LECTURE 290 – FEEDBACK CIRCUIT ANALYSIS USING RETURN RATIO

(READING: GHLM – 599-613)

Objective

The objective of this presentation is:

1.) Illustrate the method of using return ratio to analyze feedback circuits

2.) Demonstrate using examples

Outline

- Concept of return ratio
- Closed-loop gain using return ratio
- Closed-loop impedance using return ratio
- Summary

Concept of Return Ratio

Instead of using two-port analysis, return ratio takes advantage of signal flow graph theory.

The return ratio for a dependent source in a feedback loop is found as follows:

1.) Set all independent sources to zero.

2.) Change the dependent source to an independent source and define the controlling variable as, s_r, and the source variable as s_t.

3.) Calculate the return ratio designated as $RR = - s_r/s_t$.

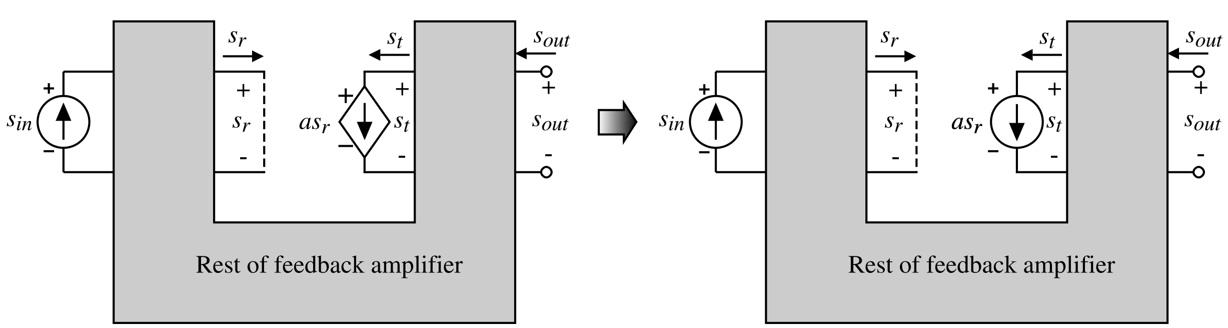

Fig. 290-01

Example 1 – Calculation of Return Ratio

Find the return ratio of the op amp with feedback shown if the input resistance of the op amp is r_i, the output resistance is r_o, and the voltage gain is a_v.

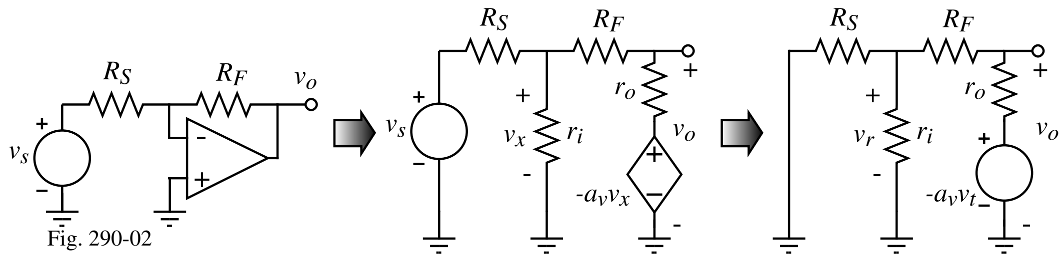

Fig. 290-02

Solution

$$v_r = \frac{(-a_v v_t) R_S || r_i}{r_o + R_F + R_S || r_i} \quad \rightarrow \quad RR = -\frac{v_r}{v_t} = \frac{(a_v) R_S || r_i}{r_o + R_F + R_S || r_i}$$

Closed-Loop Gain Using Return Ratio

Consider the following general feedback amplifier:

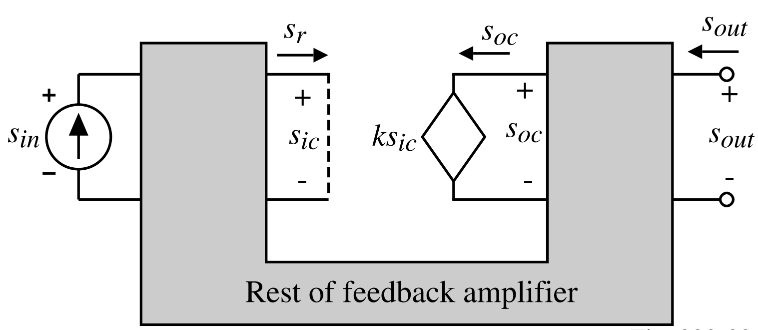

Fig. 290-03

Note that $s_{oc} = ks_{ic}$.

Assume the amplifier is linear and express s_{ic} and s_{out} as linear functions of the two sources, s_{in} and s_{oc}.

$$s_{ic} = B_1\, s_{in} - H\, s_{oc}$$

$$s_{out} = d\, s_{in} + B_2 s_{oc}$$

where B_1, B_2, and H are defined as

$$B_1 = \left.\frac{s_{ic}}{s_{in}}\right|_{s_{oc}=0} = \left.\frac{s_{ic}}{s_{in}}\right|_{k=0}, \quad B_2 = \left.\frac{s_{out}}{s_{oc}}\right|_{s_{in}=0}, \quad \text{and} \quad H = -\left.\frac{s_{ic}}{s_{oc}}\right|_{s_{in}=0}$$

Closed-Loop Gain Using Return Ratio – Continued

Interpretation:

B_1 is the transfer function from the input to the controlling signal with $k = 0$.

B_2 is the transfer function from the controlling signal to the output with $s_{in} = 0$.

H is the transfer function from the output of the dependent source to the controlling signal with $s_{in} = 0$ and multiplied times a –1.

d is defined as,

$$d = \left.\frac{s_{out}}{s_{in}}\right|_{s_{oc}=0} = \left.\frac{s_{out}}{s_{in}}\right|_{k=0}$$

d = is the direct signal feedthrough when the controlled source in A is set to zero (k=0)

Closed-loop gain (s_{out}/s_{in}) can be found as,

$$s_{ic} = B_1\, s_{in} - H\, s_{oc} = B_1\, s_{in} - kH\, s_{ic} \quad \rightarrow \quad \frac{s_{ic}}{s_{in}} = \frac{B_1}{1 + kH}$$

$$s_{out} = d\, s_{in} + B_2 s_{oc} = d\, s_{in} + kB_2 s_{ic} = d\, s_{in} + \frac{B_1 k B_2}{1 + kH}\, s_{in}$$

2.) $$A = \frac{s_{out}}{s_{in}} = \frac{B_1 k B_2}{1 + kH} + d = \frac{B_1 k B_2}{1 + RR} + d = \frac{g}{1 + RR} + d$$

where $RR = kH$ and $g = B_1 k B_2$ (gain from s_{in} to s_{out} if $H = 0$ and $d = 0$)

Closed-Loop Gain Using Return Ratio – Continued

Further simplification:

$$A = \frac{g}{1 + RR} + d = \frac{g + d(1+RR)}{1 + RR} = \frac{g + d\cdot RR}{1 + RR} + \frac{d}{1 + RR} = \frac{\left(\frac{g}{RR} + d\right)RR}{1 + RR} + \frac{d}{1 + RR}$$

Define

$$A_\infty = \frac{g}{RR} + d$$

3.) $$A = A_\infty \frac{RR}{1 + RR} + \frac{d}{1 + RR}$$

Note that as $RR \rightarrow \infty$, that $A = A_\infty$.

A_∞ is the closed-loop gain when the feedback circuit is ideal (i.e., $RR \rightarrow \infty$ or $k \rightarrow \infty$).

Block diagram of the new formulation:

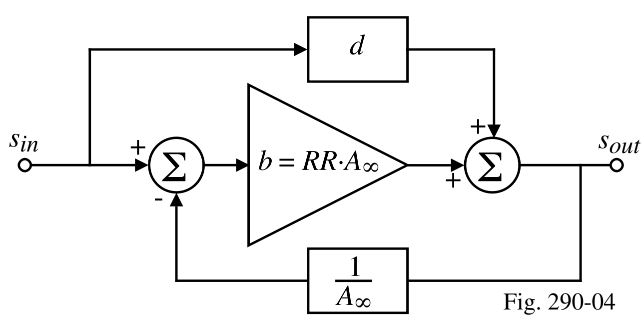

Fig. 290-04

Note that $b = RR\cdot A_\infty$ is called the effective gain of the feedback amplifier.

Example 2 – Use of Return Ratio Approach to Calculate the Closed-Loop Gain

Find the closed-loop gain and the effective gain of the transistor feedback amplifier shown using the previous formulas. Assume that the BJT g_m = 40mS, r_π = 5kΩ, and r_o = 1MΩ.

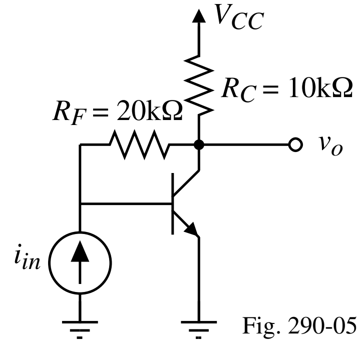

Fig. 290-05

Solution

The small-signal model suitable for calculating A_∞ and d is shown.

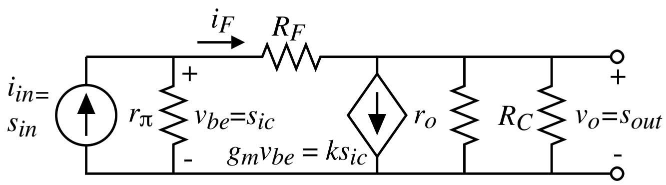

Fig. 290-06

$$A_\infty = \left.\frac{s_{out}}{s_{in}}\right|_{k=\infty} = \left.\frac{v_o}{i_{in}}\right|_{g_m=\infty} = ?$$

Remember that $A = \frac{a}{1+af} \rightarrow \frac{1}{f}$ as $a \rightarrow \infty$.

$$f = \left.\frac{v_o}{i_F}\right|_{v_{in}=0} = \frac{-1}{R_F}$$

Therefore, $A_\infty = -R_F = -20\text{k}\Omega$

$$d = \left.\frac{s_{out}}{s_{in}}\right|_{k=0} = \left.\frac{v_o}{i_{in}}\right|_{g_m=0} = \frac{r_\pi}{r_\pi + R_F + (r_o||R_C)}(r_o||R_C)$$

$$= \frac{5\text{k}\Omega}{5\text{k}\Omega + 20\text{k}\Omega + 1\text{M}\Omega||10\text{k}\Omega}(1\text{M}\Omega||10\text{k}\Omega) = 1.42\text{k}\Omega$$

Example 2 – Continued

What is left is to calculate the *RR*. A small-signal model for this is shown below.

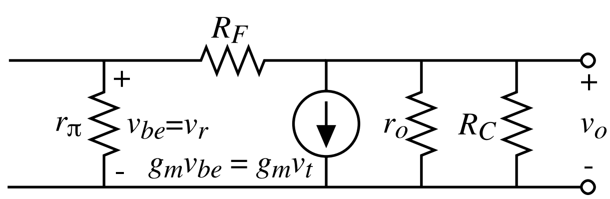

Fig. 290-07

$$v_r = (-g_m v_t)\left(\frac{r_o||R_C}{r_\pi + R_F + r_o||R_C}\right) r_\pi \quad \rightarrow \quad \frac{v_r}{v_t} = (-g_m r_\pi)\left(\frac{r_o||R_C}{r_\pi + R_F + r_o||R_C}\right)$$

$$RR = -\frac{v_r}{v_t} = (g_m r_\pi)\left(\frac{r_o||R_C}{r_\pi + R_F + r_o||R_C}\right) = (200)\left(\frac{1\text{M}\Omega||10\text{k}\Omega}{5\text{k}\Omega + 20\text{k}\Omega + 1\text{M}\Omega||10\text{k}\Omega}\right) = 56.74$$

Now, the closed loop gain is found to be,

$$A = A_\infty \frac{RR}{1+RR} + \frac{d}{1+RR} = (-20\text{k}\Omega)\left(\frac{56.74}{1+56.74}\right) + \left(\frac{1.4\text{k}\Omega}{1+56.74}\right) = -19.63\text{k}\Omega$$

The effective gain is given as,

$$b = RR \cdot A_\infty = 56.74(-20\text{k}\Omega) = -1135\text{k}\Omega$$

Closed-Loop Impedance Formula using the Return Ratio (Blackman's Formula)

Consider the following linear feedback circuit where the impedance at port X is to be calculated.

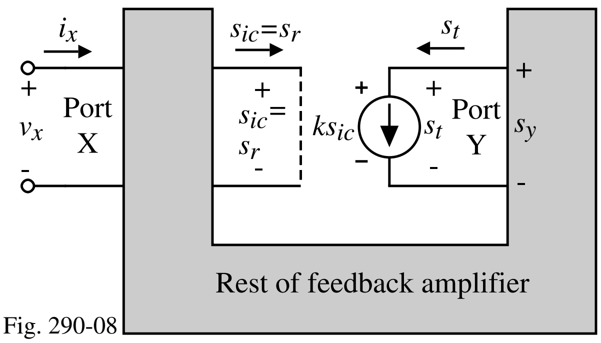

Fig. 290-08

Expressing the signals, v_x and s_{ic} as linear functions of the signals i_x and s_y gives,

$$v_x = a_1 i_x + a_2 s_y$$

$$s_{ic} = a_3 i_x + a_4 s_y$$

The impedance looking into port X when $k = 0$ is,

$$Z_{port}(k{=}0) = \left.\frac{v_x}{i_x}\right|_{k=0} = \left.\frac{v_x}{i_x}\right|_{s_y=0}$$

Closed-Loop Impedance Formula using the Return Ratio – Continued

Next, compute the *RR* for the controlled source, *k*, under two different conditions.

1.) The first condition is when port X is open ($i_x = 0$).

$$s_{ic} = a_4 s_y = a_4 s_t$$

Also,

$$s_r = ks_{ic} \quad \rightarrow \quad s_r = ka_4 s_t \quad \rightarrow \quad RR(\text{port open}) = -\frac{s_r}{s_t} = -ka_4$$

2.) The second condition is when port X is shorted ($v_x = 0$).

$$i_x = -\frac{a_2}{a_1} s_y = -\frac{a_2}{a_1} s_t$$

$$\therefore \qquad s_{ic} = a_3 i_x + a_4 s_y = \left(a_4 - \frac{a_2 a_3}{a_1}\right) s_t$$

The return signal is

$$s_r = ks_{ic} = k\left(a_4 - \frac{a_2 a_3}{a_1}\right) s_t \quad \rightarrow \quad RR(\text{port shorted}) = -\frac{s_r}{s_t} = -k\left(a_4 - \frac{a_2 a_3}{a_1}\right)$$

3.) The port impedance can be found as (Blackman's formula),

4.) $$Z_{\text{port}} = \frac{v_x}{i_x} = a_1\left(\frac{1 - k\left(a_4 - \frac{a_2 a_3}{a_1}\right)}{1 - a_4}\right) \Rightarrow \boxed{Z_{\text{port}} = Z_{\text{port}}(k{=}0)\left[\frac{1 + RR(\text{port shorted})}{1 + RR(\text{port open})}\right]}$$

Example 3 – Application of Blackman's Formula

Use Blackman's formula to calculate the output resistance of Example 2.

Solution

We must calculate three quantities. They are $R_{out}(g_m{=}0)$, RR(output port shorted), and RR(output port open). Use the following model for calculations:

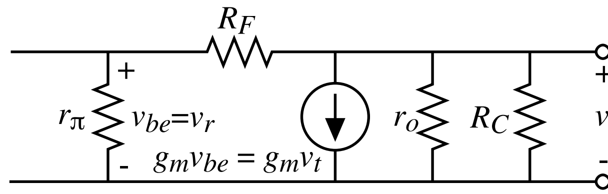

Fig. 290-07

$R_{out}(g_m{=}0) = r_o || R_C || (r_\pi + R_F) = 7.09\text{k}\Omega$

RR(output port shorted) = 0 because $v_r = 0$.

RR(output port open) = RR of Example 2 = 56.74

$$\therefore \quad R_{out} = R_{out}(g_m{=}0)\left[\frac{1 + RR(\text{port shorted})}{1 + RR(\text{port open})}\right] = 7.09\text{k}\Omega\left(\frac{1}{1+56.74}\right) = 129\Omega$$

Example 4 – Output Resistance of a Super-Source Follower

Find an expression for the small-signal output resistance of the circuit shown.

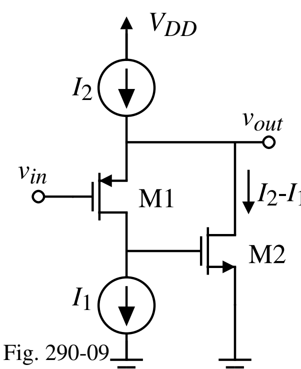

Fig. 290-09

Solution

The appropriate small-signal model is shown where $g_{m2} = k$.

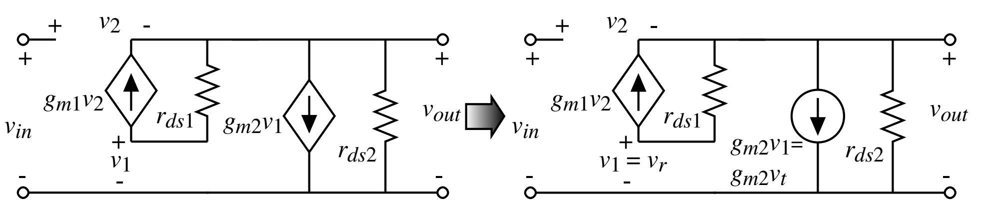

Fig. 290-10

$R_{out}(g_{m2}{=}0) = r_{ds2}$ and RR(output port shorted) = 0 because $v_t = 0$.

$$RR(\text{output port open}) = -\frac{s_r}{s_t} = -\frac{v_r}{v_t}$$

$$v_r = v_{out} - (g_{m1}v_2)r_{ds1} = v_{out} - g_{m1}r_{ds1}(-v_{out}) = v_{out}(1 + g_{m1}r_{ds1})$$

$$v_{out} = -g_{m2}r_{ds2}v_t \quad \rightarrow \quad v_r = -(1 + g_{m1}r_{ds1})g_{m2}r_{ds2}v_t$$

$$RR(\text{output port open}) = -\frac{v_r}{v_t} = (1 + g_{m1}r_{ds1})g_{m2}r_{ds2}$$

$$\therefore \quad R_{out} = R_{out}(g_{m2}{=}0)\left[\frac{1 + RR(\text{port shorted})}{1 + RR(\text{port open})}\right] = r_{ds2}\left(\frac{1+0}{1+(1+g_{m1}r_{ds1})g_{m2}r_{ds2}}\right) \approx \frac{1}{g_{m1}r_{ds1}g_{m2}}$$

SUMMARY

- Return ratio is associated with a dependent source. If the dependent source is converted to an independent source, then the return ratio is the gain from the dependent source variable to the previously controlling variable.
- The closed-loop gain of a linear, negative feedback system can be expressed as

$$A = A_\infty \frac{RR}{1+RR} + \frac{d}{1+RR}$$

where

A_∞ = the closed-loop gain when the loop gain is infinite

RR = the return ratio

d = the closed-loop gain when the amplifier gain is zero

- The resistance at a port can be found from Blackman's formula which is

$$Z_{port} = Z_{port}(k{=}0)\left[\frac{1 + RR(\text{port shorted})}{1 + RR(\text{port open})}\right]$$

where k is the gain of the dependent source chosen for the return ratio calculation

- This stuff is all great but of *little use as far as calculations are concerned*.

  Small-signal analysis is generally quicker and easier than the two-port approach or the return ratio approach.
- Why study feedback? Because it is a great tool for understanding a circuit and for knowing how to modify the performance in design.

