GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

February 23, 2001

Problem Set #6–Solutions

Problem 6.1: Express $v_{out}(t)$ in terms of $v_{in}(t)$ for the circuit in Figure 1.

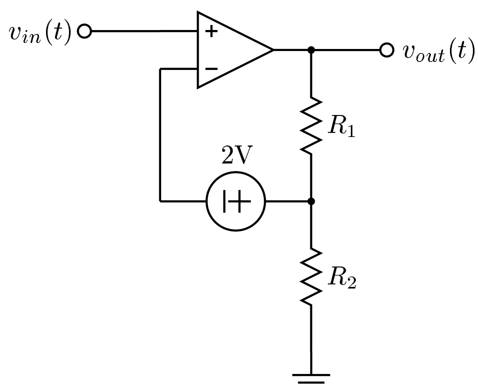


Figure 1: Circuit for Problem 6.1.

Solution: The node potential at the node where R_1, R_2, and the voltage source are connected is $v_{in}(t) + 2$. If we write a KCL equation at that node

$$\frac{v_{in}(t)+2}{R_2} + \frac{v_{in}(t)+2-v_{out}(t)}{R_1} = 0.$$

Solving for $v_{out}(t)$ gives

$$v_{out}(t) = \left(1 + \frac{R_1}{R_2}\right)(v_{in}(t)+2)$$

Problem 6.2: Determine the output voltage $v_{out}(t)$ in terms of the input voltage $v_{in}(t)$ for the circuit in Figure 2.

Solution: We can write KCL equations at the two nodes on the opposite ends of the rightmost horizontal resistor. Let the potential at its left node, where the four resistors are connected, be $e(t)$. The potential at the right node is zero, because of

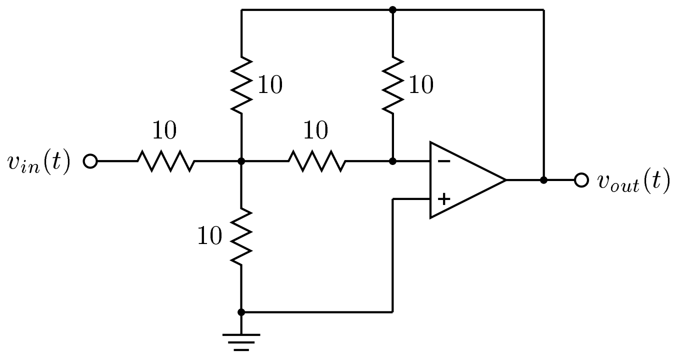


Figure 2: Circuit for Problem 6.2.

the virtual ground.

$$\text{left node:} \quad \frac{e(t) - v_{in}(t)}{10} + \frac{e(t) - v_{out}(t)}{10} + \frac{e(t)}{10} + \frac{e(t)}{10} = 0$$

$$\text{right node:} \quad -\frac{e(t)}{10} - \frac{v_{out}(t)}{10} = 0$$

The first of these equations reduces to

$$4e(t) = v_{in}(t) + v_{out}(t)$$

and the second reduces to

$$e(t) = -v_{out}(t).$$

We can substitute the latter expression into the earlier one and solve for $v_{out}(t)$ to get

$$v_{out}(t) = -5v_{in}(t).$$

Problem 6.3: The circuit in Figure 3 is a current amplifier. Compute the value of the current gain, G:

$$G = \frac{i_L(t)}{i_s(t)}.$$

Solution: The node potential at the inverting input of the operational amplifier is zero. Let the potential at the node connecting R_1, R_2, and R_L be $e(t)$. We write KCL equations at the inverting input (node a) and the node where the three resistors are connected together (node b).

$$\text{node } a: \quad -i_s(t) - \frac{1}{R_2}e(t) = 0$$

$$\text{node } b: \quad \frac{1}{R_1}e(t) - i_l(t) + \frac{1}{R_2}e(t) = 0.$$

From the first of these equations

$$e(t) = -R_2 i_s(t)$$

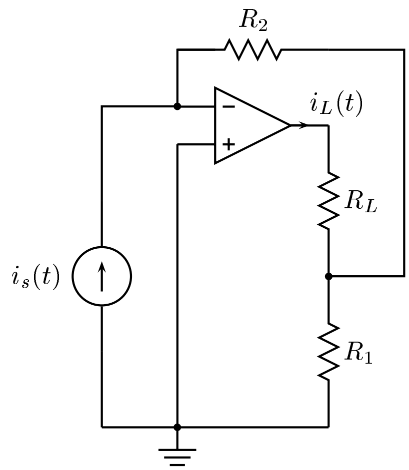


Figure 3: Circuit for Problem 6.3.

and from the second

$$e(t) = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} i_L(t).$$

If we equate these two expressions for $e(t)$, we get

$$-R_2 i_s(t) = \frac{R_1 R_2}{R_1 + R_2} i_L(t).$$

Therefore,

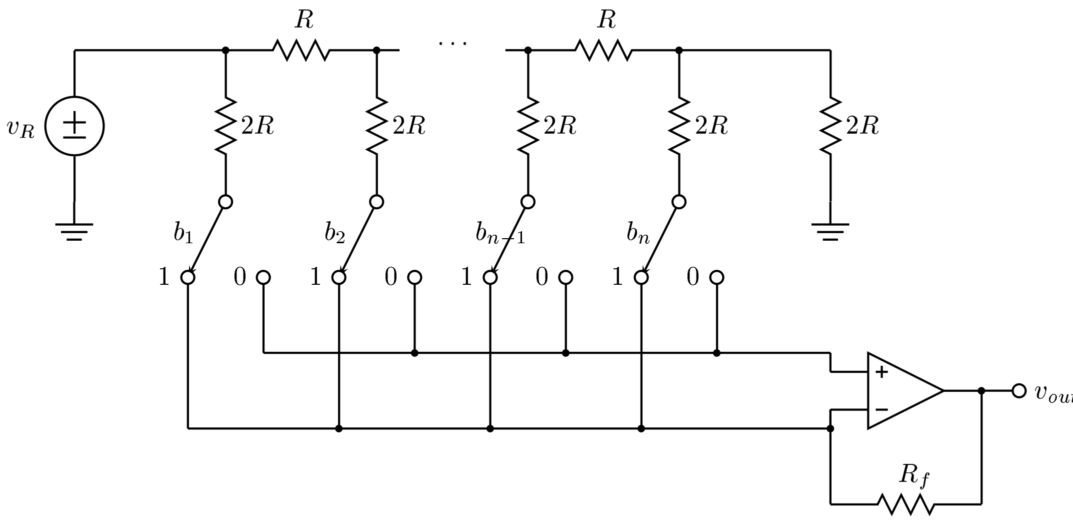
$$G = \frac{i_l(t)}{i_s(t)} = -\frac{R_1 + R_2}{R_1}.$$

Problem 6.4: The circuit shown in the next figure is called an inverted $R - 2R$ ladder digital-to-analog converter (DAC). The input to this circuit is a binary code represented by b_1, b_2, ..., b_n, where b_i is either 1 or 0. Each switch shown in the figure is controlled by one of the bits in the binary code. If $b_i = 1$, that switch will be in the '1' position; if $b_i = 0$, that switch will be in the '0' position. Depending on the position of the switch, each current i_k is diverted either to true ground (adding to the + terminal of the opamp) or to the virtual ground (adding to the − terminal.)

(a) If i is the current flowing out of the voltage source, show that $i = v_R/R$ regardless of the digital input code.

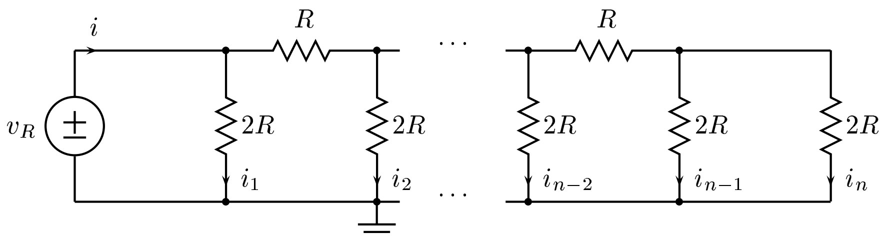
(b) Show that the output voltage can be expressed as

$$v_{out}(t) = -\frac{R_f}{R} v_R (b_1 2^{-1} + b_2 2^{-2} + \ldots + b_{n-1} 2^{-n+1} + b_n 2^{-n})$$



Solution:

(a) The voltage source sees the ladder of resistors. Because the voltages at the two input terminals of the opamp are virtually equal to each other, all of the $2R$ resistors are essentially connected to ground. Thus from the point-of-view of the voltage source the circuit looks like



This is readily seen to be equivalent to a resistance of $R\Omega$. Thus, $i = v/R$. Using current dividers

$$\begin{aligned} i_1 &= \frac{i}{2} = \frac{v}{2R} \\ i_2 &= \frac{i}{2^2} = \frac{v}{2^2 R} \\ &\vdots \\ i_n &= \frac{i}{2^n} = \frac{v}{2^n R} \end{aligned}$$

(b)

$$\frac{v_{out}(t)}{R_f} = b_1 i_1 + b_2 i_2 + \ldots + b_n i_n$$

or

$$v_{out}(t) = \frac{R_f v_R}{R} \left[b_1 2^{-1} + b_2 2^{-2} + \ldots + b_n 2^{-n} \right].$$

Problem 6.5: Express each of the following as exponentially weighted sinusoidal time functions of the form $x(t) = Ae^{\sigma t}\cos(\omega t + \phi)$.

(a) $x_a(t) = \dfrac{1+j}{2}e^{(-2+j3)t} + \dfrac{1-j}{2}e^{(-2-j3)t}$.

(b) $x_b(t) = \sqrt{3}e^{-t}\cos(2t) - e^{-t}\sin(2t)$.

(c) $x_c(t) = \dfrac{j\omega_0}{1+j\omega_0}e^{j\omega_0 t} - \dfrac{j\omega_0}{1-j\omega_0}e^{-j\omega_0 t}$.

Solution:

(a)

$$\begin{aligned}
x_a(t) &= \frac{1+j}{2}e^{(-2+j3)t} + \frac{1-j}{2}e^{(-2-j3)t} \\
&= 2\Re\mathrm{e}\left\{\frac{1+j}{2}e^{(-2+j3)t}\right\} \\
&= \Re\mathrm{e}\left\{\sqrt{2}e^{j\pi/4}e^{(-2+j3)t}\right\} \\
&= \Re\mathrm{e}\left\{\sqrt{2}e^{-2t}e^{j(3t+\pi/4)}\right\} \\
&= \sqrt{2}e^{-2t}\cos(3t + \frac{\pi}{4}).
\end{aligned}$$

(b)

$$\begin{aligned}
x_b(t) &= e^{-t}\left(\sqrt{3}\cos 2t - \sin 2t\right) \\
&= e^{-t}\cos(2t + \tan^{-1}\frac{1}{\sqrt{3}}) \\
&= e^{-t}\cos(2t + \frac{\pi}{6}).
\end{aligned}$$

(c)

$$\begin{aligned}
x_c(t) &= \frac{j\omega_0}{1+j\omega_0}e^{j\omega_0 t} - \frac{j\omega_0}{1-j\omega_0}e^{j\omega_0 t} \\
&= 2\Re\mathrm{e}\left\{\frac{j\omega_0}{1+j\omega_0}e^{j\omega_0 t}\right\} \\
&= 2\Re\mathrm{e}\left\{\frac{\omega_0}{\sqrt{1+\omega_0^2}}e^{j(\frac{\pi}{2}-\tan^{-1}\omega_0)}e^{j\omega_0 t}\right\} \\
&= 2\frac{\omega_0}{\sqrt{1+\omega_0^2}}\cos(\omega_0 + \frac{\pi}{2} - \tan^{-1}\omega_0) \\
&= -\frac{\omega_0}{\sqrt{1+\omega_0^2}}\sin(\omega_0 - \tan^{-1}\omega_0)
\end{aligned}$$

Problem 6.6: Determine the Laplace transforms of the following time waveforms.

(a) $x_a(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$

(b) $x_b(t) = t^2 e^{-3t}, \;\; t > 0$

(c) $x_c(t) = e^{-4t} \sin 5t, \;\; t > 0$

(d) $x_d(t) = t, \;\; t > 0$

Solution:
(a)

$$X_a(s) = \int_0^T 1 \cdot e^{-st}\, dt = \frac{1}{s}\left(1 - e^{-sT}\right)$$

(b) To get $X_b(s)$ we can use an indirect approach (or we could just do the integral).

$$\begin{aligned}
z(t) = e^{-3t} \quad &\longleftrightarrow \quad \frac{1}{s+3} \\
y(t) = te^{-3t} \quad &\longleftrightarrow \quad -\frac{d}{ds}\left(\frac{1}{s+3}\right) = \frac{1}{(s+3)^2} \\
x_b(t) = ty(t) \quad &\longleftrightarrow \quad -\frac{d}{ds}\left(\frac{1}{(s+3)^2}\right) = \frac{2}{(s+3)^3}
\end{aligned}$$

(c)

$$\begin{aligned}
x_c(t) &= e^{-4t} \sin 5t = \frac{1}{j2}\left(e^{-4t}e^{j5t} - e^{-4t}e^{-j5t}\right) \\
&= \frac{1}{j2}e^{-(4-j5)t} - \frac{1}{j2}e^{-(4+j5)t} \\
X_c(s) &= \frac{\frac{1}{j2}}{s+4-j5} - \frac{\frac{1}{j2}}{s+4+j5} \\
&= \frac{5}{s^2+8s+41}
\end{aligned}$$

(d)

$$\begin{aligned}
x_d(t) &= t = 1 \cdot t \\
X_d(s) &= -\frac{d}{ds}\left(\frac{1}{s}\right) = \frac{1}{s^2}
\end{aligned}$$
