GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

February 16, 2001

Problem Set #5–Solutions

Problem 5.1: (a) Determine the equivalent resistance of the two-terminal network in Figure 1a as seen at the terminals $a - a'$.

(b) Repeat for the network in Figure 1b at the terminals $b - b'$.

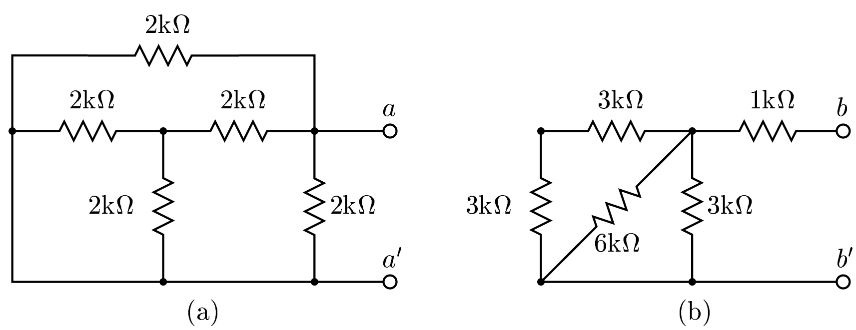


Figure 1: Networks for Problem 5.1.

Solution:

(a) The steps involved in simplifying this circuit are summarized in Figure 2.

(b) The solution to the part is performed graphically in Figure 3.

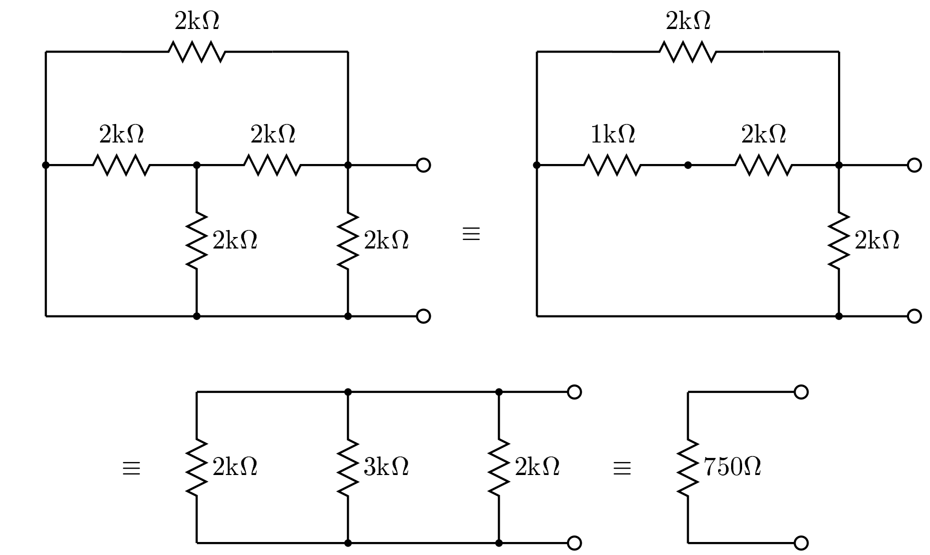


Figure 2: Solution of Problem 3.3a.

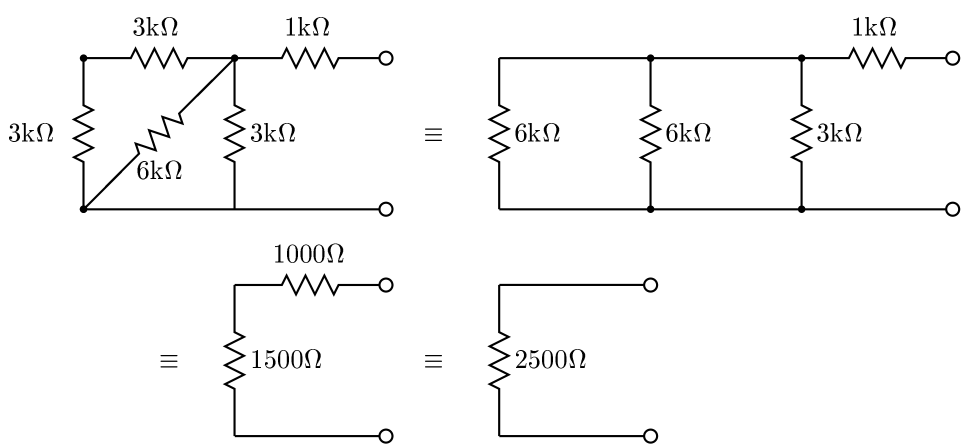


Figure 3: Graphical solution to Problem 3.3b.

Problem 5.2: You have a box containing an unlimited number of 10KΩ resistors. Show how to connect some of these together to construct equivalent resistances with the following values:

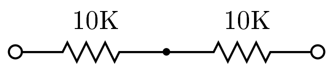
(a) 20KΩ

(b) 25KΩ

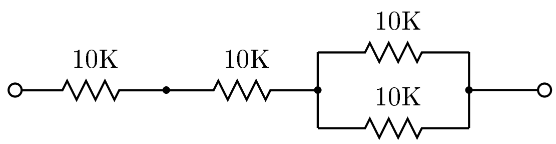
(c) 6667Ω

Solution:

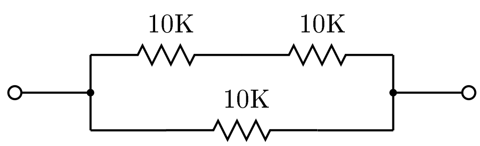
(a)



(b)



(c)



Problem 5.3: Consider a one-port network consisting of two capacitors with capacitances C_1 and C_2 connected in series, as shown in Figure 4.

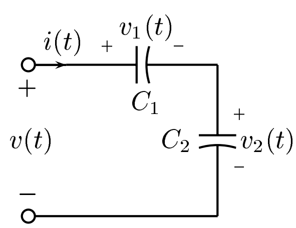


Figure 4: Two capacitors connected in series.

(a) Show that this network is equivalent to a single capacitor.

(b) Derive a formula for the equivalent capacitance C_{eq} in terms of C_1 and C_2.

(c) Derive expressions for the voltage $v_1(t)$ measured across capacitor C_1 and the voltage $v_2(t)$ measured across C_2 in terms of the voltage $v(t)$ appearing across the series connection.

Solution:

(a) We know that

$$i(t) = C_1 \frac{dv_1(t)}{dt} \Longrightarrow \frac{dv_1(t)}{dt} = \frac{i(t)}{C_1}$$
$$i(t) = C_2 \frac{dv_2(t)}{dt} \Longrightarrow \frac{dv_2(t)}{dt} = \frac{i(t)}{C_2}$$

and from KVL we know

$$v(t) = v_1(t) + v_2(t).$$

Taking a derivative of both sides gives

$$\frac{dv(t)}{dt} = \frac{dv_1(t)}{dt} + \frac{dv_2(t)}{dt} = \frac{i(t)}{C_1} + \frac{i(t)}{C_2}.$$

Therefore,

$$\frac{dv(t)}{dt} = \frac{i(t)}{C_1} + \frac{i(t)}{C_2}$$

or

$$i(t) = \left(\frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \right) \frac{dv(t)}{dt}$$

and the current is seen to be proportional to the first derivative of the voltage.

(b) From part (a) we see

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}.$$

(c) We know that

$$i(t) = \left(\frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \right) \frac{dv(t)}{dt} = C_1 \frac{dv_1(t)}{dt}.$$

Therefore,

$$\frac{dv_1(t)}{dt} = \frac{\frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2}} \frac{dv(t)}{dt}.$$

Integrating both sides gives the desired result:

$$v_1(t) = \frac{\frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2}} v(t).$$

Similarly

$$v_2(t) = \frac{\frac{1}{C_2}}{\frac{1}{C_1} + \frac{1}{C_2}} v(t).$$

Problem 5.4: A network has the $v - i$ characteristic shown graphically in Figure 5.

(a) Determine the Thévenin equivalent model corresponding to the network.

(b) Determine the Norton equivalent model corresponding to the network.

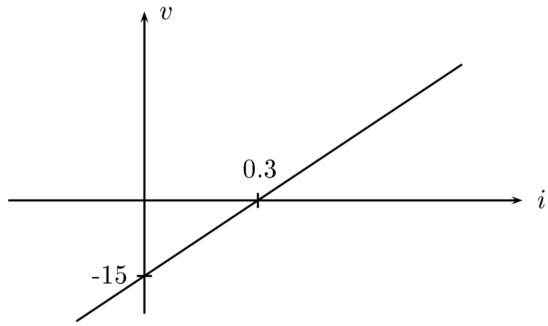


Figure 5: $v - i$ characteristic for the network in Problem 3.9.

Solution:

(a) The $v - i$ characteristic of a Thévenin equivalent network is of the form

$$v = R_T i + v_T.$$

Thus, R_T is the slope of the line and $v = v_T$ when $i = 0$. For this graph the slope is

$$R_T = \frac{15}{0.3} = 50$$

and

$$v_T = -15$$

Therefore, the Thévenin equivalent network is that shown in Figure 6a.

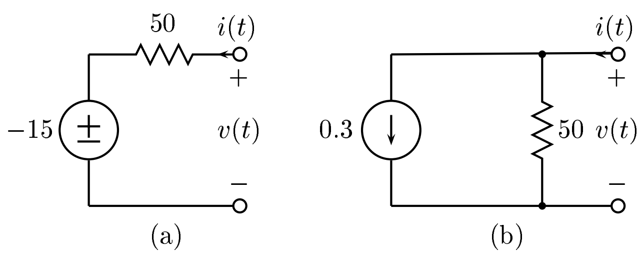


Figure 6: Equivalent circuits corresponding to the $v - i$ characteristic shown in Figure 5. (a) Thévenin equivalent circuit. (b) Norton equivalent circuit.

(b) $i_N = -i_{sc}$, where i_{sc} is the value of i when $v = 0$. Appealing to the graph, $i_{sc} = 0.3$ and therefore, $i_N = -0.3$. This gives the Norton equivalent circuit shown in Figure 6b, where we have accomodated the minus sign by reversing the normal direction of the current source.

Problem 5.5: Find the equivalent resistance, measured at the terminals, to the two-terminal network in Figure 7.

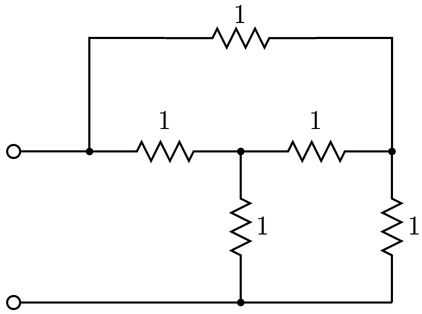
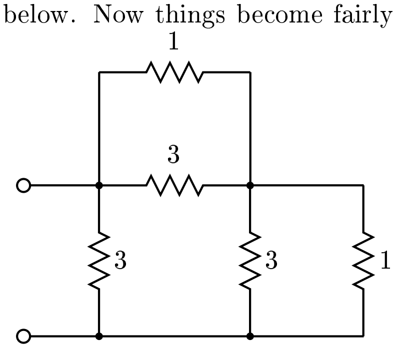
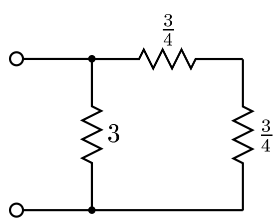


Figure 7: Two-terminal network for Problem 5.5.

Solution: There are several ways to proceed. One of the simplest is to replace the center wye connection by a delta connection. This changes the circuit to the equivalent one shown below. Now things become fairly straightforward. A 3Ω re-



sistor connected in parallel with a 1Ω resistor is equivalent to a $\frac{3}{4}\Omega$ resistor. This substitution can be made twice. This gives the figure shown below. It is seen to be equivalent to a single 1Ω resistor.



Problem 5.6: Consider the two two-terminal networks N_1 and N_2 shown in Figure 8.

The two networks have the $v - i$ relations

$$\begin{aligned} N_1: &\quad v_1(t) = 4i_1(t) - 8 \\ N_2: &\quad v_2(t) = 2i_2(t) + 3. \end{aligned}$$

Determine the equilibrium values of $v(t)$ and $i(t)$ if the two networks are connected as shown in Figure 9. *Suggested Approach:* Replace each network by its Thévenin equivalent network and then solve the resulting circuit.

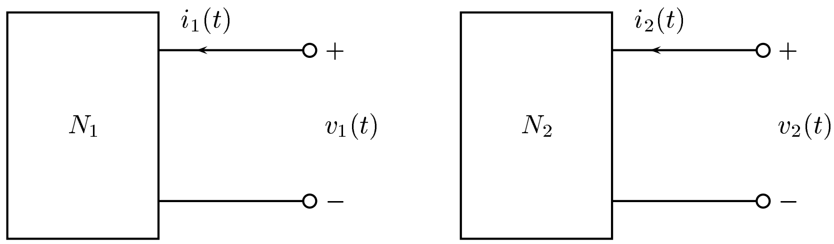


Figure 8: Networks for Problem 5.6.

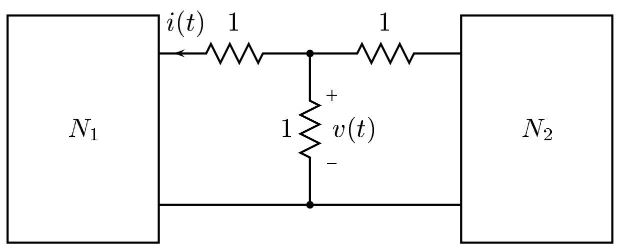


Figure 9: A circuit constructed from the two networks in Figure 8.

Solution: Following the suggestion, we replace each network by its Thévenin equivalent. The two Thévenin equivalent circuits are shown in Figure 10. Now if we

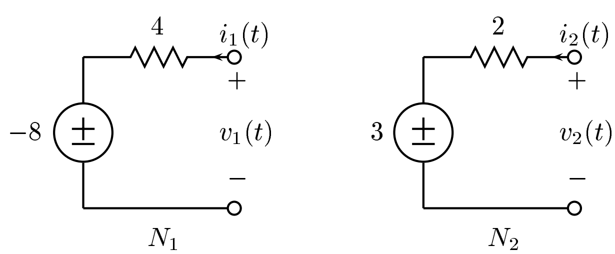


Figure 10: Thévenin equivalent networks for N_1 and N_2 in Problem 3.1.

insert the Thévenin equivalent networks into Figure 9, we get the circuit shown in Figure 11.

Let the potential at the upper node where the three resistors are joined be denoted by $v(t)$. (This effectively locates the ground at the lower node.) Then, if we write a KCL equation at that (upper) node, we get

$$\frac{1}{5}[v(t)+8]+\frac{1}{3}[v(t)-3]+v(t)=0$$

From this

$$\frac{23}{15}v(t)=-\frac{3}{5}$$

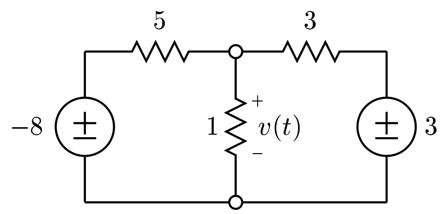


Figure 11: Circuit of Figure 9 with the Thévenin equivalent circuits included and the series resistors combined.

or

$$v(t) = -\frac{9}{23}\text{V (constant)}.$$
