## Homework Assignment No. 8 - Solutions

## Problem 1-(10 points)

This problem deals with the op amp shown in Fig. P6.5-15. All device lengths are $1 \mu \mathrm{~m}$, the slew rate is $\pm 10 \mathrm{~V} / \mu \mathrm{s}$, the GB is 10 MHz , the maximum output voltage is +2 V , the minimum output voltage is -2 V , and the input common mode range is from -1 V to +2 V .
Design all W values of all transistors in this op amp. Your design must meet or


Figure P6.5-15
exceed the specifications. When calculating the maximum or minimum output voltages, divide the voltage drop across series transistors equally. Ignore bulk effects in this problem. When you have completed your design, find the value of the small signal differential voltage gain, $A_{v d}=v_{\text {out }} / v_{i d}$, where $v_{i d}=v_{1}-v_{2}$ and the small signal output resistance, $\mathrm{R}_{\text {out }}$.

## Solution

1.) The slew rate will specify $I . \quad \therefore \quad I=C \cdot S R=10^{-11} \cdot 10^{7}=10^{-4}=100 \mu \mathrm{~A}$.
2.) Use $G B$ to define $W_{1}$ and $W_{2}$.

$$
\begin{aligned}
& G B=\frac{g_{m 1}}{C} \quad \rightarrow \quad g_{m 1}=G B \cdot C=2 \pi \times 10^{7} \cdot 10^{-11}=628 \mu \mathrm{~S} \\
& \therefore W_{1}=\frac{g_{m 1}^{2}}{2 K_{N}(0.5 I)}=\frac{(628)^{2}}{2 \cdot 110 \cdot 50}=35.85 \quad \Rightarrow \quad W_{1}=W_{2}=36 \mu \mathrm{~m}
\end{aligned}
$$

3.) Design $W_{15}$ to give $V_{T}+2 V_{O N}$ bias for M6 and M7. $V_{O N}=0.5 \mathrm{~V}$ will meet the desired maximum output voltage specification. Therefore,

$$
\begin{array}{ll}
V_{S G 15}=V_{O N 15}+\left|V_{T}\right|=2(0.5 \mathrm{~V})+\left|V_{T}\right| & \rightarrow V_{O N 15}=1 \mathrm{~V}=\sqrt{\frac{2 I}{K_{P} W_{15}}} \\
\therefore W_{15}=\frac{2 I}{K_{P} V_{O N 15}^{2}}=\frac{2 \cdot 100}{50 \cdot 1^{2}}=4 \mu \mathrm{~m} & \Rightarrow \quad \underline{\underline{W}} \underline{\underline{15}}=4 \mu \mathrm{~m}
\end{array}
$$

4.) Design $W_{3}, W_{4}, W_{6}$ and $W_{7}$ to have a saturation voltage of 0.5 V with 1.5 I current.

$$
W_{3}=W_{4}=W_{6}=W_{7}=\frac{2(1.5 I)}{K_{P} V_{O N}^{2}}=\frac{2 \cdot 150}{50 \cdot 0.5^{2}}=24 \mu \mathrm{~m} \quad \Rightarrow \underline{\underline{W}} \underline{\underline{3}}=\underline{\underline{W}}_{\underline{4}}^{\underline{=}} \underline{\underline{W}} \underline{\underline{6}}=W_{7}=24 \mu \mathrm{~m}
$$

## Problem 6.5-15 - Continued

5.) Next design $W_{8}, W_{9}, W_{10}$ and $W_{11}$ to meet the minimum output voltage specification. Note that we have not taken advantage of smallest minimum output voltage because a normal cascode current mirror is used which has a minimum voltage across it of $V_{T}{ }^{+}$ $2 V_{O N}$. Therefore, setting $V_{T}+2 V_{O N}=1 \mathrm{~V}$ gives $V_{O N}=0.15 \mathrm{~V}$. Using worst case current, we choose 1.5I. Therefore,
$W_{8}=W_{9}=W_{10}=W_{11}=\frac{2(1.51)}{K_{N} V_{O N}{ }^{2}}=\frac{2 \cdot 150}{110 \cdot 0.15^{2}}=121 \mu \mathrm{~m} \Rightarrow \underline{\underline{W}} \underline{\underline{8}} \underline{\underline{W}} \underline{\underline{9}} \underline{\underline{W}} \underline{\underline{\underline{W}}} \underline{\underline{\underline{1}}}=$ $\underline{\underline{121 \mu \mathrm{~m}}}$
6.) Check the maximum ICM voltage.

$$
V_{i c}(\max )=V_{D D}+V_{S D 3}(\mathrm{sat})+V_{T N}=3 \mathrm{~V}-0.5+0.7=3.2 \mathrm{~V} \text { which exceeds spec. }
$$

7.) Use the minimum ICM voltage to design $W_{5}$.

$$
\begin{aligned}
& V_{i c}(\min )=V_{S S}+V_{D S 5}(\mathrm{sat})+V_{G S 1}=-3+V_{D S 5}(\mathrm{sat})+\left(\sqrt{\frac{2 \cdot 50}{110 \cdot 36}}+0.7\right)=-1 \mathrm{~V} \\
\therefore & V_{D S 5}(\mathrm{sat})=1.141 \rightarrow W_{5}=\frac{2 I}{K_{N} V_{D S 5}(\mathrm{sat})^{2}}=1.39 \mu \mathrm{~m}=1.4 \mu \mathrm{~m}
\end{aligned}
$$

$$
\text { Also, let } W_{12}=W_{13}=W_{5} \quad \Rightarrow \quad W_{12}=W^{13}=W_{5}=1.4 \mu \mathrm{~m}
$$

8.) $W_{14}$ is designed as

$$
W_{14}=W_{3} \frac{I_{14}}{I_{3}}=24 \mu \mathrm{~m} \frac{I}{1.5 I}=16 \mu \mathrm{~m} \quad \Rightarrow \quad \underline{\underline{W}} \underline{\underline{14}}=16 \mu \mathrm{~m}
$$

Now, calculate the op amp small-signal performance.

$$
\begin{gathered}
R_{\text {out }} \approx r_{d s 11} g_{m 9} r_{d s 9} \| g_{m 7} r_{d s 7}\left(r_{d s 2} \| r_{d s 4}\right) \\
g_{m 9}=\sqrt{2 K_{N} \cdot \cdot \cdot W_{9}}=1632 \mu \mathrm{~S}, r_{d s 9}=r_{d s 11}=\frac{25 \mathrm{~V}}{100 \mu \mathrm{~A}}=0.25 \mathrm{M} \Omega, \\
g_{m 7}=\sqrt{2 K_{P} \cdot I \cdot W_{7}}=490 \mu \mathrm{~S}, r_{d s 7}=\frac{20 \mathrm{~V}}{100 \mu \mathrm{~A}}=0.2 \mathrm{M} \Omega, r_{d 2}=\frac{25 \mathrm{~V}}{50 \mu \mathrm{~A}}=0.5 \mathrm{M} \Omega \\
\\
r_{d s 4}=\frac{20 \mathrm{~V}}{150 \mu \mathrm{~A}}=0.1333 \mathrm{M} \Omega \quad \therefore \quad \therefore \quad \therefore \quad \underline{\underline{R_{o u t}}} \approx 102 \mathrm{M} \Omega \| 10.31 \mathrm{M} \Omega=9.3682 \mathrm{M} \Omega \\
A_{v d}=\left(\frac{2+\mathrm{k}}{2+2 \mathrm{k}}\right) g_{m 1} R_{\text {out },}, \mathrm{k}=\frac{102 \mathrm{M} \Omega}{\left(r_{d s 2} \| r_{d s 4}\right) g_{m 7} r_{d s 7}}=9.888, \quad g_{m 1}=\sqrt{K_{N} \cdot I \cdot W_{1}}=629 \mu \mathrm{~S} \\
\therefore \quad A_{v d}=(0.5459)(629 \mu \mathrm{~S})(9.3682 \mathrm{M} \Omega)=3,217 \mathrm{~V} / \mathrm{V} \quad \Rightarrow \quad \quad \underline{\underline{A_{v d}}}=3,217 \mathrm{~V} / \mathrm{V}
\end{gathered}
$$



t** BIPOLAR JUNCTION TRAMSISTORS

## SUBCKT





$6-28$
If the bias current level of 741
input stage is doubled, then
from (6.134), $G_{m_{1}}=\frac{1}{2.7 \mathrm{k} \Omega}$
From (6.138).

$$
\begin{aligned}
& R_{01}=\left.R_{\text {out }}\right|_{Q_{4}} \|\left. R_{\text {OUT }}\right|_{Q_{6}} \\
&=2 r_{04} \| r_{06}\left(1+g_{m_{6}}(1 \mathrm{k} \Omega)\right) \\
& \text { Using } \eta_{n p n}=2 \times 10^{-4}, \eta_{p n p}=5 \times 10^{-4} \\
& \text { and }\left|I_{c}\right|=194 \mathrm{~A}, \text { we have } \\
& r_{04}=\frac{1}{\eta g_{m}}=\frac{10^{4}}{5} \frac{26}{19 \times 10^{-3}}=2.74 \mathrm{M} \Omega \\
& r_{06}=\frac{10^{4}}{2} \frac{26}{19 \times 10^{-3}}=6.84 \mathrm{M} \Omega \\
& g_{m_{6}} \times 1 \mathrm{k} \Omega=0.73 \\
& \therefore R_{01}=(5.48) \|(6.84 \times 1.73) \mathrm{M} \Omega \\
&=3.75 \mathrm{~m} \Omega
\end{aligned}
$$

741 equivalent

$3.75\|5.7 \quad 2.26 \mathrm{M} \Omega ; 83 \mathrm{~K}\| 9.1 \mathrm{M}=82 \mathrm{~K} \Omega$

$$
\begin{aligned}
A_{v} & =\frac{2260}{2.7} \cdot \frac{82}{0.147}=838 \times 558 \\
& =468,000
\end{aligned}
$$

6.29

If the $100 \Omega$ emitter resistor of $Q_{17}$ is removed, then in (6.142) we have,

$$
\begin{aligned}
R_{e q_{1}} & =\gamma_{\pi_{17}} \pm \frac{\beta}{g_{m}}=250 \times \frac{26}{0.55}=11.8 \mathrm{k} \Omega \\
R_{i_{2}} & =\gamma_{\pi_{16}}+\left(1+\beta_{0}\right)\left(\gamma_{\pi_{17}} \| 50 \mathrm{k} \Omega\right) \\
& =406 \mathrm{k} \Omega+251 \times 9.55 \mathrm{k} \Omega \\
& =2.8 \mathrm{M} \Omega
\end{aligned}
$$

From (6.146)

$$
G m_{2} \simeq g m_{17}=\frac{0.55}{26}=\frac{1}{47.3 \Omega}
$$

From (6.147)

$$
\begin{aligned}
& R_{02}=\gamma_{013 \mathrm{p}} \| \gamma_{017} \\
& \gamma_{0 / 3 \beta}=\frac{1}{\eta g_{m}}=\frac{10^{4}}{5} \frac{26}{0.55}=94.5 \mathrm{k} \Omega \\
& \gamma_{017}=\frac{1}{\eta g_{m}}=\frac{10^{4}}{2} \frac{26}{0.55}=236 \mathrm{k} \Omega \\
& \therefore R_{02}=67.5 \mathrm{k} \Omega \\
& A_{v}=\frac{1980}{5.4} \times \frac{67 .}{0.047} \\
& =523,000
\end{aligned}
$$

6.30

Minimum CM input voltage:
The circuit ceases to function correctly when $Q_{3}$ and $Q_{4}$ saturate.
$Q_{3}$ and $Q_{4}$ operate in the F.A.R. when,

$$
\begin{aligned}
& V_{E C_{3}}>V_{C E(\text { sat })} \\
& V_{E_{3}}=V_{I C}-V_{B E_{1}} \quad \text { neglect } \\
& V_{C_{3}}=-V_{E E}+V_{B E_{5}}+V_{B E_{7}}+\overbrace{I_{3}}(1 K) \\
& V_{E C_{3}}=V_{1 C}-V_{B E_{1}}-\left(-V_{E E)}-V_{B E_{5}}-V_{B E_{7}}>V_{C E(\text { sat })}\right. \\
& V_{I C}>-V_{E E}+V_{B E_{1}}+V_{B E_{5}}+V_{B E_{7}}+V_{C E(\text { sat })}
\end{aligned}
$$

Maximum CM input voltage:
$Q_{1}$ and $Q_{2}$ operate in the F.A.R. when

$$
\begin{aligned}
& V_{C E_{1}}>V_{C E}(\text { sat }) \\
& V_{C 1}=V_{C C}-\left|V_{B E_{8}}\right| \\
& V_{E_{1}}=V_{1 C}-V_{B E_{1}} \\
& V_{C E_{1}}=V_{C C}-\left|V_{B E_{g}}\right|-V_{1 C}+V_{B E_{1}}>V_{C E(\text { sat. })}
\end{aligned}
$$

Assume $V_{B E_{1}}=\left|V_{B E_{8}}\right|$ Then $V_{1 C}<V_{C C}-V_{C E}$ (sat.)

| - ponitr supplies |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| vec | 100 | 01 |  |  |
| VEE | 200 | 0 |  |  |
| - intot stage |  |  |  |  |
| 01 | 7 | 8 | 10 | 189 |
| Q2 | 7 | 9 | 11 | LTP |
| Q3 | 12 | 6 | 10 | PRP |
| 94 | 16 | 6 | 11 | PRP |
| Q5 | 12 | 13 | 14 | 1P88 |
| Q6 | 16 | 13 | 15 | H29 |
| Q7 | 100 | 12 | 13 | HPR |
| Q8 | 7 | 7 | 100 | PRP |
| Q9 | 6 | 7 | 100 | P3P |
| 810 | 6 | 4 | 5 | ${ }_{4 T H}$ |
| 911 | 4 | 4 | 200 | HPY |
| 012 | 3 | 3 | 100 | PAP |
| R1 | 14 | 200 | 18 |  |
| R2 | 15 | 200 | 12 |  |
| 83 | 13 | 200 | 50 K |  |
| R5 | 3 | 4 | 39x |  |
| R4 | 5 | 200 | 5K |  |


| DARLTICTCOA CAM ETLCE |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Q13B | 19 | 3 | 100 | PRPB |  |
| 016 | 100 | 16 | 17 | ETPA |  |
| 017 | 19 | 17 | 18 | BPM $^{\text {P }}$ |  |
| 88 | 18 | 200 | 100 |  |  |
| R | 17 | 200 | 50X |  |  |
| * OUTPGT ETAGE |  |  |  |  |  |
| 0138 | 20 | 3 | 100 | 0 PSPA |  |
| Q14 | 100 | 20 | 25 | EFP |  |
| Q18 | 20 | 21 | 22 | ${ }_{2 T P 3}$ |  |
| 019 | 20 | 20 | 21 | LPE |  |
| Q20 | 200 | 22 | 23 | Ptap |  |
| $\mathbf{2 3}$ | 200 | 19 | 22 | PRTP |  |
| 86 | 25 | 9 | 21 |  |  |
| R7 | 23 | 9 | 22 |  |  |
| R10 | 21 | 22 | 40K |  |  |
|  <br> .OPTIOES BOPACE BOMOD <br> .WIDTH OOT=80 <br> . OP <br> .DC VII -15 150.5 <br>  <br> * ters mind calculantors pridict a conalos-mode range of <br> * $-12.7 \mathrm{~V}<\mathrm{VIC}<14.8 \mathrm{~V}$ <br> * In tir volutce-polioner configonation, vo = VI = VIC <br> * as lonio as the huplifitir is morkiso corpectuy. <br> - the results of tais sindation show that <br> * Vo = VI FOR thE FOLLOWING Rages: <br> - $-13 \mathrm{~V}<\mathrm{VI}<14.5 \mathrm{~V}$ <br> - thicesfore, this simmaticen ghows that the <br> - comear mode mipot ramer 1s: <br> - $-13 \mathrm{v}<(\mathrm{VO}=\mathrm{VI}=\mathrm{VIC})<14.5 \mathrm{~V}$ <br> - veich is closs to the result <br> - predicted by hard caiculations. |  |  |  |  |  |
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| * | OPERATING POİFT | InPOR | T30\% $=$ | 27.000 | TEEP= 27.000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| sodr | 3 =VOLTAGE | nods | = VOLTAGB | nods | =VOLThes |
| +0:3 | $=1.4318+01$ | 0:4 | $=-1.433 \mathrm{~B}+01$ | 0:5 | $=-1.490 \mathrm{E}+01$ |
| +0:6 | $=-1.107 \mathrm{~B}+00$ | 0:7 | = $1.4418+01$ | 0:8 | $=0$. |
| +0:9 | 2.7448-04 | $0: 10$ | $=-5.439 \mathrm{~B}-01$ | 0:11 | $=-5.4378-01$ |
| +0:12 | =-1.3898+01 | 0:13 | $=-1.446 \mathrm{~B}+01$ | 0:14 | 1.499E+01 |
| +0:15 | $=-1.499 \mathrm{E}+01$ | 0:16 | $=-1.370 \mathrm{~B}+01$ | 0:17 | $=-1.426 \mathrm{E}+01$ |
| +0:18 | $=-1.493 \mathrm{E}+01$ | 0:19 | $=-1.260 \mathrm{~B}+00$ | 0:20 | 5.9048-01 |
| +0:21 | $=2.345 \mathrm{E}-02$ | 0:22 | $=-6.088 \mathrm{E}-01$ | 0:23 | $=-2.3618-03$ |
| +0:25 | $=3.509 \mathrm{E}-03$ | 0:100 | $=1.500 \mathrm{~B}+01$ | 0:200 | $-1.500 \mathrm{~B}+01$ |

## Problem 5 - ( 10 points)

A two-stage, BiCMOS op amp is shown. For the PMOS transistors, the model parameters are $K_{P}{ }^{\prime}=50 \mu \mathrm{~A} / \mathrm{V}^{2}, V_{T P}=-0.7 \mathrm{~V}$ and $\lambda_{P}=0.05 \mathrm{~V}^{-1}$. For the NPN BJTs, the model parameters are $\beta_{F}=100, V_{C E}($ sat $)=$ $0.2 \mathrm{~V}, V_{A}=25 \mathrm{~V}, V_{t}=26 \mathrm{mV}, I_{s}=10 \mathrm{fA}$ and $n=1$. (a.) Identify which input is positive and which input is negative. (b.) Find the numerical values of differential voltage gain magnitude, $\left|A_{v}(0)\right|, G B$ (in Hertz), the slew rate, $S R$, and the location of the RHP zero. (c.) Find the numerical value of the maximum and minimum input common mode voltages.

## Solution


(a.) The plus and minus signs on the schematic show which input is positive and negative.
(b.) The differential voltage gain, $A_{v}(0)$, is given as

$$
\begin{aligned}
& A_{v}(0)=\frac{g_{m 1}}{g_{d s 2}+g_{o 4}+g_{\pi 6}} \cdot \frac{g_{m 6}}{g_{d s 7^{7}+g_{o 6}}} \quad g_{m 1}=g_{m 2}=\sqrt{2 \cdot 50 \cdot 25 \cdot 10}=158.1 \mu \mathrm{~S} \\
& r_{d s 2}=\frac{1}{\lambda_{P} I_{D}}=\frac{20}{25 \mu \mathrm{~A}}=0.8 \mathrm{M} \Omega, r_{o 4}=\frac{V_{A}}{I_{C}}=\frac{25 \mathrm{~V}}{25 \mu \mathrm{~A}}=1 \mathrm{M} \Omega, g_{m 6}=\frac{I_{C}}{V_{t}}=\frac{100 \mu \mathrm{~A}}{26 \mathrm{mV}}=3846 \mu \mathrm{~S} \\
& r_{\pi 6}=\frac{\beta_{F}}{g_{m 6}}=26 \mathrm{k} \Omega, \quad r_{d s 7}=\frac{1}{\lambda_{P} I_{D}}=\frac{20}{100 \mu \mathrm{~A}}=0.2 \mathrm{M} \Omega \text { and } r_{o 6}=\frac{V_{A}}{I_{C}}=\frac{25 \mathrm{~V}}{100 \mu \mathrm{~A}}=0.25 \mathrm{M} \Omega \\
& \therefore \quad\left|A_{\nu}(0)\right|=[158.1(0.8\|1\| 0.026)][3846(0.2 \| 0.25)]=3.888 \cdot 427.36=\underline{\underline{1,659.6 \mathrm{~V} / \mathrm{V}}} \\
& G B=\frac{g_{m 1}}{C_{c}}=\frac{158.1 \mu \mathrm{~S}}{5 \mathrm{pF}}=31.62 \times 10^{6} \mathrm{rads} / \mathrm{sec} \rightarrow \underline{\underline{G B}=5.0325 \mathrm{MHz}} \\
& S R=\frac{50 \mu \mathrm{~A}}{5 \mathrm{pF}}=\underline{\underline{10 \mathrm{~V} / \mu \mathrm{s}}}
\end{aligned}
$$

$$
\text { RHP zero }=\frac{g_{m 6}}{C_{c}}=\frac{3.846 \mathrm{mS}}{5 \mathrm{pF}}=\underline{\underline{769.24 \times 10^{6} \mathrm{rads} / \mathrm{sec}} .(122 \mathrm{MHz}) ~}
$$

(c.) The maximum input common mode voltage is given as

$$
\begin{aligned}
& v_{i c m^{+}}=V_{C C^{-}} V_{D S 5}(\mathrm{sat})-V_{S G 1}=1.5-\sqrt{\frac{2 \cdot 50}{50 \cdot 10}}-0.7-\sqrt{\frac{2 \cdot 25}{50 \cdot 10}}=0.8-0.447-0.316= \\
& \therefore \quad v_{i c m^{+}}=\underline{\underline{0.0367 \mathrm{~V}}} \\
& v_{i c m^{-}}=-1.5+V_{B E 3}-V_{T 1}=-1.5+V_{t} \ln \left(\frac{25 \mu \mathrm{~A}}{10 \mathrm{fA}}\right)-0.7=-2.2+0.5626=\underline{\underline{-1.6374 \mathrm{~V}}}
\end{aligned}
$$

