GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

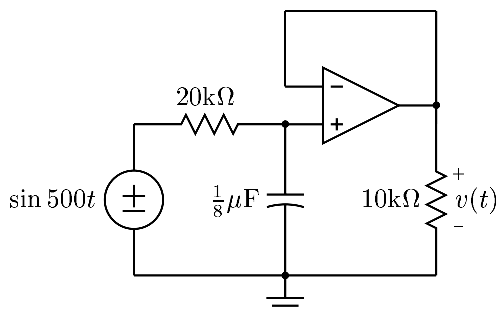
Course ECE 2040

Circuit Analysis

April 13, 2001

Problem Set #11–Solutions

Problem 11.1: For the circuit below find $v(t)$.



Solution: The opamp acts like a voltage follower, i.e., $v(t)$ is equal to the voltage across the capacitor. Furthermore, we know that $v(t)$ is of the form

$$v(t) = \Im\mathrm{m}\left[Ve^{j500t}\right].$$

To determine V, replace the source by a complex exponential time function, the elements by their equivalent impedances, and use the voltage divider.

$$V = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}.$$

Letting $R = 20 \times 10^3$; $C = 0.125 \times 10^{-6}$, $\omega = 500$, we get

$$\begin{aligned} V &= \frac{1}{1 + j(20 \times 10^3)(0.125 \times 10^{-6})(500)} = \frac{1}{1 + j(1.25)} \\ &= 0.625e^{-j\tan^{-1} 1.25} \\ v(t) &= 0.625 \sin(500t - \tan^{-1} 1.25). \end{aligned}$$

Problem 11.2: For the circuit in Figure 1 find $v(t)$ when $v_s(t) = \cos(\omega t)$.

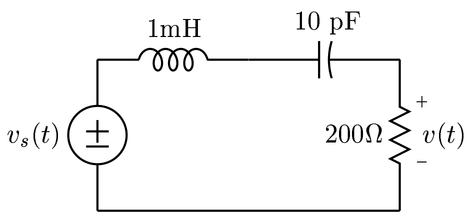


Figure 1: Circuit for Problem 11.2.

Solution: If the input were a complex exponential time function with a complex amplitude V_s, then by using the voltage divider we would have

$$\begin{aligned} V &= V_s \frac{200}{200 + \frac{1}{j10^{-8}\omega} + j10^{-3}\omega} \\ &= V_s \frac{j(2 \times 10^{10})\omega}{(1 - 10^5\omega^2) + j(2 \times 10^{10})\omega} \end{aligned}$$

Since $v_s(t) = \cos(\omega t)$, we recognize that $V_s = 1$ and $v(t)$ is

$$\begin{aligned} v(t) &= \Re\mathrm{e}\left\{ \frac{j(2 \times 10^{10})\omega}{1 - 10^5\omega^2 + j(2 \times 10^10)} e^{j\omega t} \right\} \\ &= \frac{2 \times 10^{10}\omega}{\sqrt{(1 - 10^5\omega^2)^2 + (2 \times 10^{10}\omega)^2}} \cos(\omega t + \frac{\pi}{2} - \left(\tan^{-1} \frac{2 \times 10^{10}\omega}{1 - 10^5\omega^2}\right)) \\ &= \frac{2 \times 10^{10}\omega}{\sqrt{(1 - 10^5\omega^2)^2 + (2 \times 10^{10}\omega)^2}} \sin(\omega t - \left(\tan^{-1} \frac{2 \times 10^{10}\omega}{1 - 10^5\omega^2}\right)) \end{aligned}$$

Problem 11.3: For the circuit in Figure 2 what is the average power supplied by the current source?

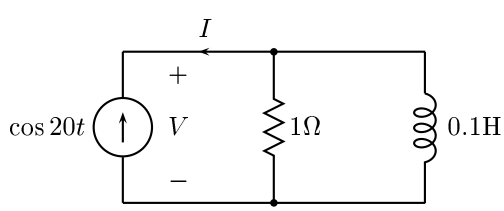


Figure 2: Circuit for Problem 11.3.

Solution: Let I and V be the complex amplitudes shown on the figure.

$$P_{ave} = \frac{1}{2}\Re\mathrm{e}\{VI^*\},$$

but $I^* = -1$. To calculate V, we have

$$V = -I \cdot \left\{ \frac{0.1s}{0.1s+1} \right\} = -I \left\{ \frac{j2}{j2+1} \right\}$$

$$P_{ave} = \frac{1}{2} \Re \left\{ -\frac{j2}{j2+1} \right\} = -\Re \left\{ \frac{2+j}{5} \right\} - -\frac{2}{5}.$$

The power supplied by the current source is $\frac{2}{5}$ Watt.

Problem 11.4: Determine the values of R and L for the circuit in Figure 3 that cause the maximum amount of power to be delivered to the load.

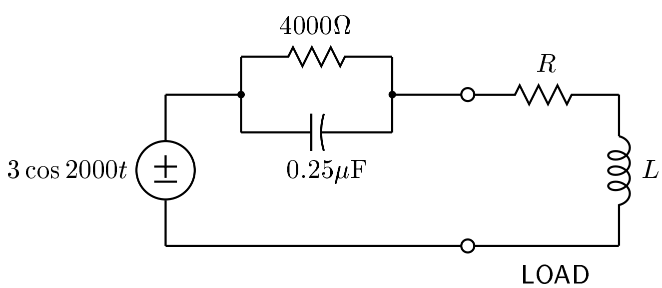


Figure 3: Circuit for Problem 11.4.

Solution: We know for maximum power delivery we should have

$$Z_L = R + j2000L = Z_T^*.$$

Computing the Thévenin equivalent source impedance

$$\begin{aligned} Z_T &= \frac{\frac{4000}{j(.25\times10^{-6}(2000)}}{4000 + \frac{1}{j(.25\times10^{-6})(2000)}} \\ &= \frac{4000}{j(8\times10^6)(.25\times10^{-6})+1} \\ &= \frac{4000}{1+j2} = 800 - j1600. \end{aligned}$$

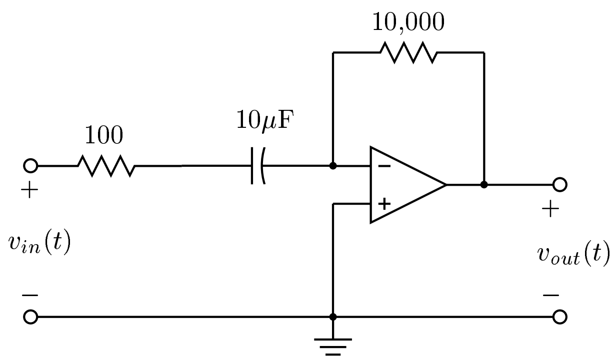
Therefore, we want

$$R + j2000L = 800 + j1600,$$

so

$$R = 800\Omega \quad L = 0.8H.$$

Problem 11.5:



(a) Calculate the frequency response of the above circuit.

(b) Plot the magnitude response.

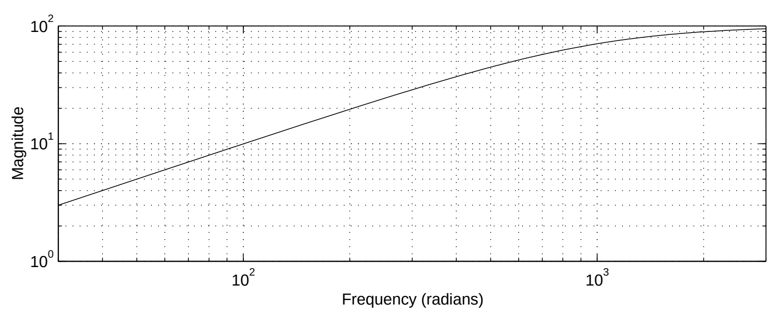
(c) Plot the phase response.

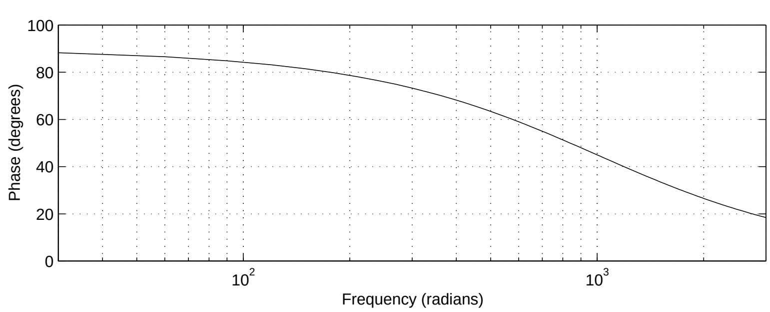
Solution:

(a)

$$H(j\omega) = -\frac{Z_f(j\omega)}{Z_i(j\omega)} = -\frac{10^4}{100 + \frac{1}{j10^{-5}\omega}}$$

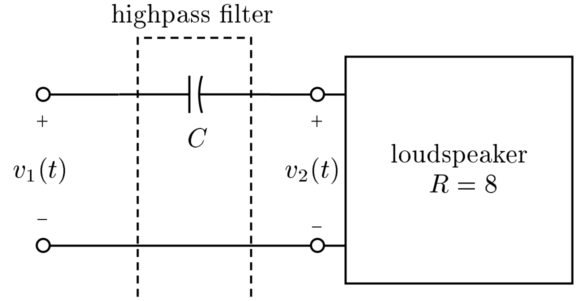
$$= -\frac{j(0.1)\omega}{j0.001\omega + 1}$$





(b,c)

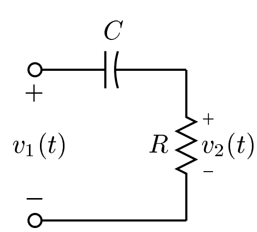
Problem 11.6: Many loudspeaker systems consist of two loudspeakers: the woofer, which reproduces the low frequency part of the signal, and the tweeter, which reproduces the high frequency part of the signal. A crossover network is used to select the high frequency part of the signal and feed it into the tweeter. Such a network functions as a highpass filter. The entire audio signal is applied at the terminals $a - a'$.



(a) Assuming that the equivalent circuit for the tweeter consists of just a resistor with a resistance of R, plot the pole-zero pattern of the system function that relates $v_2(t)$ to $v_1(t)$ and sketch the frequency response curves (magnitude and angle).

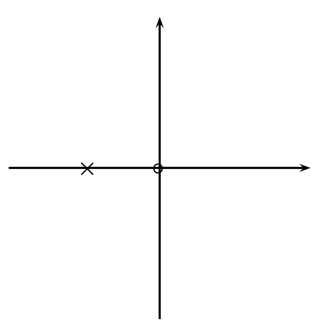
(b) If $R = 8\Omega$, find the value of the capacitance C so that the half-power frequency of the highpass filter is 5 kHz ($= 2\pi(5000)$ rad/s).

Solution:

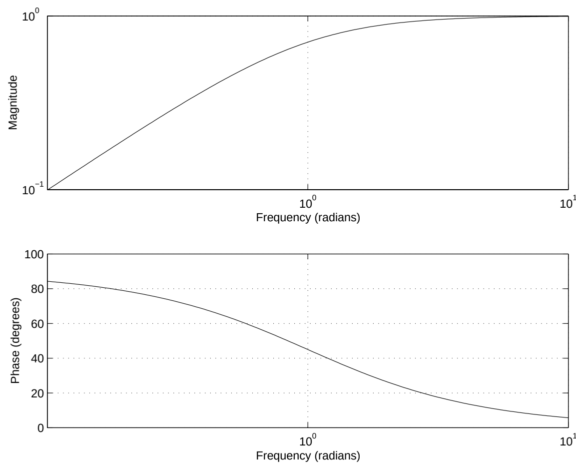


$$H(s) = \frac{R}{R + \frac{1}{Cs}} = \frac{s}{s + \frac{1}{RC}}$$

(a)



Using the MATLAB function **freqs** again gives the following plot (for positive frequencies) if $RC = 1$.



(b)

$$\frac{1}{RC} = 2\pi(5000)$$

Thus,

$$C = \frac{1}{80,000\pi} \approx 4\mu F$$
