LECTURE 010 – ECE 4430 REVIEW I
(READING: GHLM - Chap. 1)

Objective

The objective of this presentation is:

1.) Identify the prerequisite material as taught in ECE 4430

2.) Insure that the students of ECE 6412 are adequately prepared

Outline

- Models for Integrated-Circuit Active Devices
- Bipolar, MOS, and BiCMOS IC Technology
- Single-Transistor and Multiple-Transistor Amplifiers
- Transistor Current Sources and Active Loads

MODELS FOR INTEGRATED-CIRCUIT ACTIVE DEVICES

PN Junctions - Step Junction

Barrier potential-

$$\psi_o = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) = V_t \ln\left(\frac{N_A N_D}{n_i^2}\right) = U_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Depletion region widths-

$$\left.\begin{aligned} W_1 &= \sqrt{\frac{2\varepsilon_{si}(\psi_o - v_D)N_D}{qN_A(N_A+N_D)}} \\ W_2 &= \sqrt{\frac{2\varepsilon_{si}(\psi_o - v_D)N_A}{qN_D(N_A+N_D)}} \end{aligned}\right\} \quad W \propto \sqrt{\frac{1}{N}}$$

Depletion capacitance-

$$C_j = A\sqrt{\frac{\varepsilon_{si} q N_A N_D}{2(N_A+N_D)}}\frac{1}{\sqrt{\psi_o - v_D}} = \frac{C_{j0}}{\sqrt{1 - \frac{v_D}{\psi_o}}}$$

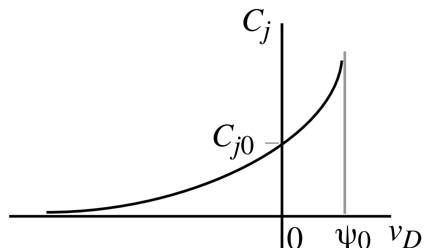

Fig. 010-01

PN-Junctions - Graded Junction

Graded junction:

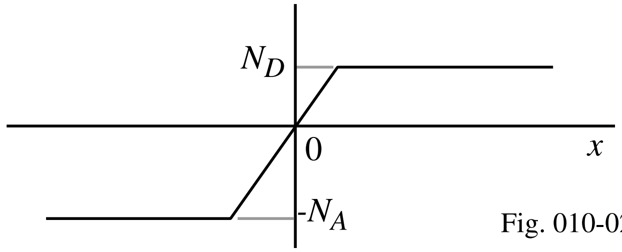

Fig. 010-02

Above expressions become:

Depletion region widths-

$$W_1 = \left(\frac{2\varepsilon_{si}(\psi_o - v_D)N_D}{qN_D(N_A+N_D)}\right)^m$$

$$W_2 = \left(\frac{2\varepsilon_{si}(\psi_o - v_D)N_A}{qN_D(N_A+N_D)}\right)^m$$

$$W \propto \left(\frac{1}{N}\right)^m$$

Depletion capacitance-

$$C_j = A\left(\frac{\varepsilon_{si}qN_AN_D}{2(N_A+N_D)}\right)^m \frac{1}{(\psi_o - v_D)^m} = \frac{C_{j0}}{\left(1 - \frac{v_D}{\psi_o}\right)^m}$$

where $0.33 \leq m \leq 0.5$.

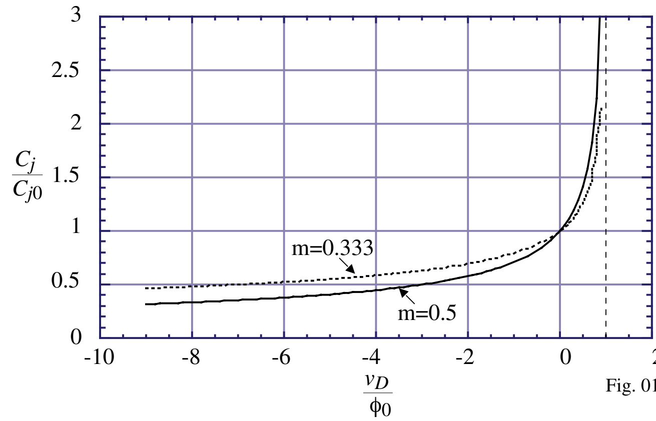

Fig. 010-03

Large Signal Model for the BJT in the Forward Active Region

Large-signal model for a *npn* transistor:

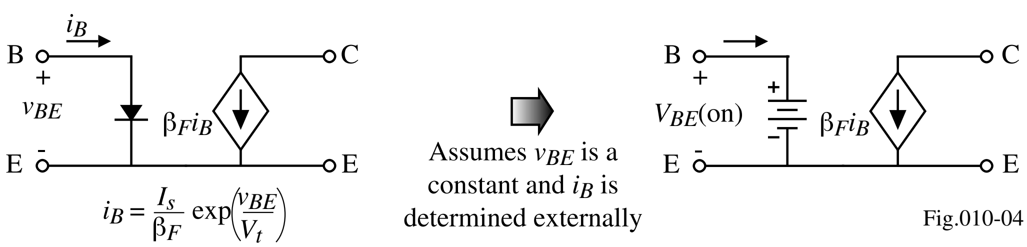

Fig.010-04

Large-signal model for a *pnp* transistor:

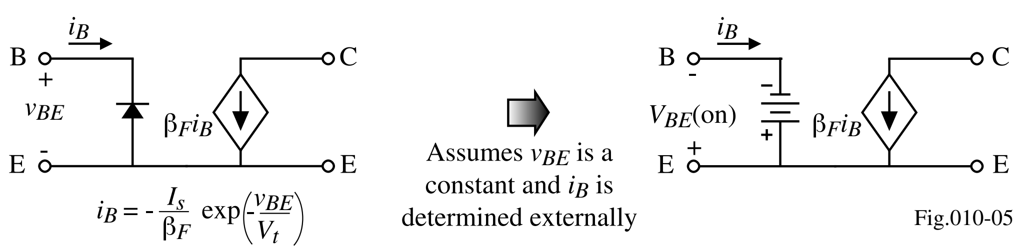

Fig.010-05

Early Voltage:

Modified large signal model becomes

$$i_C = I_S\left(1 + \frac{v_{CE}}{V_A}\right)\exp\left(\frac{v_{BE}}{V_t}\right)$$

The Ebers-Moll Equations

The reciprocity condition allows us to write,

$$\alpha_F I_{EF} = \alpha_R I_{CR} = I_S$$

Substituting into a previous form of the Ebers-Moll equations gives,

$$i_C = I_S\left(\exp\frac{v_{BE}}{V_t} + 1\right) - \frac{I_S}{\alpha_R}\left(\exp\frac{v_{BC}}{V_t} + 1\right)$$

and

$$i_E = -\frac{I_S}{\alpha_F}\left(\exp\frac{v_{BE}}{V_t} + 1\right) + I_S\left(\exp\frac{v_{BC}}{V_t} + 1\right)$$

These equations are valid for all four regions of operation of the BJT.

Also:

- Dependence of β_F as a function of collector current
- The temperature coefficient of β_F is,

$$\mathrm{TC_F} = \frac{1}{\beta_F}\frac{\partial \beta F}{\partial T} \approx +7000\mathrm{ppm/°C}$$

Simple Small Signal BJT Model

Implementing the above relationships, $i_c = g_m v_i + g_o v_{ce}$, and $v_i = r_\pi i_b$, into a schematic model gives,

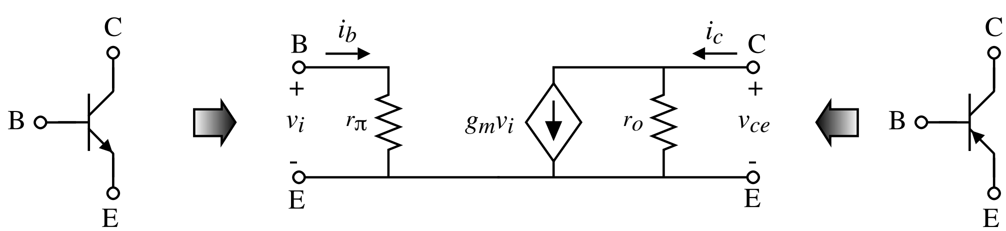

Fig. 010-06

Note that the small signal model is the same for either a *npn* or a *pnp* BJT.

Example:

Find the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain of the common emitter BJT if the BJT is unloaded ($R_L = \infty$), v_{out}/v_{in}, the dc collector current is 1mA, the Early voltage is 100V, and $\beta_o = 100$ at room temperature.

$$g_m = \frac{I_C}{V_t} = \frac{1\mathrm{mA}}{26\mathrm{mV}} = \frac{1}{26}\text{ mhos or Siemans} \qquad R_{in} = r_\pi = \frac{\beta_o}{g_m} = 100\cdot 26 = 2.6\mathrm{k\Omega}$$

$$R_{out} = r_o = \frac{V_A}{I_C} = \frac{100\mathrm{V}}{1\mathrm{mA}} = 100\mathrm{k\Omega} \qquad \frac{v_{out}}{v_{in}} = -g_m r_o = -26\mathrm{mS}\cdot 100\mathrm{k\Omega} = -2600\mathrm{V/V}$$

Complete Small Signal BJT Model

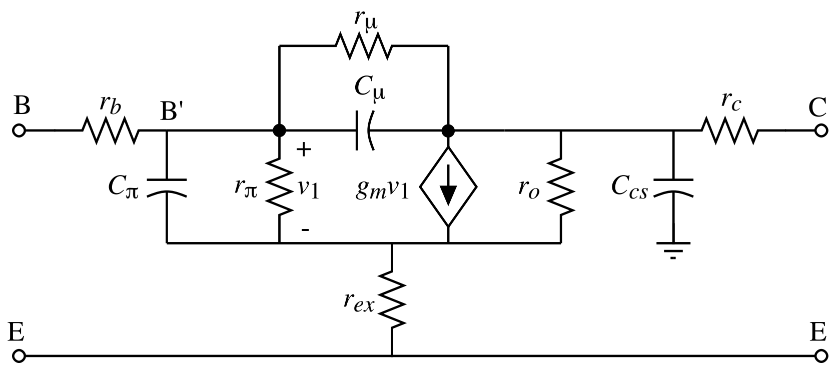

Fig. 010-07

The capacitance, C_π, consists of the sum of C_{je} and C_b.

$$C_\pi = C_{je} + C_b$$

Example 1

Derive the complete small signal equivalent circuit for a BJT at I_C = 1mA, V_{CB} = 3V, and V_{CS} = 5V. The device parameters are $Cje0$ = 10fF, n_e = 0.5, ψ_{0e} = 0.9V, $C_{\mu 0}$ = 10fF, n_c = 0.3, ψ_{0c} = 0.5V, C_{cs0} = 20fF, n_s = 0.3, ψ_{0s} = 0.65V, β_o = 100, τ_F = 10ps, V_A = 20V, r_b = 300Ω, r_c = 50Ω, r_{ex} = 5Ω, and $r_\mu = 10\beta_o r_o$.

Solution

Because C_{je} is difficult to determine and usually an insignificant part of C_π, let us approximate it as $2C_{je0}$.

$\therefore\ C_{je}$ = 20fF

$$C_\mu = \frac{C_{\mu 0}}{\left(1+\frac{V_{CB}}{\psi_{0c}}\right)^{n_e}} = \frac{10\text{fF}}{\left(1+\frac{3}{0.5}\right)^{0.3}} = 5.6\text{fF} \quad \text{and} \quad C_{cs} = \frac{C_{cs0}}{\left(1+\frac{V_{CS}}{\psi_{0s}}\right)^{n_s}} = \frac{20\text{fF}}{\left(1+\frac{5}{0.65}\right)^{0.3}} = 10.5\text{fF}$$

$$g_m = \frac{I_C}{V_t} = \frac{1\text{mA}}{26\text{mV}} = 38\text{mA/V} \qquad C_b = \tau_F\, g_m = (10\text{ps})(38\text{mA/V}) = 0.38\text{pF}$$

$$\therefore\ C_\pi = C_b + C_{je} = 0.38\text{pF} + 0.02\text{pF} = 0.4\text{pF}$$

$$r_\pi = \frac{\beta_o}{g_m} = 100\cdot 26\Omega = 2.6\text{k}\Omega, \quad r_o = \frac{V_A}{I_C} = \frac{20\text{V}}{1\text{mA}} = 20\text{k}\Omega \quad \text{and} \quad r_\mu = 10\beta_o r_o = 20\text{M}\Omega$$

Transition Frequency, f_T

f_T is the frequency where the magnitude of the short-circuit, common-emitter current =1.

Circuit and model:

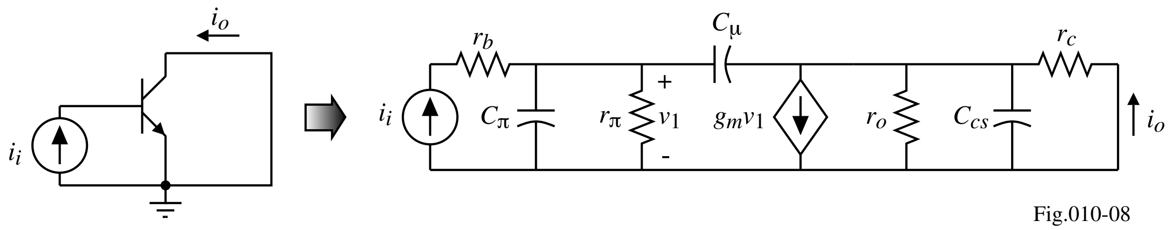

Fig.010-08

Assume that $r_c \approx 0$. As a result, r_o and C_{cs} have no effect.

$$V_1 \approx \frac{r_\pi}{1+r_\pi(C_\pi+C_\mu)s} I_i \quad \text{and} \quad I_o \approx g_m V_1 \quad \Rightarrow \quad \frac{I_o(j\omega)}{I_i(j\omega)} = \frac{g_m r_\pi}{1+g_m r_\pi \frac{(C_\pi+C_\mu)s}{g_m}} = \frac{\beta_o}{1+\beta_o \frac{(C_\pi+C_\mu)s}{g_m}}$$

Now,

$$\beta(j\omega) = \frac{I_o(j\omega)}{I_i(j\omega)} = \frac{\beta_o}{1+\beta_o \frac{(C_\pi+C_\mu)j\omega}{g_m}}$$

At high frequencies,

$$\beta(j\omega) \approx \frac{g_m}{j\omega\,(C_\pi+C_\mu)} \quad \Rightarrow \quad \text{When } |\beta(j\omega)| = 1 \text{ then } \omega_T = \frac{g_m}{C_\pi+C_\mu} \text{ or } f_T = \frac{1}{2\pi}\frac{g_m}{C_\pi+C_\mu}$$

JFET Large Signal Model

Large signal model:

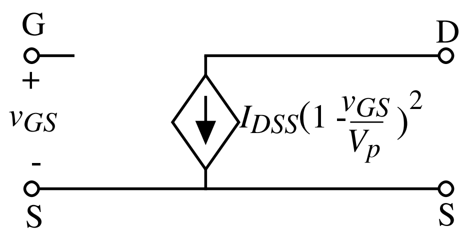

Fig. 010-09

Incorporating the channel modulation effect:

$$i_D = I_{DSS}\left(1 - \frac{v_{GS}}{V_p}\right)^2 (1+\lambda v_{DS}) \quad , \qquad v_{DS} \geq v_{GS}-V_p$$

Signs for the JFET variables:

Type of JFET	V_p	I_{DSS}	v_{GS}
p-channel	Positive	Negative	Normally positive
n-channel	Negative	Positive	Normally negative

Frequency Independent JFET Small Signal Model

Schematic:

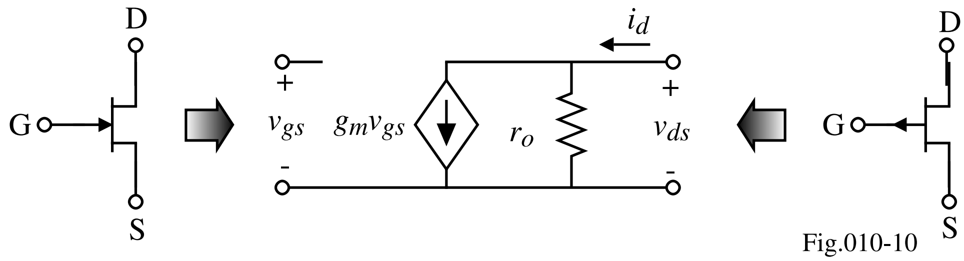

Fig.010-10

Parameters:

$$g_m = \frac{di_D}{dv_{GS}}\bigg|_Q = -\frac{2I_{DSS}}{V_p}\left(1-\frac{V_{GS}}{V_p}\right) = g_{m0}\left(1-\frac{V_{GS}}{V_p}\right)$$

where

$$g_{m0} = -\frac{2I_{DSS}}{V_p}$$

$$r_o = \frac{di_D}{dv_{DS}}\bigg|_Q = \lambda I_{DSS}\left(1-\frac{V_{GS}}{V_p}\right)^2 \approx \frac{1}{\lambda I_D}$$

Typical values of I_{DSS} and V_p for a *p*-channel JFET are -1mA and 2V, respectively. With $\lambda = 0.02\text{V}^{-1}$ and I_D = 1mA we get g_m = 1mA/V or 1mS and $r_o = 50\text{k}\Omega$.

Frequency Dependent JFET Small Signal Model

Complete small signal model:

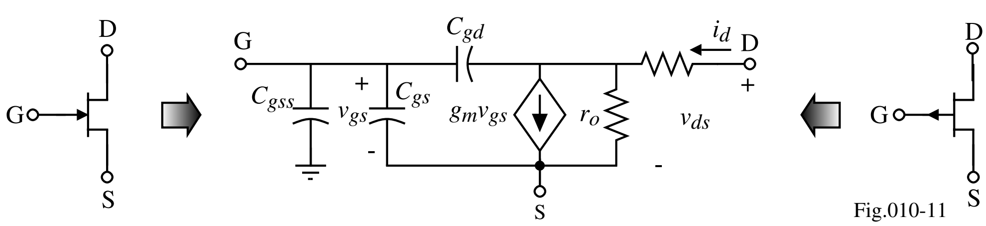

Fig.010-11

All capacitors are reverse biased depletion capacitors given as,

$$C_{gs} = \frac{C_{gs0}}{\left(1+\frac{V_{GS}}{\psi_o}\right)^{1/3}}$$ (capacitance from source to *top* and *bottom* gates)

$$C_{gd} = \frac{C_{gd0}}{\left(1+\frac{V_{GD}}{\psi_o}\right)^{1/3}}$$ (capacitance from drain to *top* and *bottom* gates)

$$C_{gss} = \frac{C_{gss0}}{\left(1+\frac{V_{GSS}}{\psi_o}\right)^{1/2}}$$ (capacitance from the gate (p-base) to substrate)

$$\therefore \quad f_T = \frac{1}{2\pi}\frac{g_m}{C_{gs}+C_{gd}+C_{gss}} = 30\text{MHz}$$ if g_m = 1mA/V and $C_{gs}+C_{gd}+C_{gss}$ = 5pF

Simple Large Signal MOSFET Model

N-channel reference convention:

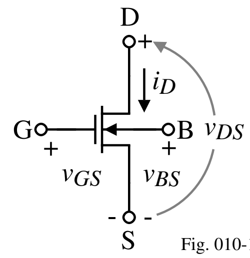

Fig. 010-12

Non-saturation-

$$i_D = \frac{W\mu_o C_{ox}}{L}\left[(v_{GS} - V_T)v_{DS} - \frac{v_{DS}^2}{2}\right](1 + \lambda v_{DS}),\ \ 0 < v_{DS} < v_{GS} - V_T$$

Saturation-

$$i_D = \frac{W\mu_o C_{ox}}{L}\left[(v_{GS} - V_T)v_{DS}(\text{sat}) - \frac{v_{DS}(\text{sat})^2}{2}\right](1 + \lambda v_{DS})$$

$$= \frac{W\mu_o C_{ox}}{2L}(v_{GS} - V_T)^2(1 + \lambda v_{DS}), \qquad 0 < v_{GS} - V_T < v_{DS}$$

where:

μ_o = zero field mobility (cm^2/volt·sec)

C_{ox} = gate oxide capacitance per unit area (F/cm^2)

λ = channel-length modulation parameter (volts^{-1})

$$V_T = V_{T0} + \gamma\left(\sqrt{2|\phi_f| + |v_{BS}|} - \sqrt{2|\phi_f|}\right)$$

V_{T0} = zero bias threshold voltage

γ = bulk threshold parameter (volts$^{-0.5}$)

$2|\phi_f|$ = strong inversion surface potential (volts)

For p-channel MOSFETs, use n-channel equations with p-channel parameters and invert current.

MOSFET Small-Signal Model

Complete schematic model:

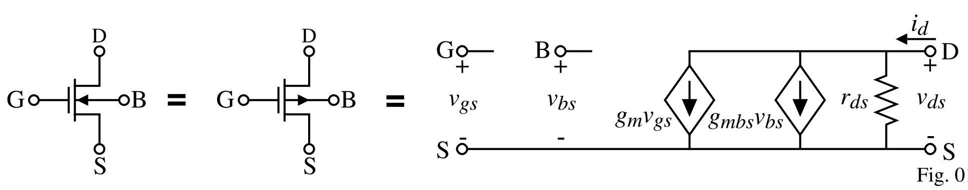

Fig. 010-13

where

$$g_m \equiv \left.\frac{di_D}{dv_{GS}}\right|_Q = \beta(V_{GS} - V_T) = \sqrt{2\beta I_D} \qquad g_{ds} \equiv \left.\frac{di_D}{dv_{DS}}\right|_Q = \frac{\lambda i_D}{1 + \lambda v_{DS}} \approx \lambda i_D$$

and $$g_{mbs} = \left.\frac{\partial i_D}{\partial v_{BS}}\right|_Q = \left.\left(\frac{\partial i_D}{\partial v_{GS}}\right)\left(\frac{\partial v_{GS}}{\partial v_{BS}}\right)\right|_Q = \left.\left(-\frac{\partial i_D}{\partial v_T}\right)\left(\frac{\partial v_T}{\partial v_{BS}}\right)\right|_Q = \frac{g_m \gamma}{2\sqrt{2|\phi_F| - V_{BS}}} = \eta g_m$$

Simplified schematic model:

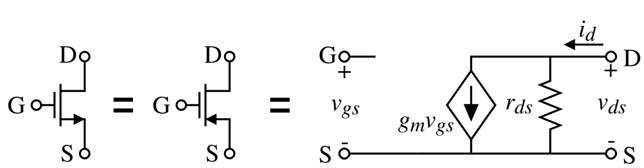

Fig. 4.2-2

Extremely important assumption: $g_m \approx 10 g_{mbs} \approx 100 g_{ds}$

MOSFET Depletion Capacitors - C_{BS} and C_{BD}

Model:

$$C_{BS} = \frac{CJ \cdot AS}{\left(1 - \frac{v_{BS}}{PB}\right)^{MJ}} + \frac{CJSW \cdot PS}{\left(1 - \frac{v_{BS}}{PB}\right)^{MJSW}}, \quad v_{BS} \le FC \cdot PB$$

and

$$C_{BS} = \frac{CJ \cdot AS}{(1 - FC)^{1+MJ}}\left(1 - (1+MJ)FC + MJ\frac{V_{BS}}{PB}\right)$$
$$+ \frac{CJSW \cdot PS}{(1 - FC)^{1+MJSW}}\left(1 - (1+MJSW)FC + MJSW\frac{V_{BS}}{PB}\right),$$

$v_{BS} > FC \cdot PB$

where

AS = area of the source

PS = perimeter of the source

$CJSW$ = zero bias, bulk source sidewall capacitance

$MJSW$ = bulk-source sidewall grading coefficient

For the bulk-drain depletion capacitance replace "S" by "D" in the above equations.

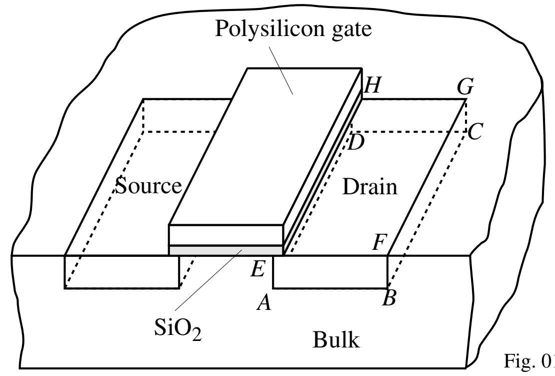

Fig. 010-14

Drain bottom = $ABCD$

Drain sidewall = $ABFE + BCGF + DCGH + ADHE$

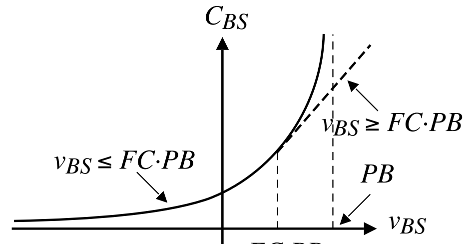

Fig 010-15

MOSFET Intrinsic Capacitors - C_{GD}, C_{GS} and C_{GB}

Cutoff Region:

$C_{GB} = C_2 + 2C_5 = C_{ox}(W_{eff})(L_{eff}) + 2\text{CGBO}(L_{eff})$

$C_{GS} = C_1 \approx C_{ox}(\text{LD})W_{eff}) = \text{CGSO}(W_{eff})$

$C_{GD} = C_3 \approx C_{ox}(\text{LD})W_{eff}) = \text{CGDO}(W_{eff})$

Saturation Region:

$C_{GB} = 2C_5 = \text{CGBO}(L_{eff})$

$C_{GS} = C_1 + (2/3)C_2 = C_{ox}(\text{LD}+0.67L_{eff})(W_{eff})$

$= \text{CGSO}(W_{eff}) + 0.67C_{ox}(W_{eff})(L_{eff})$

$C_{GD} = C_3 \approx C_{ox}(\text{LD})W_{eff}) = \text{CGDO}(W_{eff})$

Active Region:

$C_{GB} = 2\,C_5 = 2\text{CGBO}(L_{eff})$

$C_{GS} = C_1 + 0.5C_2 = C_{ox}(\text{LD}+0.5L_{eff})(W_{eff})$

$= (\text{CGSO} + 0.5C_{ox}L_{eff})W_{eff}$

$C_{GD} = C_3 + 0.5C_2 = C_{ox}(\text{LD}+0.5L_{eff})(W_{eff})$

$= (\text{CGDO} + 0.5C_{ox}L_{eff})W_{eff}$

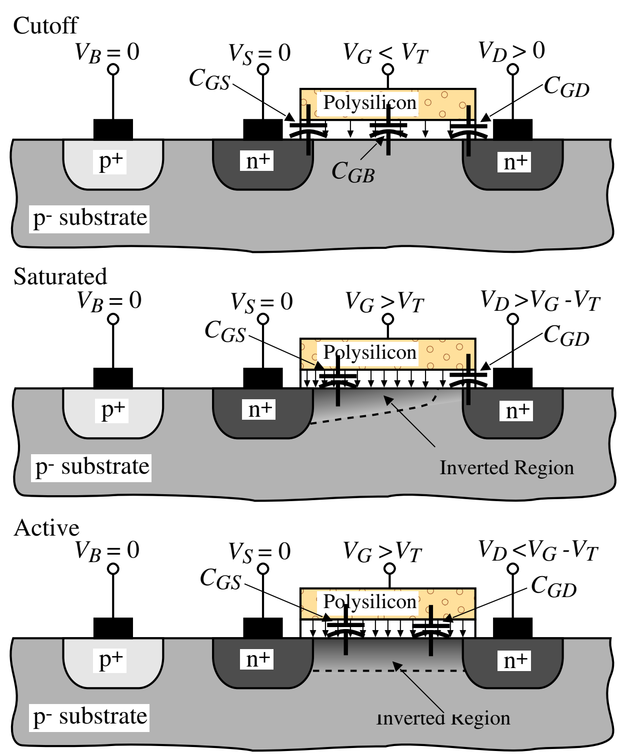

Fig 010-16

Small-Signal Frequency Dependent Model

The depletion capacitors are found by evaluating the large signal capacitors at the DC operating point.

The charge storage capacitors are constant for a specific region of operation.

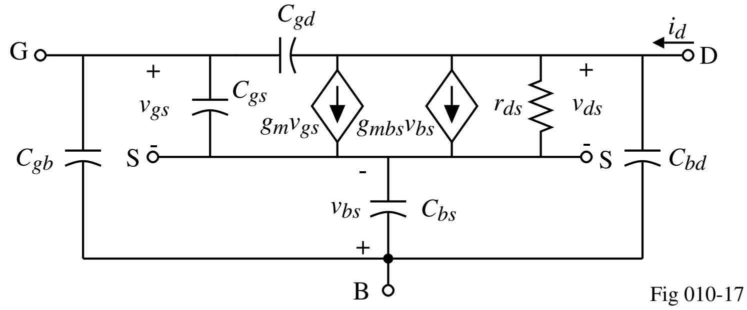

Fig 010-17

Gainbandwidth of the MOSFET:

Assume $V_{SB} = 0$ and the MOSFET is in saturation,

$$f_T = \frac{1}{2\pi}\frac{g_m}{C_{gs}+C_{gd}} \approx \frac{1}{2\pi}\frac{g_m}{C_{gs}}$$

Recalling that

$$C_{gs} \approx \frac{2}{3} C_{ox}WL \quad \text{and} \quad g_m = \mu_o C_{ox}\frac{W}{L}(V_{GS}\text{-}V_T)$$

gives

$$f_T = \frac{3}{4\pi}\frac{\mu_o}{L^2}(V_{GS}\text{-}V_T)$$

Subthreshold MOSFET Model

Weak inversion operation occurs when the applied gate voltage is below V_T and pertains to when the surface of the substrate beneath the gate is weakly inverted.

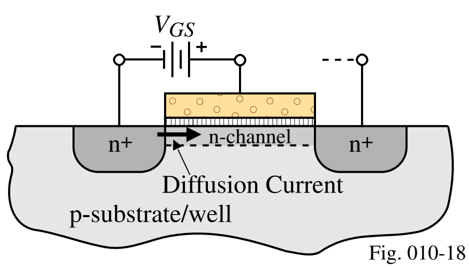

Fig. 010-18

Regions of operation according to the surface potential, ϕ_S.

$\phi_S < \phi_F$: Substrate not inverted

$\phi_F < \phi_S < 2\phi_F$: Channel is weakly inverted (diffusion current)

$2\phi_F < \phi_S$: Strong inversion (drift current)

Drift current versus diffusion current in a MOSFET:

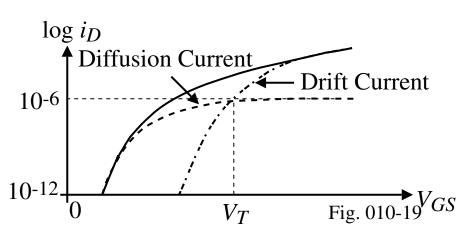

Fig. 010-19

Large-Signal Model for Subthreshold

Model:

$$i_D = K_x \frac{W}{L} e^{v_{GS}/nV_t}(1 - e^{-v_{DS}/V_t})(1 + \lambda v_{DS})$$

where

K_x is dependent on process parameters and the bulk-source voltage

$n \approx 1.5 - 3$

and

$$V_t = \frac{kT}{q}$$

If $v_{DS} > 0$, then

$$i_D = K_x \frac{W}{L} e^{v_{GS}/nV_t} (1 + \lambda v_{DS})$$

Small-signal model:

$$g_m = \frac{\partial i_D}{\partial v_{GS}}\Big|_Q = \frac{qI_D}{nkT}$$

$$g_{ds} = \frac{\partial i_D}{\partial v_{DS}}\Big|_Q \approx \frac{I_D}{V_A}$$

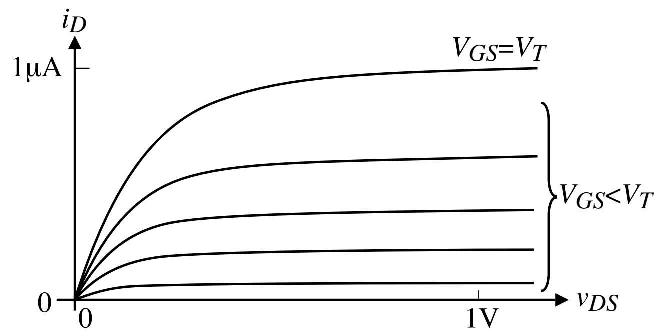

Fig 010-20

SUMMARY

- Models
  - Large-signal
  - Small-signal
- Components
  - pn Junction
  - BJT
  - MOSFET
    - Strong inversion
    - Weak inversion
  - JFET
- Capacitors
  - Depletion
  - Parallel plate