Homework No. 5 - Solutions

Problem 1 - (10 points) (Problem 6.2-8 of A&H)

A two-stage, Miller-compensated CMOS op amp has a RHP zero at 20*GB*, a dominant pole due to the Miller compensation, a second pole at p_2 and a mirror pole at -3*GB*. (a) If *GB* is 1MHz, find the location of p_2 corresponding to a 45° phase margin. (b) Assume that in part (a) that $|p_2| = 2GB$ and a nulling resistor is used to cancel p_2. What is the new phase margin assuming that *GB* = 1MHz? (c) Using the conditions of (b), what is the phase margin if C_L is increased by a factor of 4?

Solution

a.) Since the magnitude of the op amp is unity at GB, then let ω = GB to evaluate the phase.

$$\text{Phase margin} = \text{PM} = 180° - \tan^{-1}\left(\frac{GB}{|p_1|}\right) - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - \tan^{-1}\left(\frac{GB}{|p_3|}\right) - \tan^{-1}\left(\frac{GB}{|z_1|}\right)$$

But, $p_1 = GB/A_o$, $p_3 = -3GB$ and $z_1 = -20GB$ which gives

$$\text{PM} = 45° = 180° - \tan^{-1}(A_o) - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - \tan^{-1}(0.33) - \tan^{-1}(0.05)$$

$$45° \approx 90° - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - \tan^{-1}(0.33) - \tan^{-1}(0.05) = 90° - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - 18.26° - 2.86°$$

$$\tan^{-1}\left(\frac{GB}{|p_2|}\right) = 45° - 18.26° - 2.86° = 23.48° \rightarrow \frac{GB}{|p_2|} = \tan(23.84°) = 0.442$$

$$\boxed{p_2 = -2.26 \cdot GB = -14.2\text{x}10^6 \text{ rads/sec}}$$

b.) The only roots now are p_1 and p_3. Thus,

$$\boxed{\text{PM} = 180° - 90° - \tan^{-1}(0.33) = 90° - 18.3° = 71.7°}$$

c.) In this case, z_1 is at -2GB and p_2 moves to -0.5GB. Thus the phase margin is now,

$$\boxed{\text{PM} = 90° - \tan^{-1}(2) + \tan^{-1}(0.5) - \tan^{-1}(0.33) = 90° - 63.43° + 26.57° - 18.3° = 34.4°}$$

Problem 2 – (Problem 6.2-10 of A&H)

For the two-stage op amp of Fig. 6.2-8, find W_1/L_1, W_6/L_6, and C_c if GB = 1 MHz, $|p_2|$ = 5 GB, z = 3 GB and C_L = C_2 = 20 pF. Use the parameter values of Table 3.1-2 and consider only the two-pole model of the op amp. The bias current in M5 is 40 μA and in M7 is 320 μA.

Solution

Given

$GB = 1\ \text{MHz}.$

$p_2 = 5GB$

$z = 3GB$

$C_L = C_2 = 20\ \text{pF}$

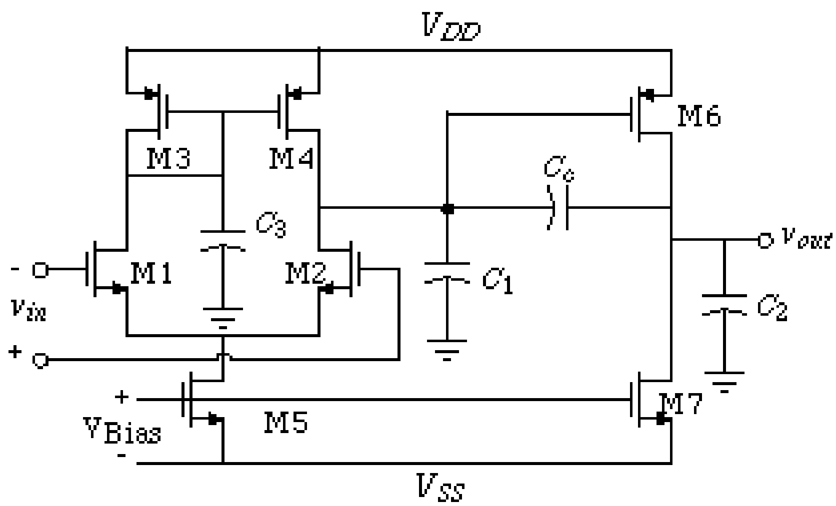


Figure 6.2-8 A two-stage op amp with various parasitic and circuit capacitances shown.

Now, $p_2 = \frac{g_{m6}}{C_2}$

or,

$g_{m6} = 628.3\,\mu\text{S}$

or,

$$\boxed{\left(\frac{W}{L}\right)_6 = \frac{g_{m6}^2}{2K_P' I_{D6}} \quad 12.33}$$

RHP zero is given by

$$z = \frac{g_{m6}}{C_C}$$

or, $$\boxed{C_C = \frac{g_{m6}}{z} = 33.3\text{pF}}$$

Finally, Gain-bandwidth is given by

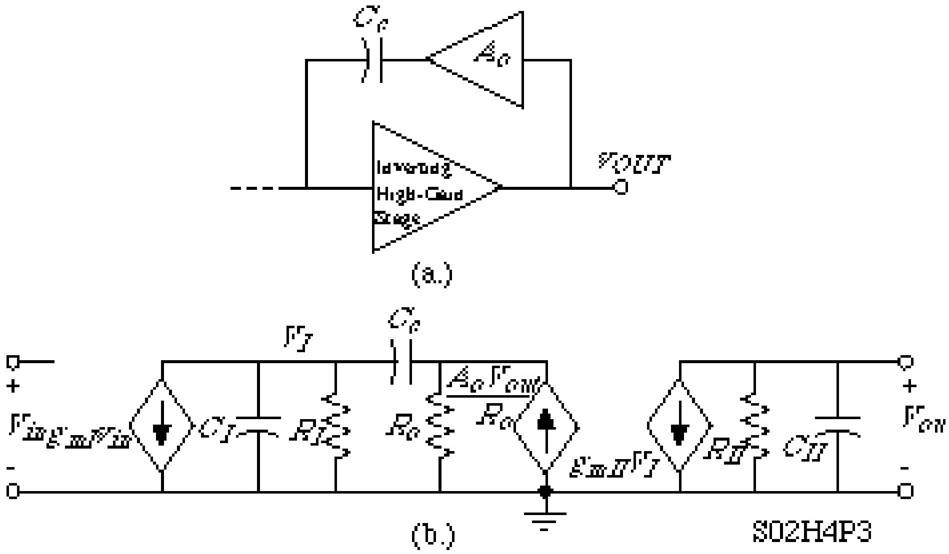
$$GB = \frac{g_{m1}}{C_C}$$

or, $g_{m1} = 209.4\ \ \mu S$

or, $$\boxed{\left(\frac{W}{L}\right)_1 = \frac{g_{m1}^2}{2K_N' I_{D1}} \quad 10}$$

<u>Problem 3 - (10 points) (Problem 6.2-11 of A&H)</u>

In the figure shown, assume that R_I = 150 kΩ, R_{II} = 100 kΩ, g_{mII} = 500 μS, C_I = 1 pF, C_{II} = 5 pF, and C_c = 30 pF. Find the value of R_z and the locations of all roots for (a) the case where the zero is moved to infinity and (b) the case where the zero cancels the highest pole.



<u>Soluiton</u>

(a.) Zero at infinity.

$$R_z = \frac{1}{g_{mII}} = \frac{1}{500\mu S}$$

$$\boxed{R_z = \ 2\,k\Omega}$$

Check pole due to R_z.

$$p_4 = \frac{-1}{R_z C_I} = \frac{-1}{2k\Omega\cdot 1pF} \ = -500\text{x}10^6 \text{ rps or } 79.58 \text{ MHz}$$

The pole at p_2 is

$$p_2 \approx \frac{-g_{mII}C_c}{C_I C_{II} + C_c C_I + C_c C_{II}} \approx \frac{-g_{mII}}{C_{II}} = \frac{-500\mu S}{5pF} \ = 100\text{x}10^6 \text{ rps or } 15.9 \text{ MHz}$$

Therefore, p_2 is the next highest pole.

(b.) Zero at p_2.

$$R_z = \left(\frac{C_c + C_{II}}{C_c}\right)(1/g_{mII}) = \left(\frac{30+5}{30}\right)\frac{1}{500\mu S} \ = 2.33k\Omega$$

$$\boxed{R_z = \ 2.33k\Omega}$$

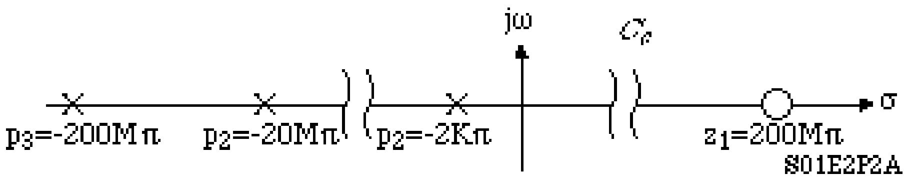
Problem 4 – (10 points)

The poles and zeros of a Miller compensated, two-stage op amp are shown below.

(a.) If the influence of p_3 and z_1 are ignored, what is the GB in MHz of this op amp for 60° phase margin?

(b.) What is the value of $A_v(0)$? What is the value of C_c if g_{m1}=g_{m2}=500µS?

(c.) If p_2 is moved to p_3, what is the new GB in MHz for 60° phase margin? What is the new C_c if the input transconductances are the same as in (b.)?



Solution

(a.) The phase margin, PM, can be written as

$$\text{PM} = 180 - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - \tan^{-1}\left(\frac{GB}{|p_3|}\right) - \tan^{-1}\left(\frac{GB}{z_1}\right) \approx 90° - \tan^{-1}\left(\frac{GB}{|p_2|}\right) = 60°$$

$$\tan^{-1}\left(\frac{GB}{|p_2|}\right) = 30° \qquad \rightarrow \qquad GB = 0.5774\cdot|p_2| = \underline{\underline{5.774\text{MHz}}}$$

(b.) $$A_v(0) = \frac{GB}{|p_1|} = \frac{5.774\text{MHz}}{1\text{kHz}} = \underline{\underline{5{,}774\text{V/V}}}$$

$$\frac{g_{m1}}{C_c} = GB \qquad \rightarrow \qquad C_c = \frac{g_{m1}}{GB} = \frac{500\mu\text{S}}{2\pi\cdot 5.774\text{x}10^6} = \underline{\underline{13.78\text{pF}}}$$

(c.) The phase margin, PM, can be written as

$$\text{PM} = 180 - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - \tan^{-1}\left(\frac{GB}{|p_3|}\right) - \tan^{-1}\left(\frac{GB}{z_1}\right) \approx 90° - 3\cdot\tan^{-1}\left(\frac{GB}{|p_2|}\right) = 60°$$

$$\tan^{-1}\left(\frac{GB}{|p_2|}\right) = 10° \qquad \rightarrow \qquad GB = 0.1763\cdot|p_2| = 0.01763\cdot 100\text{MHz} = \underline{\underline{17.63\text{MHz}}}$$

$$C_c = \frac{g_{m1}}{GB} = \frac{500\mu\text{S}}{2\text{p}\cdot 17.63\text{x}10^6} = \underline{\underline{4.514\text{pF}}}$$