## LECTURE 180 - POWER SUPPLY REJECTION RATIO <br> (READING: GHLM - 434-439, AH - 286-293)

## Objective

The objective of this presentation is:
1.) Illustrate the calculation of PSRR
2.) Examine the PSRR of the two-stage, Miller compensated op amp

## Outline

- Definition of PSRR
- Calculation of PSRR for the two-stage op amp
- Conceptual reason for PSRR
- Summary

What is PSRR?

$$
\operatorname{PSRR}=\frac{A_{v}\left(V_{d d}=0\right)}{A_{d d}\left(V_{i n}=0\right)}
$$



How do you calculate PSRR?
You could calculate $A_{v}$ and $A_{d d}$ and divide, however

$V_{\text {out }}=A_{d d} V_{d d}+A_{\nu}\left(V_{1}-V_{2}\right)=A_{d d} V_{d d}-A_{v} V_{\text {out }} \quad \rightarrow \quad V_{\text {out }}\left(1+A_{v}\right)=A_{d d} V_{d d}$
$\therefore \frac{V_{\text {out }}}{V_{d d}}=\frac{A_{d d}}{1+A_{v}} \approx \frac{A_{d d}}{A_{v}}=\frac{1}{P S R R^{+}}($Good for frequencies up to $G B)$

## Positive PSRR of the Two-Stage Op Amp



The nodal equations are:

$$
\begin{aligned}
& \left(g_{d s 1}+g_{d s 4}\right) V_{d d}=\left(g_{d s 2}+g_{d s 4}+s C_{c}+s C_{I}\right) V_{1}-\left(g_{m 1}+s C_{c}\right) V_{\text {out }} \\
& \left(g_{m 6}+g_{d s 6}\right) V_{d d}=\left(g_{m 6}-s C_{c}\right) V_{1}+\left(g_{d s 6}+g_{d s 7}+s C_{c}+s C_{I I}\right) V_{\text {out }}
\end{aligned}
$$

Using the generic notation the nodal equations are:

$$
G_{I} V_{d d}=\left(G_{I}+s C_{c}+s C_{I}\right) V_{1}-\left(g_{m I}+s C_{c}\right) V_{\text {out }}
$$

$$
\left(g_{m I I}+g_{d s 6}\right) V_{d d}=\left(g_{m I I}-s C_{c}\right) V_{1}+\left(G_{I I}+s C_{c}+s C_{I I}\right) V_{\text {out }}
$$

$$
\text { where } G_{I}=g_{d s 1}+g_{d s 4}=g_{d s 2}+g_{d s 4}, G_{I I}=g_{d s 6}+g_{d s 7}, g_{m I}=g_{m 1}=g_{m 2} \text { and } g_{m I I}=g_{m 6}
$$

## Positive PSRR of the Two-Stage Op Amp - Continued

Using Cramers rule to solve for the transfer function, $V_{o u t} / V_{d d}$, and inverting the transfer function gives the following result.

$$
\frac{V_{d d}}{V_{o u t}}=\frac{s 2\left[C_{c} C_{I^{+}} C_{I} C_{I I}+C_{I I} C_{c}\right]+s\left[G_{I}\left(C_{c^{+}} C_{I I}\right)+G_{I I}\left(C_{c^{+}} C_{I}\right)+C_{c}\left(g_{m I I}-g_{m I}\right)\right]+G_{I} G_{I I} g_{m I} g_{m I I}}{s\left[C_{c}\left(g_{m I I}+G_{I}+g_{d s 6}\right)+C_{I}\left(g_{m I I}+g_{d s 6}\right)\right]+G_{I I} g_{d s 6}}
$$

We may solve for the approximate roots of numerator as

$$
P S R R^{+}=\frac{V_{d d}}{V_{\text {out }}} \cong\left(\frac{g_{m I} g_{m I I}}{G_{I} g_{d s 6}}\right)\left[\frac{\left(\frac{s C_{c}}{g_{m I}}+1\right)\left(\frac{s\left(C_{c} C_{I+} C_{I} C_{I I^{+}} C_{c} C_{I I}\right)}{g_{m I I} C_{c}}+1\right)}{\left(\frac{s g_{m I I} C_{c}}{G_{I} g_{d s 6}}+1\right)}\right]
$$

where $g_{m I I}>g_{m I}$ and that all transconductances are larger than the channel conductances.
$\therefore \quad$ PSRR $^{+}=\frac{V_{d d}}{V_{\text {out }}}=\left(\frac{g_{m I} g_{m I I}}{G_{I} g_{d s 6}}\right)\left[\frac{\left(\frac{s C_{c}}{g_{m I}}+1\right)\left(\frac{s C_{I I}}{g_{m I I}}+1\right)}{\frac{s g_{m I I} C_{c}}{G_{I} g_{d s 6}}+1}\right]=\left(\frac{G_{I I} A_{v o}}{g_{d s 6}}\right) \frac{\left(\frac{s}{G B}+1\right)\left(\frac{s}{\left|p_{2}\right|}+1\right)}{\left(\frac{s G_{I I} A_{v o}}{g_{d s 6} G B}+1\right)}$

## Positive PSRR of the Two-Stage Op Amp - Continued



At approximately the dominant pole, the $P S R R$ falls off with a $-20 \mathrm{~dB} /$ decade slope and degrades the higher frequency $P S R R^{+}$of the two-stage op amp.
Using the values of Example 6.3-1 we get:

$$
\operatorname{PSRR}^{+}(0)=68.8 \mathrm{~dB}, \quad z_{1}=-5 \mathrm{MHz}, \quad z_{2}=-15 \mathrm{MHz} \quad \text { and } p_{1}=-906 \mathrm{~Hz}
$$

## Concept of the PSRR ${ }^{+}$for the Two-Stage Op Amp



Fig. 180-05
1.) The M7 current sink causes $V_{S G 6}$ to act like a battery.
2.) Therefore, $V_{d d}$ couples from the source to gate of M6.
3.) The path to the output is through any capacitance from gate to drain of M6.

## Conclusion:

The Miller capacitor $C_{C}$ couples the positive power supply ripple directly to the output. Must reduce or eliminate $C_{C}$.

## Negative PSRR of the Two-Stage Op Amp with $V_{\text {Bias }}$ Grounded



Nodal equations for $V_{\text {Bias }}$ grounded:

$$
\begin{aligned}
& 0=\left(G_{I}+s C_{c}+s C_{I}\right) V_{1}-\left(g_{m I}+s C_{c}\right) V_{o} \\
& g_{m 7} V_{s s}=\left(g_{M I I}-s C_{c}\right) V_{1}+\left(G_{I I}+s C_{C}+s C_{I I}\right) V_{o}
\end{aligned}
$$

Solving for $V_{\text {out }} / V_{s s}$ and inverting gives

$$
\frac{V_{s s}}{V_{\text {out }}}=\frac{s 2\left[C_{c} C_{I}+C_{I} C_{I I}+C_{I I} C_{c}\right]+s\left[G_{I}\left(C_{C^{+}}+C_{I I}\right)+G_{I I}\left(C_{C^{+}} C_{I}\right)+C_{c}\left(g_{m I I}-g_{m I}\right)\right]+G_{I} G_{I I}+g_{m I} g_{m I I}}{\left[s\left(C_{C^{+}}+C_{I}\right)+G_{I}\right] g_{m 7}}
$$

Negative PSRR of the Two-Stage Op Amp with $V_{\text {Bias }}$ Grounded - Continued
Again using techniques described previously, we may solve for the approximate roots as

$$
P S R R^{-}=\frac{V_{s s}}{V_{\text {out }}} \cong\left(\frac{g_{m I} g_{m I I}}{G_{I} g_{m 7}}\right)\left[\frac{\left(\frac{s C_{c}}{g_{m I}}+1\right)\left(\frac{\mathrm{s}\left(C_{c} C_{I}+C_{I} C_{I I} C_{c} C_{I I}\right)}{g_{m I I} C_{c}}+1\right)}{\left(\frac{s\left(C_{c}+C_{I}\right)}{G_{I}}+1\right)}\right]
$$

This equation can be rewritten approximately as

$$
P S R R^{-}=\frac{V_{s s}}{V_{\text {out }}} \cong\left(\frac{g_{m I} g_{m I I}}{G_{I g} g_{m 7}}\right)\left[\frac{\left(\frac{s C_{c}}{g_{m I}}+1\right)\left(\frac{s C_{I I}}{g_{m I I}}+1\right)}{\left(\frac{s C_{c}}{G_{I}}+1\right)}\right]=\left(\frac{G_{I I} A_{\nu 0}}{g_{m 7}}\right)\left[\frac{\left(\frac{s}{G B}+1\right)\left(\frac{s}{\left|p_{2}\right|}+1\right)}{\left(\frac{s}{G B} \frac{g_{m I}}{G_{I}}+1\right)}\right]
$$

Comments:
$P S R R^{-}$zeros $=P S R R^{+}$zeros
DC gain $\approx$ Second-stage gain,
$P S R R^{-}$pole $\approx\left(\right.$ Second-stage gain) $\times\left(P S R R^{+}\right.$pole $)$
Assuming the values of Ex. 6.3-1 gives a gain of 23.7 dB and a pole -147 kHz . The dc value of $P S R R$ - is very poor for this case, however, this case can be avoided by correctly implementing $V_{\text {Bias }}$ which we consider next.

Negative PSRR of the Two-Stage Op Amp with $V_{\text {Bias }}$ Connected to $V_{\underline{S S}}$


Fig. 180-07
If the value of $V_{\text {Bias }}$ is independent of $V_{s s}$, then the model shown results. The nodal equations for this model are

$$
0=\left(G_{I}+s C_{c}+s C_{I}\right) V_{1}-\left(g_{m I}+s C_{c}\right) V_{\text {out }}
$$

and

$$
\left(g_{d s 7}+s C_{g d 7}\right) V_{s s}=\left(g_{m I I}-s C_{c}\right) V_{1}+\left(G_{I I}+s C_{c}+s C_{I I}+s C_{g d 7}\right) V_{\text {out }}
$$

Again, solving for $V_{\text {out }} / V_{s s}$ and inverting gives
$\frac{V_{s S}}{V_{\text {out }}}=\frac{s 2\left[C_{c} C_{I}+C_{I} C_{I I}+C_{I I} C_{c}+C_{I} C_{g d} I+C_{c} C_{g d}\right]+s\left[G_{I}\left(C_{c}+C_{I I}+C_{g d}\right)+G_{I I}\left(C_{C}+C_{I}\right)+C_{c}\left(g_{m I I}-g_{m l}\right)\right]+G_{I} G_{I I}+g_{m I I} g_{m I I}}{\left(s C_{g d 7}+g_{d s 7}\right)\left(s\left(C_{I}+C_{C}\right)+G_{I D}\right)}$

Negative PSRR of the Two-Stage Op Amp with $V_{\text {Bias }}$ Connected to $V_{S S}$ - Continued
Assuming that $g_{m I I}>g_{m I}$ and solving for the approximate roots of both the numerator and denominator gives

$$
P S R R^{-}=\frac{V_{s s}}{V_{\text {out }}} \cong\left(\frac{\left(g_{m I} g_{m I I}\right.}{G_{I g} g_{d s} 7}\right)\left[\frac{\left(\frac{s C_{c}}{g_{m I}}+1\right)\left(\frac{\mathrm{s}\left(C_{c} C_{I} C_{I} C_{I I} C_{c} C_{I I}\right)}{g_{m I I} C_{c}}+1\right)}{\left(\frac{s C_{g d 7}}{g_{d s 7}}+1\right)\left(\frac{s\left(C_{I}+C_{c}\right)}{G_{I}}+1\right)}\right]
$$

This equation can be rewritten as

$$
P S R R^{-}=\frac{V_{s s}}{V_{\text {out }}} \approx\left(\frac{G_{I I} A_{\nu 0}}{g_{d s 7}}\right)\left[\frac{\left(\frac{s}{G B}+1\right)\left(\frac{s}{\left|p_{2}\right|}+1\right)}{\left(\frac{s C_{g d 7}}{g_{d s 7}}+1\right)\left(\frac{s C_{c}}{G_{I}}+1\right)}\right]
$$

Comments:

- DC gain has been increased by the ratio of $G_{I I}$ to $g_{d s 7}$
- Two poles instead of one, however the pole at $-g_{d s} 7 / C_{g d 7}$ is large and can be ignored.

Using the values of Ex. 6.3-1 and assume that $C_{d s}=10 \mathrm{fF}$, gives,

$$
\operatorname{PSRR}^{-}(0)=76.7 \mathrm{~dB} \quad \text { and } \quad \text { Poles at }-71.2 \mathrm{kHz} \text { and }-149 \mathrm{MHz}
$$

## Frequency Response of the Negative PSRR of the Two-Stage Op Amp with $V_{\text {Bias }}$ Connected to $V_{\underline{S S}}$



## Approximate Model for Negative PSRR with $V_{\text {Bias }}$ Connected to Ground



Fig. 180-09
Path through the input stage is not important as long as the CMRR is high.
Path through the output stage:

$$
\left.\begin{array}{rl} 
& v_{\text {out }} \approx i_{\text {ss }} Z_{\text {out }}=g_{m 7} Z_{\text {out }} V_{S S} \\
\therefore \quad & \frac{V_{\text {out }}}{V_{\text {sS }}}=g_{m 7} Z_{\text {out }}=g_{m 7} R_{\text {out }}\left(\frac{1}{s R_{\text {out }} C_{\text {out }}+1}\right)
\end{array}\right)
$$



## Approximate Model for Negative PSRR with $V_{B i a s}$ Connected to $V_{S S}$

What is $Z_{\text {out }}$ ?



Path through $C_{g d 7}$ is negligible
$Z_{\text {out }}=\frac{V_{t}}{I_{t}} \Rightarrow$
Fig. 180-11
$I_{t}=g_{m I I} V_{1}=g_{m I I}\left(\frac{g_{m I} V_{t}}{G_{I}+s C_{I}+s C_{c}}\right)$
Thus, $Z_{\text {out }}=\frac{G_{I}+s\left(C_{I}+C_{\mathcal{C}}\right)}{g_{m I} g_{M I I}}$

$\therefore$

$$
\frac{V_{S s}}{V_{\text {out }}}=\frac{1+\frac{r_{d s} 7}{Z_{\text {out }}}}{1}=\frac{s\left(C_{C}+C_{I}\right)+G_{I}+g_{m I I} g_{m I I} r_{d s} 7}{s\left(C_{C}+C_{I}\right)+G_{I}} \quad \Rightarrow \quad \text { Pole at } \frac{-G_{I}}{C_{C^{+}}+C_{I}}
$$

The two-stage op amp will never have good PSRR because of the Miller compensation.

## SUMMARY

- PSRR is a measure of the influence of power supply ripple on the op amp output voltage
- PSRR can be calculated by putting the op amp in the unity-gain configuration with the input shorted.
- The Miller compensation capacitor allows the power supply ripple at the output to be large
- The two-stage op amp will never have good PSRR unless some modifications are made.

