GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

February 9, 2001

Problem Set #4–Solutions

Problem 4.1:

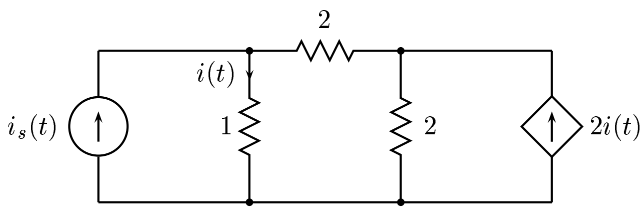

Figure 1: Circuit for Problem 4.1.

(a) For the circuit in Figure 1, what is the minimum number of KVL equations that need to be written to specify the equilibrium solution?

(b) Define a mesh current for each mesh of the basic network. Write a sufficient set of KVL equations in terms of the mesh currents and the source waveforms.

(c) Solve your equations and use the results to determine $i(t)$.

Solution:

(a) There is one mesh in the basic network. Therefore, we need only a single KVL equation.

(b) Let $i_\alpha(t)$ be the clockwise mesh current in the center mesh as shown in Figure 2. Then

$$[i_\alpha(t) - i_s(t)] + 2i_\alpha(t) + 3[i_\alpha(t) + 2(i_s(t) - i_\alpha(t))] = 0$$

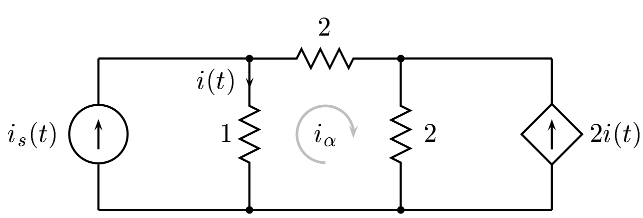

Figure 2: Circuit for Problem 4.1 with the mesh current $i_\alpha(t)$ inserted.

(c) Multiplying out the terms gives

$$i_\alpha(t) - i_s(t) + 2i_\alpha(t) + 2i_\alpha(t) + 4i_s(t) - 4i_\alpha(t) = 0,$$

which implies

$$i_\alpha(t) = -3i_s(t).$$

The variable of interest, however, is $i(t)$. Since

$$i(t) = i_s(t) - i_\alpha(t),$$

we can substitute to get

$$i(t) = 4i_s(t).$$

Problem 4.2: Find all of the element voltages and currents in the circuit of Figure 3 using the mesh method. Be sure to identify the variables clearly.

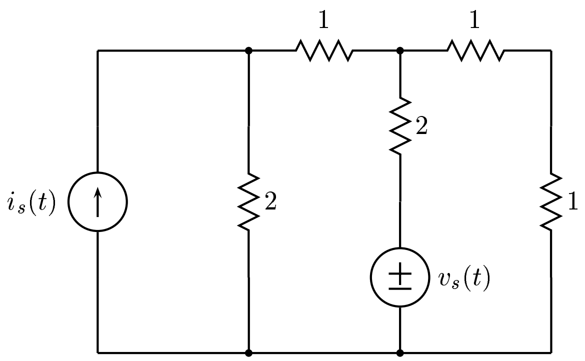

Figure 3: Circuit for Problem 4.2.

Solution: The first step is to identify the meshes in the basic network and to identify

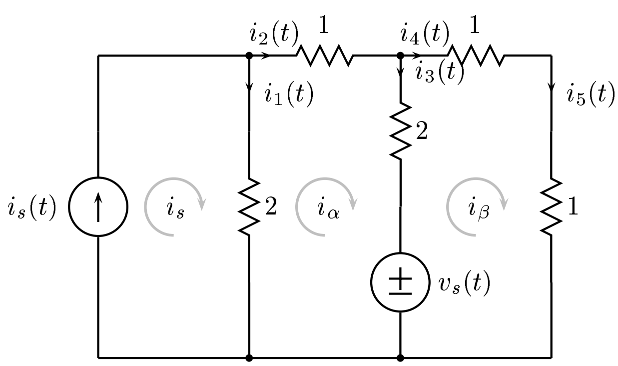

Figure 4: Circuit for Problem 4.2 with the meshes indicated and the currents defined.

the variables in the circuit. This is done in Figure 4. The voltages are implied by the currents using the default sign convention. The two mesh equations are

$$\text{mesh } \alpha: \quad 2[i_\alpha(t) - i_s(t)] + i_\alpha(t) + 2[i_\alpha - i_\beta] + v_s(t) = 0$$
$$\text{mesh } \beta: \quad -v_s(t) + 2[i_\beta(t) - i_\alpha(t)] + i_\beta(t) + i_\beta(t) = 0.$$

These can be put into matrix-vector form as:

$$\begin{bmatrix} 5 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} i_\alpha(t) \\ i_\beta(t) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} i_s(t) + \begin{bmatrix} -1 \\ 1 \end{bmatrix} v_s(t)$$

The solution is

$$i_\alpha(t) = \frac{1}{2}i_s(t) - \frac{1}{8}v_s(t)$$
$$i_\beta(t) = \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t).$$

From these we can compute all of the element variables.

$$i_1(t) = \frac{1}{2}i_s(t) + \frac{1}{8}v_s(t)$$
$$i_2(t) = \frac{1}{2}i_s(t) - \frac{1}{8}v_s(t)$$
$$i_3(t) = \frac{1}{4}i_s(t) - \frac{5}{16}v_s(t)$$
$$i_4(t) = \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t)$$
$$i_5(t) = \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t)$$
$$v_1(t) = i_s(t) + \frac{1}{4}v_s(t)$$
$$v_2(t) = \frac{1}{2}i_s(t) - \frac{1}{8}v_s(t)$$
$$v_3(t) = \frac{1}{2}i_s(t) - \frac{5}{8}v_s(t)$$
$$v_4(t) = \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t)$$
$$v_5(t) = \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t)$$

Problem 4.3: (a) Which method, the mesh method or the node method, will result in fewer equations to solve in order to determine $v(t)$ in the circuit in Figure 5?

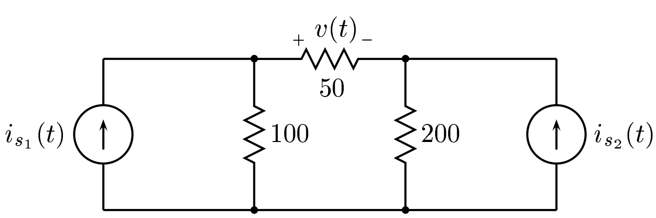

Figure 5: Circuit for Problem 4.3.

(b) Determine $v(t)$ using the method that you selected in (a).

Solution:

(a) The mesh method requires writing only one KVL equation, because the basic network contains only one mesh. The node method, on the other hand requires writing two KCL equations because the basic network contains three nodes. Therefore, the mesh method will result in fewer equations.

(b) Let $i_{s_1}(t)$ be a clockwise mesh current in the left mesh of the complete circuit, $i(t)$ be a clockwise mesh current in the center mesh (the only one around which we write a KVL equation), and $i_{s_2}(t)$ be the counterclockwise mesh current around the right mesh. Then the KVL equation is

$$100[i(t) - i_{s_1}(t)] + 50i(t) + 200[i(t) + i_{s_2}(t)] = 0.$$

Solving for $i(t)$ gives

$$350i(t) = 100i_{s_1} - 200i_{s_2}(t)$$
$$i(t) = \frac{2}{7}i_{s_1} - \frac{4}{7}i_{s_2}(t)$$

From this

$$v(t) = \frac{100}{7}i_{s_1}(t) - \frac{200}{7}i_{s_2}(t).$$

Problem 4.4: Find $v(t)$ in the circuit in Figure 6 by any method that you choose.

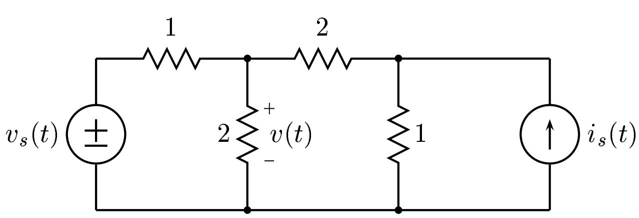

Figure 6: Circuit for Problem 4.4.

Solution: The node method and the mesh method represent an equal amount of work. Here we will work the problem using the node method because the variable that we are trying to find, $v(t)$, is a node potential if we let the bottom node be the ground. Let the remaining (upper right) node potential be $e(t)$. Let node a denote the upper left node and node b denote the upper right one. At node a:

$$[v(t) - v_s(t)] + \frac{1}{2}v(t) + \frac{1}{2}[v(t) - e(t)] = 0$$

or

$$2v(t) - \frac{1}{2}e(t) = v_s(t). \tag{1}$$

At node b:

$$\frac{1}{2}[e(t) - v(t)] + e(t) = i_s(t)$$

or

$$-\frac{1}{2}v(t) + \frac{3}{2}e(t) = i_s(t). \qquad (2)$$

Multiplying (1) by 3 and adding to (2) gives

$$\frac{11}{2}v(t) = 3v_s(t) + i_s(t)$$
$$v(t) = \frac{6}{11}v_s(t) + \frac{2}{11}i_s(t).$$

Problem 4.5: In our derivation of the mesh method, we stressed its duality with the node method, i.e., the similarity of the two methods if the roles of voltages and currents, and nodes and meshes are reversed. This problem lets you explore this issue further. Consider the network in Figure 7.

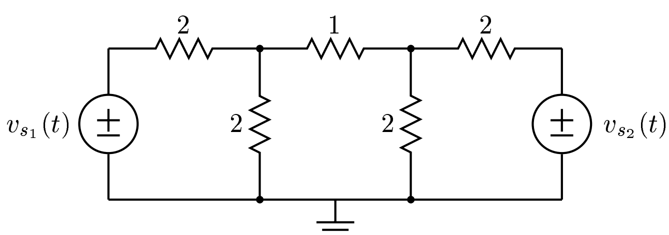

Figure 7: Circuit for Problem 4.5.

(a) Use the node method to determine the set of equations that must be solved to find the equilibrium solution. Omit the ground node when writing your equations. Express these equations in the form

$$\boldsymbol{C}\boldsymbol{v}(t) = \boldsymbol{s}_1 v_{s_1}(t) + \boldsymbol{s}_2 v_{s_2}(t).$$

Here $\boldsymbol{v}(t)$ is a vector of node potentials, $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ are column vectors of constants, and $\boldsymbol{C}$ is a constant matrix.

(b) Now design a *different* network containing two *current* sources with currents $i_{s_1}(t)$ and $i_{s_2}(t)$, such that the set of *mesh* equations that need to be solved to find the equilibrium solution is

$$\boldsymbol{C}\boldsymbol{i}(t) = \boldsymbol{s}_1 i_{s_1}(t) + \boldsymbol{s}_2 i_{s_2}(t).$$

and $\boldsymbol{i}(t)$ is the vector of mesh currents.

(c) Solve your equations from part (b).

Solution:

(a) There are two nodes, which are located at the terminals of the 1Ω resistor. These lead to the two equations

$$\text{node } a: \quad \frac{1}{2}[e_a(t) - v_{s_1}(t)] + \frac{1}{2}e_a(t) + [e_a(t) - e_b(t)] = 0$$
$$\text{node } b: \quad [e_b(t) - e_a(t)] + \frac{1}{2}e_b(t) + \frac{1}{2}[e_b(t) - v_{s_2}(t)] = 0,$$

which can be written in matrix-vector form as

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} e_a(t) \\ e_b(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} v_{s_1}(t) + \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix} v_{s_2}(t).$$

(b) The mesh equations for the desired circuit should look like the node equations for the previous one. Thus, they should have the form

$$\text{mesh } a\text{:} \quad \frac{1}{2}[i_a(t) - i_{s_1}(t)] + \frac{1}{2}i_a(t) + [i_a(t) - i_b(t)] = 0$$
$$\text{mesh } b\text{:} \quad [i_b(t) - i_a(t)] + \frac{1}{2}i_b(t) + \frac{1}{2}[i_b(t) - i_{s_2}(t)] = 0$$

One circuit that is consistent with these mesh equations is the one drawn in Figure 8.

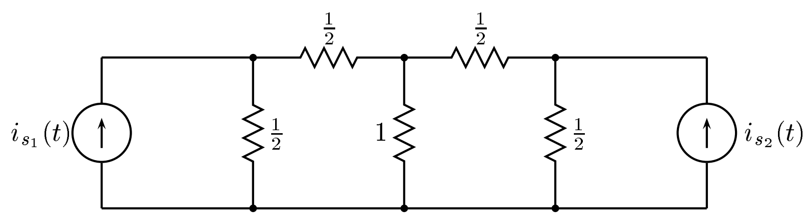

Figure 8: The dual to the circuit in Figure 7.

(c)

$$i_a(t) = \frac{1}{3}i_{s_1}(t) + \frac{1}{6}i_{s_2}(t)$$
$$i_b(t) = \frac{1}{6}i_{s_1}(t) + \frac{1}{3}i_{s_2}(t)$$

Problem 4.6: Find the $v - i$ relation of the two-terminal network shown in Figure 9.

Solution: For this circuit, we can find the $v - i$ relation by using the node method. Let the node connected to the $-$ terminal be the ground. Then the potential at the node connected to the $+$ terminal is $v(t)$. Writing a KCL at this node provides the relation

$$\frac{1}{2}[v(t) - v_s(t)] + \frac{1}{2}v(t) - i_s(t) - i(t) + [v(t) - v_s(t)] = 0.$$

This simplifies to

$$2v(t) = \frac{3}{2}v_s(t) + i_s(t) + i(t)$$

or

$$v(t) = \frac{1}{2}i(t) + \frac{3}{4}v_s(t) + \frac{1}{2}i_s(t)$$

which is the desired $v - i$ relation. This problem could also be solved by a number of other means including using source superposition.

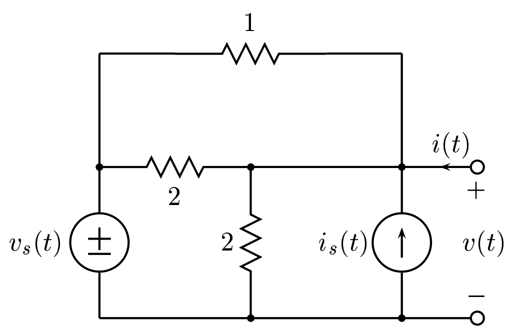

Figure 9: Two-port network for Problem 4.6.