GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 2040
Circuit Analysis

Quiz #4 – Solutions

Problem Q4.1:

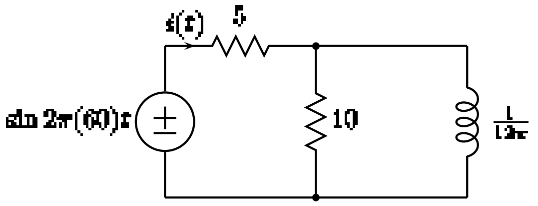


Figure 1: Figure for Problem Q4.1.

(a) Redraw the circuit in the complex amplitude domain.
(b) Determine the complex amplitude I of the current $i(t)$.
(c) Determine the current $i(t)$ generated by the voltage source.
(d) Determine the average power supplied by the voltage source.

Solution:
(a) The circuit in the complex amplitude domain is drawn in **Figure 2**.

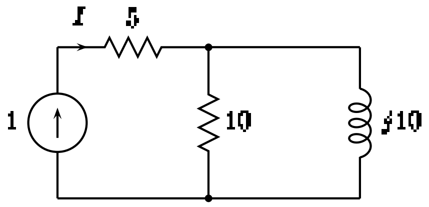


Figure 2: The circuit of Figure 1 redrawn in the complex amplitude domain.

(b)

$$I = 1 \cdot \frac{1}{5 + \frac{j100}{10 - j10}} = \frac{10 + j10}{50 + j150} = \frac{1 + j}{5 + j15}$$

(c)

$$\begin{aligned} i(t) &= \Im m \left[\frac{1 + j}{5 + j15} e^{j2\pi(60)t} \right] \\ &= \Im m \left[\frac{1}{5\sqrt{5}} e^{j(\frac{\pi}{4} - \tan^{-1} 3)} e^{j2\pi(60)t} \right] \\ &= \frac{1}{5\sqrt{5}} \sin(2\pi(60)t + \frac{\pi}{4} - \tan^{-1} 3) \end{aligned}$$

(d)

$$\begin{aligned} P_{avg} &= \frac{1}{2}\Re e V I^* = \frac{1}{2}\Re e\left\{\frac{1-j}{5-j15}\right\} \\ &= \frac{1}{2}\Re e\left\{\frac{(1-j)(5+j15)}{250}\right\} = \frac{1}{2}\Re e\left\{\frac{20+j10}{250}\right\} = 0.04W. \end{aligned}$$

Problem Q4.2:

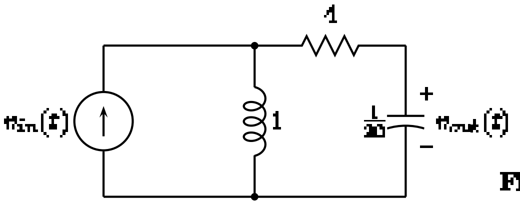


Figure 3: Circuit for Problem Q4.2.

(a) Find the system function of the circuit in Figure 3. The circuit is at initial rest.

(b) Find and sketch the impulse response.

(c) Sketch the response of the circuit to the input sketched in Figure 4.

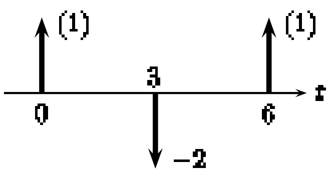


Figure 4: Waveform for Problem Q4.4.

Solution:
(a)

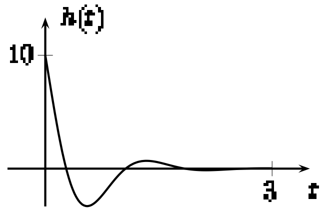
$$H(s) = \frac{\frac{20}{s}\cdot s}{s+4+\frac{20}{s}} = \frac{20s}{s^2+4s+20} = \frac{20s}{(s+2)^2+16}$$

(b) The impulse response is the inverse Laplace transform of the system function.

$$\begin{aligned} H(s) &= \frac{10+j5}{s+2-j4} + \frac{10-j5}{s+2+j4} \\ &= \frac{5\sqrt{5}e^{j\tan^{-1}\frac{1}{2}}}{s+2-j4} + \frac{5\sqrt{5}e^{-j\tan^{-1}\frac{1}{2}}}{s+2+j4}. \end{aligned}$$

Therefore,

$$h(t) = 5\sqrt{5}e^{-2t}\cos(4t+\tan^{-1}\frac{1}{2})u(t).$$



(c) Since this is a linear, time-invariant system

$$v_{out}(t) = h(t) - 2h(t-3) + h(t-6),$$

which is easily sketched.

Problem Q4.3: Compute the current $i(t)$ for the circuit in **Figure 5**. The solution will require several steps. To receive partial credit, you must clearly indicate your reasoning for each of these.

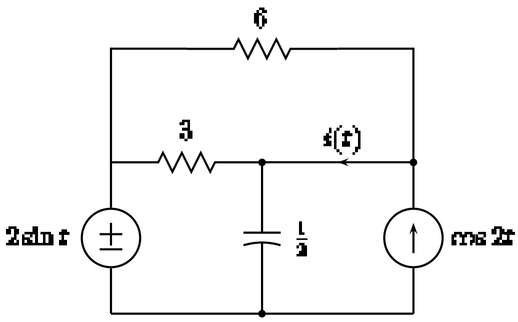
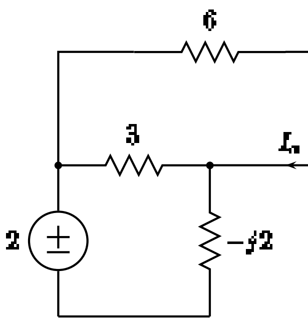


Figure 5: Circuit for Problem Q4.3.

Solution: Since there are two sinusoidal inputs at different frequencies, we will need to apply superposition to analyse the circuit. Turning off the current source, gives the following circuit (in the complex amplitude domain):



The complex amplitude of the current I_a is the same as for the current flowing through the 6Ω resistor, which is 1/3 of the current leaving the voltage source.

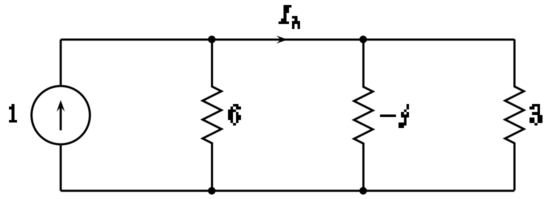
Therefore,

$$I_a = \frac{1}{2} \cdot \frac{2}{2 - j2} = \frac{1 + j}{6} = \frac{\sqrt{2}}{6} e^{j\pi/4}.$$

and

$$i_a(t) = \frac{\sqrt{2}}{6} \sin(t + \frac{\pi}{4}).$$

To get the second component of the current, we turn off the current source.



We can get I_b by using a current divider.

$$I_b = \frac{6}{6 + \frac{-j3}{3-j}} = \frac{6 - 2j}{6 - 3j} = \frac{\sqrt{8}}{3} e^{j(\tan^{-1} \frac{1}{2} - \tan^{-1} \frac{1}{3})}.$$

Therefore,

$$i_b(t) = \frac{\sqrt{8}}{3} \cos(2t + \tan^{-1} \frac{1}{2} - \tan^{-1} \frac{1}{3})$$

and

$$i(t) = \frac{\sqrt{2}}{6} \sin(t + \frac{\pi}{4}) + \frac{\sqrt{8}}{3} \cos(2t + \tan^{-1} \frac{1}{2} - \tan^{-1} \frac{1}{3})$$
