

7. Find $S_{6}$ by letting the second pole $\left(p_{2}\right)$ be equal to 2.2 times GB and assuming that $V_{S G 4}=V_{S G 6}$.

$$
g_{m 6}=2.2 g_{m 2}\left(C_{L} / C_{c}\right) \rightarrow S_{6}=S_{4} \frac{g_{m 6}}{g_{m 4}}
$$

8. Calculate $I_{6}$ from

$$
I_{6}=\frac{g_{m 6^{2}}}{2 K_{6}^{\prime} S_{6}}
$$

Check to make sure that $S_{6}$ satisfies the $V_{\text {out }}(\max )$ requirement and adjust as necessary. 9. Design $S_{7}$ to achieve the desired current ratios between $I_{5}$ and $I_{6}$.

$$
\left.S_{7}=\left(I_{6} / I_{5}\right) S_{5} \quad \text { (Check the minimum output voltage requirements }\right)
$$

## Unbuffered Op Amp Design Procedure - Continued

10. Check gain and power dissipation specifications.

$$
A_{v}=\frac{2 g_{m 2} g_{m 6}}{I_{5}\left(\lambda_{2}+\lambda_{3}\right) I_{6}\left(\lambda_{6}+\lambda_{7}\right)} \quad P_{\text {diss }}=\left(I_{5}+I_{6}\right)\left(V_{D D}+\left|V_{S S}\right|\right)
$$

11. If the gain specification is not met, then the currents, $I_{5}$ and $I_{6}$, can be decreased or the W/L ratios of M2 and/or M6 increased. The previous calculations must be rechecked to insure that they are satisfied. If the power dissipation is too high, then one can only reduce the currents $I_{5}$ and $I_{6}$. Reduction of currents will probably necessitate increase of some of the W/L ratios in order to satisfy input and output swings.
12. Simulate the circuit to check to see that all specifications are met.

Example 1 - Design of a Two-Stage Op Amp
Using the material and device parameters given in Tables 3.1-1 and 3.1-2, design an amplifier similar to that shown in Fig. 6.3-1 that meets the following specifications. Assume the channel length is to be $1 \mu \mathrm{~m}$.

$$
\begin{array}{lll}
A_{V}>3000 \mathrm{~V} / \mathrm{V} & V_{D D}=2.5 \mathrm{~V} & V_{S S}=-2.5 \mathrm{~V} \\
G B=50^{\circ} \text { phase margin } \\
G B=5 \mathrm{MHz} & C_{L}=10 \mathrm{pF} & S R>10 \mathrm{~V} / \mu \mathrm{s} \\
V_{\text {out }} \text { range }= \pm 2 \mathrm{~V} & \text { ICMR }=-1 \text { to } 2 \mathrm{~V} & P_{\text {diss }} \leq 2 \mathrm{~mW}
\end{array}
$$

## Solution

1.) The first step is to calculate the minimum value of the compensation capacitor $C_{c}$,

$$
C_{c}>(2.2 / 10)(10 \mathrm{pF})=2.2 \mathrm{pF}
$$

2.) Choose $C_{c}$ as 3 pF . Using the slew-rate specification and $C_{c}$ calculate $I_{5}$.

$$
I_{5}=\left(3 \times 10^{-12}\right)(10 \times 106)=30 \mu \mathrm{~A}
$$

3.) Next calculate $(W / L)_{3}$ using ICMR requirements.

$$
(W / L)_{3}=\frac{30 \times 10^{-6}}{\left(50 \times 10^{-6}\right)[2.5-2-.85+0.55]^{2}}=15 \quad \rightarrow \quad(W / L)_{3}=(W / L)_{4}=15
$$

## Example 1-Continued

4.) Now we can check the value of the mirror pole, $p_{3}$, to make sure that it is in fact greater than $10 G B$. Assume the $C_{o x}=0.4 \mathrm{fF} / \mu \mathrm{m}^{2}$. The mirror pole can be found as

$$
p_{3} \approx \frac{-g_{m 3}}{2 C_{g s 3}}=\frac{-\sqrt{2 K_{p}^{\prime} S_{3} I_{3}}}{2(0.667) W_{3} L_{3} C_{o x}}=2.81 \times 10^{9}(\mathrm{rads} / \mathrm{sec})
$$

or 448 MHz . Thus, $p_{3}$, is not of concern in this design because $p_{3} \gg 10 G B$.
5.) The next step in the design is to calculate $g_{m 1}$ to get

$$
g_{m 1}=\left(5 \times 10^{6}\right)(2 \pi)\left(3 \times 10^{-12}\right)=94.25 \mu \mathrm{~S}
$$

Therefore, $(W / L)_{1}$ is

$$
(W / L)_{1}=(W / L)_{2}=\frac{g_{m 1^{2}}}{2 K_{N}^{\prime} I_{1}}=\frac{(94.25)^{2}}{2 \cdot 110 \cdot 15}=2.79 \approx 3.0 \quad \Rightarrow \quad(W / L)_{1}=(W / L)_{2}=3
$$

6.) Next calculate $V_{D S 5}$,

$$
V_{D S 5}=(-1)-(-2.5)-\sqrt{\frac{30 \times 10^{-6}}{110 \times 10^{-6} 3}}-.85=0.35 \mathrm{~V}
$$

Using $V_{D S 5}$ calculate $(W / L)_{5}$ from the saturation relationship.

$$
(W / L)_{5}=\frac{2(30 \times 10-6)}{(110 \times 10-6)(0.35)^{2}}=4.49 \approx 4.5 \quad \rightarrow \quad \quad(W / L)_{5}=4.5
$$

## Example 1 - Continued

7.) For $60^{\circ}$ phase margin, we know that

$$
g_{m 6} \geq 10 g_{m 1} \geq 942.5 \mu \mathrm{~S}
$$

Assuming that $g_{m 6}=942.5 \mu \mathrm{~S}$ and knowing that $g_{m 4}=150 \mu \mathrm{~S}$, we calculate $(W / L)_{6}$ as

$$
(W / L)_{6}=15 \frac{942.5 \times 10-6}{(150 \times 10-6)}=94.25 \approx 94
$$

8.) Calculate $I_{6}$ using the small-signal $g_{m}$ expression:

$$
I_{6}=\frac{(942.5 \times 10-6) 2}{(2)(50 \times 10-6)(94.25)}=94.5 \mu \mathrm{~A} \approx 95 \mu \mathrm{~A}
$$

If we calculate $(W / L)_{6}$ based on $V_{\text {out }}(\max )$, the value is approximately 15 . Since 94 exceeds the specification and maintains better phase margin, we will stay with $(W / L)_{6}=$ 94 and $I_{6}=95 \mu \mathrm{~A}$.
With $I_{6}=95 \mu \mathrm{~A}$ the power dissipation is

$$
P_{\text {diss }}=5 \mathrm{~V} \cdot(30 \mu \mathrm{~A}+95 \mu \mathrm{~A})=0.625 \mathrm{~mW} .
$$

## Example 1-Continued

9.) Finally, calculate $(W / L)_{7}$

$$
(W / L)_{7}=4.5\left(\frac{95 \times 10-6}{30 \times 10-6}\right)=14.25 \approx 14 \quad \rightarrow \quad \quad(W / L)_{7}=14
$$

Let us check the $V_{\text {out }}(\mathrm{min})$ specification although the W/L of M7 is so large that this is probably not necessary. The value of $V_{\text {out }}(\mathrm{min})$ is

$$
V_{\text {out }}(\min )=V_{D S 7}(\mathrm{sat})=\sqrt{\frac{2.95}{110 \cdot 35}}=0.351 \mathrm{~V}
$$

which is less than required. At this point, the first-cut design is complete.
10.) Now check to see that the gain specification has been met

$$
A_{v}=\frac{(92.45 \times 10-6)(942.5 \times 10-6)}{15 \times 10-6(.04+.05) 95 \times 10-6(.04+.05)}=7,697 \mathrm{~V} / \mathrm{V}
$$

which exceeds the specifications by a factor of two. An easy way to achieve more gain would be to increase the W and L values by a factor of two which because of the decreased value of $\lambda$ would multiply the above gain by a factor of 20 .

## Incorporating the Nulling Resistor into the Miller Compensated Two-Stage Op Amp

 Circuit:

We saw earlier that the roots were:

$$
\begin{array}{ll}
p_{1}=-\frac{g_{m 2}}{A_{v} C_{c}}=-\frac{g_{m 1}}{A_{\nu} C_{c}} & p_{2}=-\frac{g_{m 6}}{C_{L}} \\
p_{4}=-\frac{1}{R_{z} C_{I}} & z_{1}=\frac{-1}{R_{z} C_{c}-C_{c} / g_{m 6}}
\end{array}
$$

where $A_{v}=g_{m 1} g_{m 6} R_{I} R_{I I}$.
(Note that $p_{4}$ is the pole resulting from the nulling resistor compensation technique.)

## Design of the Nulling Resistor (M8)

In order to place the zero on top of the second pole ( $p_{2}$ ), the following relationship must hold

$$
R_{z}=\frac{1}{g_{m 6}}\left(\frac{C_{L}+C_{c}}{C_{C}}\right)=\left(\frac{C_{c}+C_{L}}{C_{c}}\right) \frac{1}{\sqrt{2 K^{\prime}{ }_{P} S_{6} I_{6}}}
$$

The resistor, $R_{z}$, is realized by the transistor M8 which is operating in the active region because the dc current through it is zero. Therefore, $R_{z}$, can be written as

$$
R_{z}=\frac{\partial v_{D S 8}}{\partial i_{D 8}} V_{D S 8}=0=\frac{1}{K^{\prime}{ }_{P} S_{8}\left(V_{S G 8}-\left|V_{T P}\right|\right)}
$$

The bias circuit is designed so that voltage $V_{A}$ is equal to $V_{B}$.

$$
\therefore \quad\left|V_{G S 10}\right|-\left|V_{T}\right|=\left|V_{G S 8}\right|-\left|V_{T}\right| \Rightarrow \quad V_{S G 11}=V_{S G 6} \quad \Rightarrow \quad\left(\frac{W_{11}}{L_{11}}\right)=\left(\frac{I_{10}}{I_{6}}\right)\left(\frac{W_{6}}{L_{6}}\right)
$$

In the saturation region

$$
\begin{array}{ll} 
& \left|V_{G S 10}\right|-\left|V_{T}\right|=\sqrt{\frac{2\left(I_{10}\right)}{K_{P}^{\prime}\left(W_{10} / L_{10}\right)}}=\left|V_{G S 8}\right|-\left|V_{T}\right| \\
\therefore & R_{z}=\frac{1}{K_{P}^{\prime} S_{8}} \sqrt{\frac{K_{P}^{\prime} S_{10}}{2 I_{10}}}=\frac{1}{S_{8}} \sqrt{\frac{S_{10}}{2 K_{P}^{\prime} I_{10}}}
\end{array}
$$

Equating the two expressions for $R_{z}$ gives

$$
\left(\frac{W_{8}}{L_{8}}\right)=\left(\frac{C_{c}}{C_{L}+C_{c}}\right) \sqrt{\frac{S_{10} S_{6} I_{6}}{I_{10}}}
$$

## Example 2-RHP Zero Compensation

Use results of Ex. 1 and design compensation circuitry so that the RHP zero is moved from the RHP to the LHP and placed on top of the output pole $p_{2}$. Use device data given in Ex. 1.

## Solution

The task at hand is the design of transistors M8, M9, M10, M11, and bias current $I_{10}$. The first step in this design is to establish the bias components. In order to set $V_{A}$ equal to $V_{B}$, then $V_{S G 11}$ must equal $V_{S G 6}$. Therefore,

$$
S_{11}=\left(I_{11} / I_{6}\right) S_{6}
$$

Choose $I_{11}=I_{10}=I_{9}=15 \mu \mathrm{~A}$ which gives $S_{11}=(15 \mu \mathrm{~A} / 95 \mu \mathrm{~A}) 94=14.8 \approx 15$.
The aspect ratio of M10 is essentially a free parameter, and will be set equal to 1 . There must be sufficient supply voltage to support the sum of $V_{S G 11}, V_{S G 10}$, and $V_{D S 9}$. The ratio of $I_{10} / I_{5}$ determines the $(W / L)$ of M9. This ratio is

$$
(W / L)_{9}=\left(I_{10} / I_{5}\right)(W / L)_{5}=(15 / 30)(4.5)=2.25 \approx 2
$$

Now $(W / L)_{8}$ is determined to be

$$
(W / L)_{8}=\left(\frac{3 \mathrm{pF}}{3 \mathrm{pF}+10 \mathrm{pF}}\right) \sqrt{\frac{1.94 .95 \mu \mathrm{~A}}{15 \mu \mathrm{~A}}}=5.63 \approx 6
$$

## Example 2-Continued

It is worthwhile to check that the RHP zero has been moved on top of $p_{2}$. To do this, first calculate the value of $R_{z} \cdot V_{S G 8}$ must first be determined. It is equal to $V_{S G 10}$, which is

$$
V_{S G 10}=\sqrt{\frac{2 I_{10}}{K_{P}^{\prime} S_{10}}}+\left|V_{T P}\right|=\sqrt{\frac{2 \cdot 15}{50 \cdot 1}}+0.7=1.474 \mathrm{~V}
$$

Next determine $R_{z}$.

$$
R_{z}=\frac{1}{K_{P}{ }_{P} S_{8}\left(V_{S G 10}-\left|V_{T P}\right|\right)}=\frac{10^{6}}{50 \cdot 5.63(1.474-.7)}=4.590 \mathrm{k} \Omega
$$

The location of $z_{1}$ is calculated as

$$
z_{1}=\frac{-1}{\left(4.590 \times 10^{3}\right)\left(3 \times 10^{-12}\right)-\frac{3 \times 10^{-12}}{942.5 \times 10^{-6}}}=-94.46 \times 10^{6} \mathrm{rads} / \mathrm{sec}
$$

The output pole, $p_{2}$, is

$$
p_{2}=\frac{942.5 \times 10^{-6}}{10 \times 10^{-12}}=-94.25 \times 10^{6} \mathrm{rads} / \mathrm{sec}
$$

Thus, we see that for all practical purposes, the output pole is canceled by the zero that has been moved from the RHP to the LHP.

The results of this design are summarized below.

$$
W_{8}=6 \mu \mathrm{~m} \quad W_{9}=2 \mu \mathrm{~m} W_{10}=1 \mu \mathrm{~m} \quad W_{11}=15 \mu \mathrm{~m}
$$

## SUMMARY

Programmability of the Two-Stage Op Amp:
The following relationships depend on the bias current, $I_{\text {bias }}$, in the following manner and allow for programmability after fabrication.
$A_{v}(0)=g_{m I B_{m I I}} R_{I} R_{I I} \propto \frac{1}{I_{\text {Bias }}}$
$G B=\frac{g_{m I}}{C_{c}} \propto \sqrt{I_{\text {Bias }}}$
$P_{\text {diss }}=\left(V_{D D}+\left|V_{S S}\right|\right)\left(1+K_{1}+K_{2}\right) I_{\text {Bias }} \propto I_{\text {bias }}$
$S R=\frac{K_{1} I_{\text {Bias }}}{C_{c}} \propto I_{\text {Bias }}$
$R_{\text {out }}=\frac{1}{2 \lambda K_{2} I_{\text {Bias }}} \propto \frac{1}{I_{\text {Bias }}}$
$\left|p_{1}\right|=\frac{1}{g_{\text {mII }} R_{I} R_{I I} C_{c}} \propto \frac{I_{\text {Bias }}{ }^{2}}{\sqrt{I_{\text {Bias }}}} \propto I_{\text {Bias }}{ }^{1.5}$
$|z|=\frac{g_{m I I}}{C_{c}} \propto \sqrt{I_{\text {Bias }}}$
Illustration of the $I_{\text {bias }}$ dependence $\rightarrow$


