GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

April 6, 2001

Problem Set #10a

Problem 10a.1: The circuit in Figure 1 is at initial rest.

(a) Find the system function $H(s)$ that relates the output $I_{out}(s)$ to the input $V_{in}(s)$.

(b) Find the impulse response of the system $h(t)$.

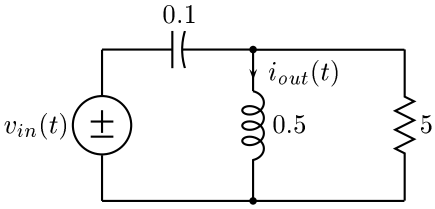


Figure 1: Circuit for Problem 10a.1.

Solution:

(a) Let $I_{in}(s)$ denote the current flowing out of the voltage source. Then we can use a current divider to relate $I_{in}(s)$ to $I_{out}(s)$.

$$I_{out}(s) = I_{in}(s) \cdot \frac{5}{\frac{s}{2}+5} = I_{in}(s) \cdot \frac{10}{s+10}$$

To determine $I_{in}(s)$ we can determine the equivalent impedance of the combined elements, since

$$I_{in}(s) = \frac{V_{in}(s)}{Z_{eq}(s)}.$$

But,

$$\begin{aligned} Z_{eq}(s) &= \frac{10}{s} + \frac{\frac{s}{2} \cdot 5}{\frac{s}{2}+5} = \frac{10}{s} + \frac{5s}{s+10} \\ &= \frac{5(s^2+2s+20)}{s(s+10)} \end{aligned}$$

Therefore,

$$I_{out}(s) = V_{in}(s) \cdot \frac{10}{s+10} \cdot \frac{s(s+10)}{5(s^2+2s+20)} = \frac{2s}{s^2+2s+20}$$

and

$$H(s) = \frac{2s}{s^2 + 2s + 20}$$

(b) We can manipulate $H(s)$ into the form

$$H(s) = \frac{2(s+1)}{(s+1)^2 + 19} - \frac{\frac{2}{\sqrt{19}} \cdot \sqrt{19}}{(s+1)^2 + 19}$$

from which we see that

$$h(t) = 2e^{-t} \cos \sqrt{19}t - \frac{2}{\sqrt{19}} e^{-t} \sin \sqrt{19}t.$$

Problem 10a.2: The network of Figure 2 is initially at rest. Determine the voltage $v(t)$ for each of the inputs below:

(a) $i_s(t) = u(t)$

(b) $i_s(t) = (\sin t)u(t)$

(c) $i_s(t) = tu(t)$

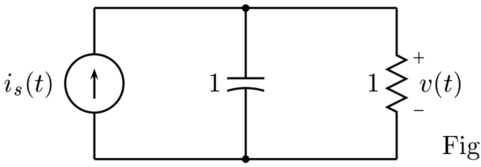


Figure 2: Circuit for Problem 10a.2.

Solution: For this circuit

$$H(s) = \frac{V(s)}{I_s(s)} = Z_{eq}(s) = \frac{1}{s+1}.$$

Since the circuit is at initial rest, all three outputs will be zero for $t < 0$.

(a) If $i_s(t) = u(t)$, then $I_s(s) = \frac{1}{s}$ and

$$V_a(s) = H(s)I_s(s) = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}.$$

Therefore,

$$v_a(t) = (1 - e^{-t})u(t).$$

(b) If $i_s(t) = (\sin t)u(t)$, then $I_s(s) = \frac{1}{s^2+1}$ and

$$V_b(s) = H(s)I_s(s) = \frac{1}{(s+1)(s^2+1)} = \frac{\frac{1}{2}}{s+1} - \frac{\frac{1}{2}s}{s^2+1} + \frac{\frac{1}{2}}{s^2+1}.$$

Therefore,

$$v_b(t) = \frac{1}{2}\left(1 - \cos t + \sin t\right)u(t)$$

(c) If $i_s(t) = tu(t)$, then $I_s(s) = \frac{1}{s^2}$ and

$$V_c(s) = \frac{1}{s^2(s+1)} = \frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}.$$

Therefore,

$$v_c(t) = \left[t - 1 + e^{-t}\right] u(t)$$

Problem 10a.3: (a) The system function of the circuit in Figure 3, which is at initial rest, has the form

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{as+b}{s+c}.$$

Determine the values of a, b, and c.

(b) Determine $v_{out}(t)$ for all t, if $v_{in}(t) = e^{-2t}u(t)$.

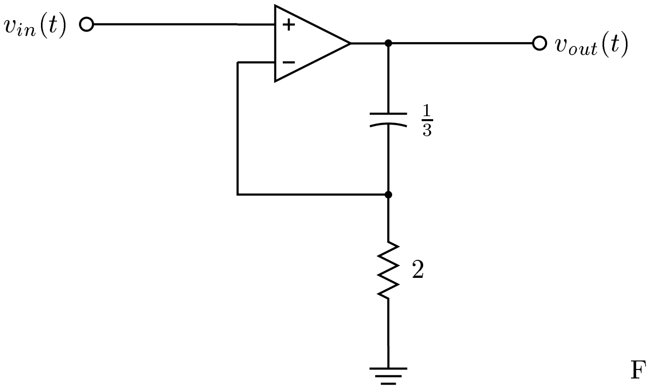


Figure 3: Circuit for Problem 10a.3.

Solution:

(a) The potential at the node that connects the resistor to the capacitor is $V_{in}(s)$. Writing a KCL equation at that node gives

$$\frac{V_{in}(s)}{2} + \frac{s}{3}[V_{in}(s) - V_{out}(s)] = 0.$$

Simplifying, we get

$$\begin{aligned} V_{in}(s)\left[\frac{1}{2} + \frac{s}{3}\right] &= \frac{s}{3}V_{out}(s) \\ V_{out}(s) &= \frac{3}{s}\left[\frac{2s+3}{6}\right] V_{in}(s) \end{aligned}$$

and

$$H(s) = \frac{s+\frac{3}{2}}{s}.$$

Therefore, $a = 1$, $b = 3/2$, and $c = 0$.

(b)

$$V_{in}(s) = \frac{1}{s+2}$$

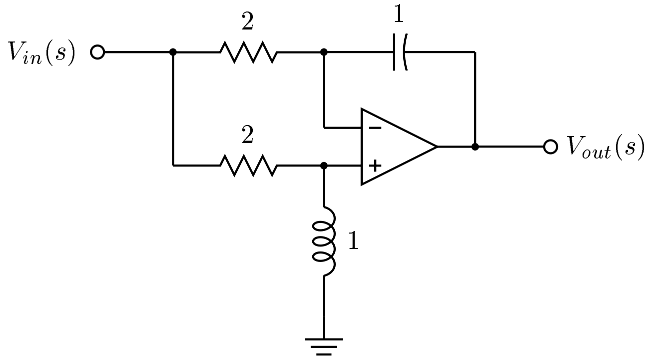
Therefore,

$$V_{out}(s) = \frac{s+\frac{3}{2}}{s(s+2)} = \frac{\frac{3}{4}}{s} + \frac{\frac{1}{4}}{s+2}.$$

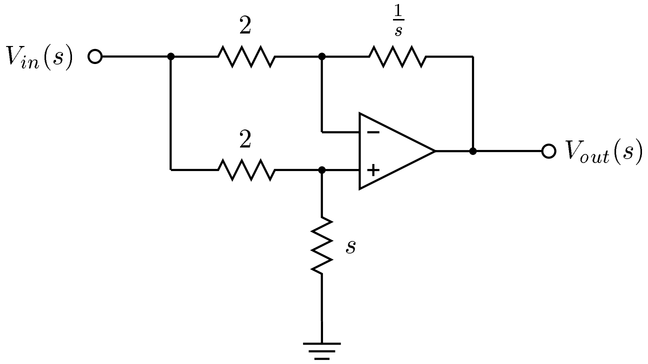
$$v_{out}(t) = \left[\frac{3}{4} + \frac{1}{4}e^{-2t}\right] u(t)$$

Notice that the output is zero before $t = 0$.

Problem 10a.4: Find the system function that relates $V_{out}(s)$ to $v_{in}(s)$.



Solution: The first step is to redraw the circuit in the Laplace domain.



Let the node potential at each of the inputs of the opamp (they must be the same) be $E(s)$. Writing a KCL equation at each of these input nodes gives

$$s[E(s) - V_{out}(s)] + \frac{1}{2}[E(s) - V_{in}(s)] = 0$$

$$\frac{1}{s}E(s) + \frac{1}{2}[E(s) - V_{in}(s)] = 0$$

From the second of these equations we get

$$E(s) = V_{in}(s)\left[\frac{s}{s+2}\right].$$

Substituting this result into the first equation gives (after algebraic simplification)

$$V_{out}(s) = V_{in}(s)\left[\frac{s}{s+2} + \frac{\frac{1}{2}}{s+2} - \frac{1}{2s}\right],$$

from which

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{s^2-1}{s(s+2)}.$$