GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

January 26, 2001

Problem Set #2–Solutions

Problem 2.1: Find the current $i_1(t)$ for the circuit shown in Figure 1.

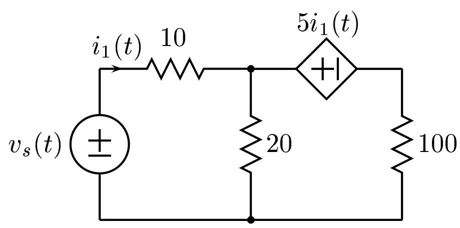


Figure 1: Circuit for Problem 2.1.

Solution: Define the auxiliary currents shown in Figure 2. From KCL

$$i_1(t) - i_2(t) - i_3(t) = 0 \tag{1}$$

From KVL applied to the left mesh

$$-v_s(t) + 10i_1(t) + 20i_2(t) = 0 \tag{2}$$

and from KVL applied to the right mesh

$$-20i_2(t) + 5i_1(t) + 100i_3(t) = 0. \tag{3}$$

Remember the dependent source is a *voltage* source with a voltage drop that is equal to $5i_1(t)$. From the first equation $i_3(t) = i_1(t) - i_2(t)$. Substituting into the second

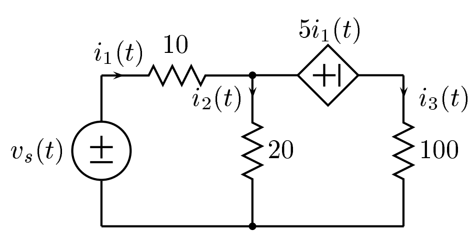


Figure 2: Circuit for Problem 2.1 with additional currents labeled.

and third equations, this gives

$$10i_1(t) + 20i_2(t) = v_s(t)$$
$$105i_1(t) - 120i_2(t) = 0.$$

We can now multiply the first equation by 6 and add the two equations together.

$$165i_1(t) = 6v_s(t)$$

or

$$i_1(t) = \frac{6}{165}v_s(t) = \frac{1}{55}v_s(t).$$

Problem 2.2: Determine the voltage $v(t)$ in the circuit in Figure 3.

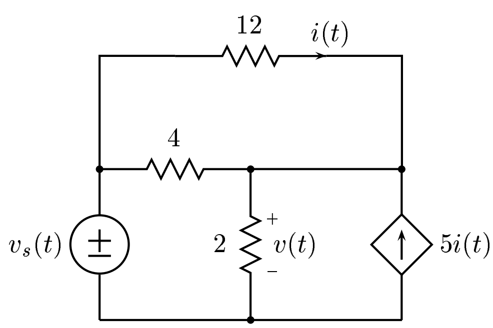
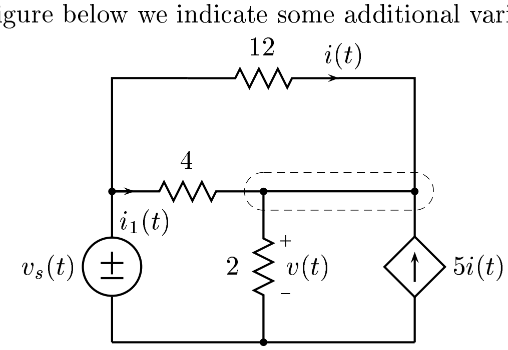


Figure 3: Figure for Problem 2.2.

Solution: In the figure below we indicate some additional variables.



Notice that we have let the current flowing through the 4Ω resistor be denoted by $i_1(t)$. The current flowing downward through the 2Ω resistor is $v(t)/2$. We can set up 1 KCL equation (at the circled node), and two KVL equations to solve for the three variables $i(t)$, $v(t)$, and $i_1(t)$.

$$\text{KCL:} \quad i_1(t) + 6i(t) - \frac{1}{2}v(t) = 0$$
$$\text{KVL } \alpha\text{:} \quad 4i_1(t) + v(t) = v_s(t)$$
$$\text{KVL } \beta\text{:} \quad 12i(t) - 4i_1(t) = 0.$$

From the third equation

$$i_1(t) = 3i(t).$$

Substituting this fact into the first equation gives

$$9i(t) - \frac{1}{2}v(t) = 0 \Longrightarrow i(t) = \frac{1}{18}v(t).$$

Finally, substituting this result into the second equation gives

$$\frac{12}{18}v(t) + v(t) = v_s(t)$$

or

$$v(t) = \frac{3}{5}v_s(t).$$

Problem 2.3: This problem is concerned with the three networks shown in Figure 4.

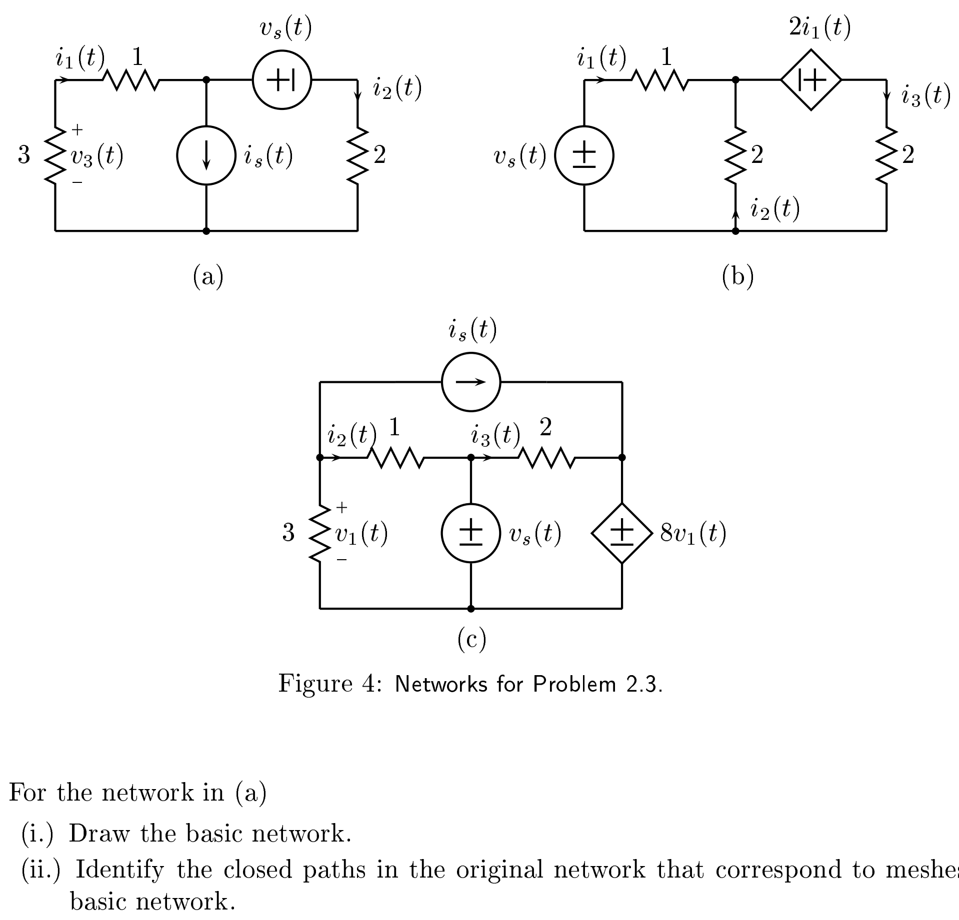


Figure 4: Networks for Problem 2.3.

(a) For the network in (a)

   (i.) Draw the basic network.

   (ii.) Identify the closed paths in the original network that correspond to meshes in the basic network.

   (iii.) Identify the closed surfaces in the original network that correspond to nodes in the basic network.

(b) Repeat for the network in (b).

(c) Repeat for the network in (c).

Solution:

(a) (i) The basic network is shown in Figure 5.

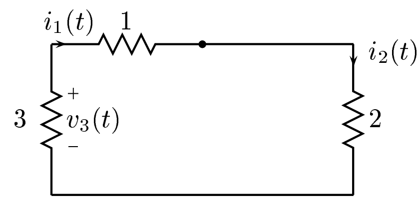


Figure 5: The basic network for the circuit in Figure 4(a).

(ii) There is only one mesh in the basic network and it corresponds to the path around the outside of the original network, i.e. it is the path that contains both the 3Ω and the 2Ω resistors.

(iii) The surfaces in the complete network that correspond to nodes in the basic network are indicated in Figure 6 as dashed lines.

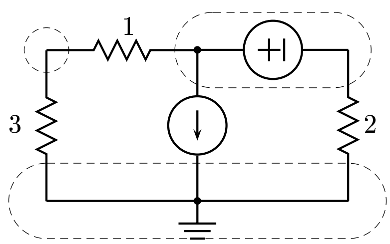


Figure 6: The surfaces in the complete network in Figure 4(a) corresponding to nodes in the basic network.

(b) (i) The basic network is shown in Figure 7.

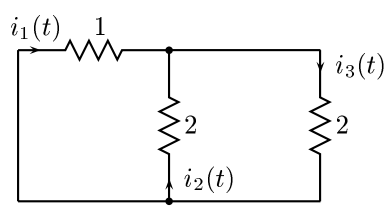


Figure 7: The basic network for the circuit in Figure 4(b).

(ii) The basic network contains two meshes. The paths corresponding to those meshes in the complete network are the two meshes in the complete network.

(iii) The basic network contains two nodes. In the complete network, those correspond to the two closed surfaces illustrated in Figure 8.

(c) (i) The basic network is shown in Figure 9.

(ii) The meshes in the basic network correspond to the two lower meshes in the complete network.

(iii) The basic network contains only two nodes. In the complete network these correspond to the dashed surfaces shown in Figure 10. Since both terminals of the 2Ω resistor are included in that node, the resistor itself can be incorporated in it as well.

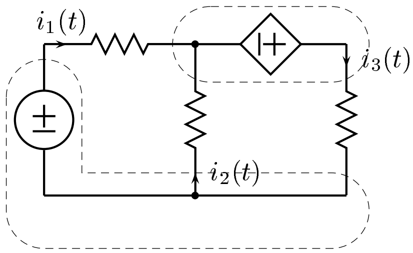


Figure 8: The surfaces in the complete network of Figure 4(b) corresponding to nodes in the basic network of Figure 7.

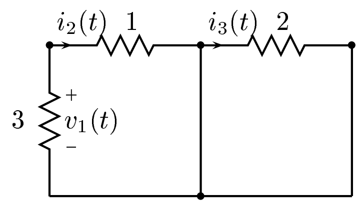


Figure 9: The basic network for the circuit in Figure 4(c).

Problem 2.4: For the circuit in Figure 11

(a) Write a minimal set of KCL equations to specify the equilibrium solution, based on the basic network associated with the circuit. These should be expressed in terms of the variables indicated on the figure.

(b) Write a minimal set of KVL equations to specify the equilibrium solution, based on the basic network. These should be expressed in terms of the indicated variables.

(c) Solve for $v_1(t)$ and $i_2(t)$.

Solution:

(a) The basic network contains two nodes. In the complete circuit one of these becomes a supernode surrounding the dependent voltage source.

$$\frac{1}{10,000}v_1(t) + i_2(t) = i_s(t).$$

(b) The basic network contains a single mesh—the right mesh of the complete circuit. Its KVL equation is

$$-v_1(t) + 2v_1(t) + 20,000i_2(t) = 0$$

or

$$v_1(t) + 20,000i_2(t) = 0.$$

(c) After multiplying both sides of the KCL equation by 10,000, these two equations can be written as

$$\begin{bmatrix} 1 & 10,000 \\ 1 & 20,000 \end{bmatrix} \begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix} = \begin{bmatrix} 10,000 \\ 0 \end{bmatrix} i_s(t).$$

From these

$$v_1(t) = 20,000i_s(t)$$
$$i_2(t) = -i_s(t).$$

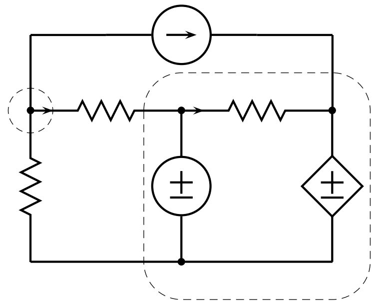


Figure 10: The surfaces for the network in Figure 4(c) corresponding to nodes in the basic network of Figure 7.

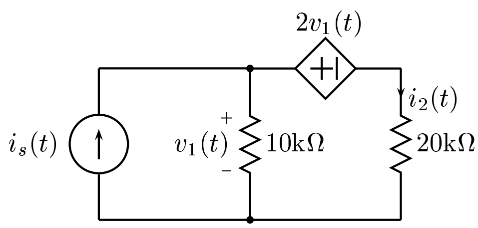


Figure 11: Circuit for Problem 2.4.

Problem 2.5: Set up a set of minimal KCL and KVL equations to find the element voltages in the circuit of Figure 12 using the Simplified Exhaustive Method.

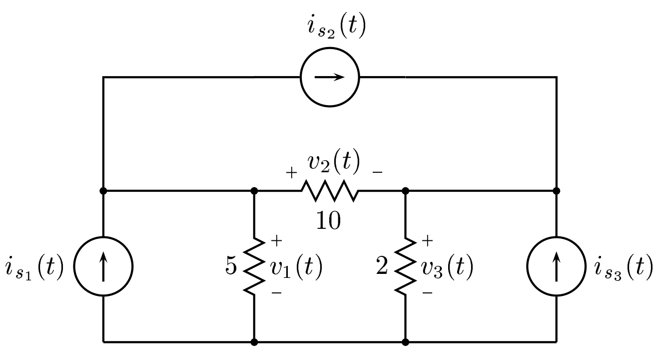


Figure 12: Circuit for Problems 2.5.

Solution: There is one mesh in the basic network. Therefore, we need only a single KVL equation.

$$-v_1(t) + v_2(t) + v_3(t) = 0$$

The basic network has three nodes, so we need to write two KCL equations. Any two of the following three (or their negatives) is satisfactory.

$$\frac{1}{5}v_1(t) + \frac{1}{10}v_2(t) = i_{s_1}(t) - i_{s_2}(t)$$

$$-\frac{1}{10}v_2(t) + \frac{1}{2}v_3(t) = i_{s_2}(t) + i_{s_3}(t)$$
$$\frac{1}{5}v_1(t) + \frac{1}{2}v_3(t) = i_{s_1}(t) + i_{s_3}(t).$$

Problem 2.6: Set up a set of minimal KCL and KVL equations to find the element currents in the circuit of Figure 13 using the Simplified Exhaustive Method.

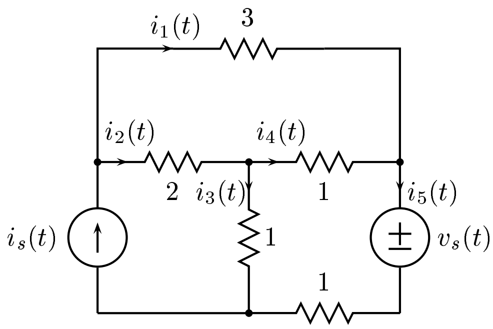


Figure 13: Circuit for Problems 2.6.

Solution: The basic network contains four nodes and two meshes. We, therefore, need three KCL equations and two KVL equations. Any three of the following four KCL equations along with the two KVL equations are sufficient.

$$\begin{aligned}
\text{KCL1:}\quad & i_1(t) + i_2(t) = i_s(t) \\
\text{KCL2:}\quad & i_2(t) - i_3(t) - i_4(t) = 0 \\
\text{KCL3:}\quad & i_1(t) + i_4(t) - i_5(t) = 0 \\
\text{KCL4:}\quad & i_3(t) + i_5(t) = i_s(t) \\
\text{KVL1:}\quad & 3i_1(t) - 2i_2(t) - i_4(t) = 0 \\
\text{KVL2:}\quad & -i_3(t) + i_4(t) + i_5(t) = v_s(t)
\end{aligned}$$
