

GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

January 19, 2001

Problem Set #1—Solutions

Problem 1.1: (a) The voltage drop across a 1F capacitor is

$$v(t) = \begin{cases} \frac{1}{2}(1 - \cos pt), & 0 < t < 2 \\ 0, & \text{otherwise.} \end{cases}$$

This waveform is shown in Figure 1.

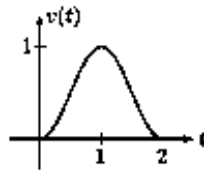


Figure 1: Waveform for Problem 1.6.

- (i) Sketch the current flowing through the device, $i(t)$.
 - (ii) Sketch the energy stored in the device as a function of t .
 - (iii) For what values of t is the device supplying power?
 - (iv) For what values of t is the device dissipating power?
- (b) Repeat the questions asked in part (a) if $v(t)$ is the voltage drop across a 1H inductor.
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Solution:

(a) (i)

$$i_c(t) = C \frac{dv_c(t)}{dt} = \begin{cases} \frac{p}{2} \sin pt, & 0 < t < 2 \\ 0, & \text{otherwise.} \end{cases}$$

This waveform is shown on the graph in Figure 2.

(ii) The energy stored in a capacitor is given by

$$E_c(t) = \frac{1}{2} C v_c^2(t) = \begin{cases} \frac{1}{8} (1 - \cos pt)^2, & 0 < t < 2 \\ 0, & \text{otherwise.} \end{cases}$$

This is shown in Figure 3.

(iii) It supplies power when the slope of the energy curve is negative, which occurs for $1 < t < 2$.

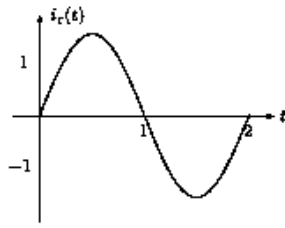


Figure 2: Capacitor current as a function of time.

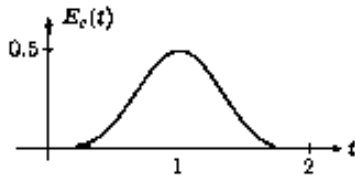


Figure 3: Energy stored in the capacitor as a function of time.

- (iv) It dissipates power when the slope of the energy curve is positive, which occurs for $0 < t < 1$.
- (b) (i) For values of t in the range $0 < t < 2$ we can write

$$\begin{aligned}
 i_c(t) &= \int_0^t v_c(\tau) d\tau + i_c(0) \\
 &= \int_0^t \frac{1}{2}(1 - \cos \pi\tau) d\tau \\
 &= \frac{1}{2}t - \frac{1}{2\pi} \sin \pi t.
 \end{aligned}$$

For $t > 2$, the current remains constant at 1A and for $t < 0$ it is constant at 0A. Therefore,

$$i_c(t) = \begin{cases} 0, & t < 0 \\ \frac{1}{2}t + \frac{1}{2\pi} \sin \pi t, & 0 < t < 2 \\ 1, & 2 < t. \end{cases}$$

This is shown in Figure 4.

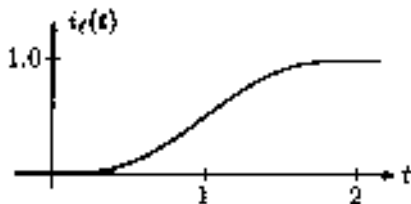


Figure 4: Inductor current as a function of time.

- (ii) The energy stored in the inductor is $\frac{1}{2} Li_l^2(t) = \frac{1}{2} i_l^2(t)$, which is shown in Figure 5.
- (iii) The inductor dissipates power (absorbs energy) for $0 < t < \infty$.
- (iv) There are no values of t for which the inductor is supplying power.

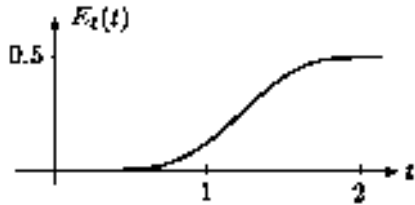


Figure 5: Energy stored in the inductor as a function of time.

Problem 1.2: For the circuit in Figure 6 write a KCL equation at every node in the circuit in terms of the current variables whose reference directions are given.

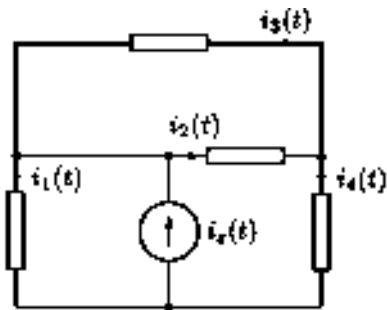
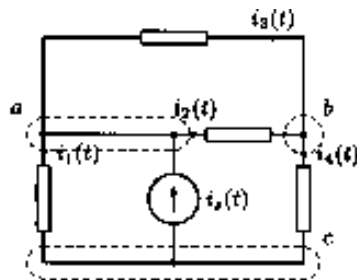


Figure 6: Circuit for Problems 1.7 and 1.13.

Solution: There are three nodes in this circuit as shown below.



If we sum the currents *entering* the node, then the three KCL equations are:

$$\text{node } a: \quad -i_1(t) - i_2(t) + i_3(t) + i_8(t) = 0$$

$$\text{node } b: \quad i_2(t) - i_3(t) - i_4(t) = 0$$

$$\text{node } c: \quad i_1(t) + i_4(t) - i_8(t) = 0.$$

Problem 1.3: For the circuit in Figure 7 write a sufficient set of KCL equations in terms of the voltage variables that are labeled. You should incorporate Ohm's Law for the resistors. Exploit any obvious series and parallel connections to reduce the number of equations and variables.

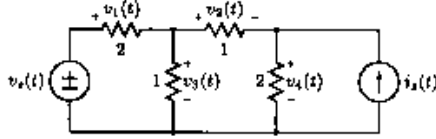
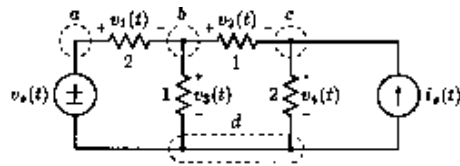


Figure 7: Circuit for Problems 1.9 and 1.21.

Solution: We begin by redrawing the circuit to identify the four nodes.



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Since the circuit contains four nodes, it will be sufficient to write only three KCL equations. The equation at node a , will tell us only that the current flowing upwards through the voltage source is the same as $i_1(t)$, since these two “elements” are connected in series; we can ignore this one also. That leaves only nodes b and c .

$$\text{node } b: \quad \frac{v_1(t)}{2} - v_2(t) - v_3(t) = 0$$

$$\text{node } c: \quad v_2(t) - \frac{v_4(t)}{2} = -i_8(t)$$

Problem 1.4: For the circuit in Figure 8 write a sufficient set of KVL equations to constrain all of the element variables in terms of the current variables that are indicated. You should incorporate Ohm's Law for the resistors. Use any obvious series and parallel connections to reduce the number of equations and variables.

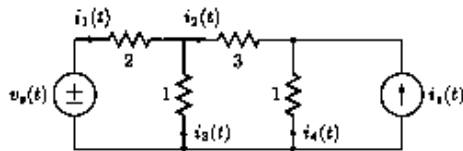


Figure 8: Circuit for Problems 1.14 and 1.22.

Solution: The right mesh KVL equation tells us only that the voltage drop across the current source is the same as the voltage drop across the resistor connected in parallel to it. The remaining two KVL equations are:

$$\text{left mesh: } 2i_1(t) - i_3(t) = v_8(t)$$

$$\text{center mesh: } i_3(t) + i_2(t) - i_4(t) = 0$$

Problem 1.5: (a) If a $1\text{k}\Omega$ resistor and a $2\text{k}\Omega$ resistor are connected in series, which will dissipate more power?

(b) If a $1\text{k}\Omega$ resistor and a $2\text{k}\Omega$ resistor are connected in parallel, which will dissipate more power?

Solution:

(a) If two resistors are connected in series, the same current passes through them. Let that current be $i(t)$. Then the power dissipated in the $1\text{k}\Omega$ resistor is

$$P_1(t) = 1000i^2(t)$$

and the power dissipated in the $2\text{k}\Omega$ resistor is

$$P_2(t) = 2000i^2(t),$$

which is twice as great. Therefore, more power is dissipated in the $2\text{k}\Omega$ resistor.

(b) If two resistors are connected in parallel, the same voltage drop appears across both. Let that voltage drop be $v(t)$. Then the power dissipated in the $1\text{k}\Omega$ resistor is

$$P_1(t) = \frac{1}{1000} v^2(t)$$

and the power dissipated in the $2\text{k}\Omega$ resistor is

$$P_2(t) = \frac{1}{2000} v^2(t),$$

which is only half as much. Thus in this case more power is dissipated in the $1\text{k}\Omega$ resistor, which is just the opposite of what we saw in (a).

Problem 1.6: Two resistors connected in series act like a single resistor. Similarly, two resistors connected in parallel behave like a single resistor. In this problem, we derive these basic results.

- (a) Consider two resistors connected in series and connected across a voltage source, as in Figure 9a. Show that the current flowing through them is proportional to the source voltage.

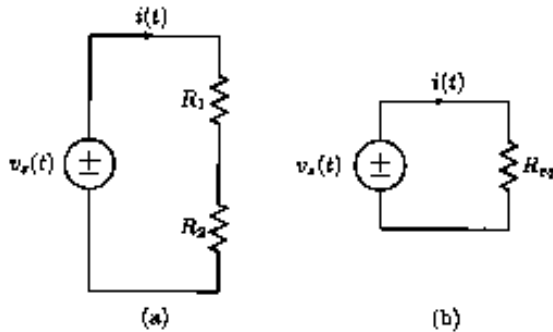


Figure 9: Two resistors connected in series and their equivalent resistance.

- (b) This implies that from the point-of-view of the voltage source, the series connection of resistors is equivalent to a single resistor as shown in Figure 9b. Express R_{eq} in terms of R_1 and R_2 .
- (c) We can similarly consider two resistors connected in parallel across a voltage source, as in Figure 10a. Show again that the current flowing through them is proportional to the source voltage.

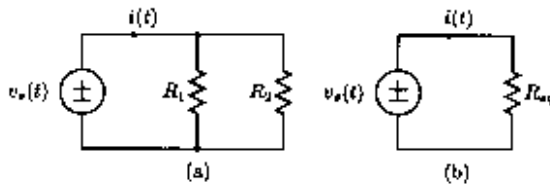


Figure 10: Two resistors connected in parallel and their equivalent resistance.

- (d) Find the equivalent resistance of the parallel combination as you did in part (b).

Solution:

- (a) From KVL

$$\begin{aligned} v_8(t) &= v_1(t) + v_2(t) \\ &= R_1 i(t) + R_2 i(t) \\ &= (R_1 + R_2) i(t). \end{aligned}$$

or

$$i(t) = \frac{1}{R_1 + R_2} v_8(t).$$

- (b) From the equivalent circuit

$$i(t) = \frac{1}{R_{eq}} v_8(t).$$

Therefore,

$$R_{eq} = R_1 + R_2$$

(c) From KCL

$$\begin{aligned}i(t) &= i_1(t) + i_2(t) \\ &= \frac{v_s(t)}{R_1} + \frac{v_s(t)}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right) v_s(t).\end{aligned}$$

(d)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$