

GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040
Circuit Analysis

Assigned: February 23, 2001
Due: March 2, 2001

Problem Set #7

Reading: Read the following sections from the class notes:
Chapter 5, Sections 5.5

Reading: Some of same topics are discussed in Dorf and Svoboda:
Chapter 14, Sections 14.4–14.5, 14.12 (inverse Laplace transforms)

Problem 7.1: This problem derives a number of identities concerning the complex conjugate.

- (a) Using either the rectangular or polar representation for each complex variable, show that:
- (i) $(s_1 + s_2)^* = s_1^* + s_2^*$.
 - (ii) $(as_1)^* = as_1^*$, where a is any real number.
 - (iii) $(s_1s_2)^* = s_1^*s_2^*$.
 - (iv) $(e^s)^* = e^{s^*}$.
 - (iv) $(s^a)^* = (s^*)^a$, where a is any real number.
 - (vi) $\left(\frac{s_1}{s_2}\right)^* = \frac{s_1^*}{s_2^*}$.

- (b) Using the results that you derived in (a), show that

$$[P(s)]^* = P(s^*),$$

where $P(s)$ is a polynomial of degree n in s with real coefficients

$$P(s) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0.$$

- (c) Using the results of (a) and (b), show that

$$[Q(s)]^* = Q(s^*),$$

where $Q(s)$ is the ratio of two polynomials with real coefficients

$$Q(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}.$$

- (d) Using the results of (b), show that the complex roots of a polynomial with real coefficients occur in complex conjugate pairs, i.e., show that if $P(s_1) = 0$, where $P(s)$ is a polynomial with real coefficients, then $P(s_1^*) = 0$.

Problem 7.2: Find the inverse Laplace transform of

$$X(s) = \frac{2s + 6}{s(s^2 + 3s + 2)}.$$

Problem 7.3: The inverse Laplace transform of

$$X(s) = \frac{s + 2}{(s + 2)^2 + 9}$$

has the form

$$x(t) = K e^{-at} \cos(\omega_0 t + \phi).$$

Determine the values of K , a , ω_0 , and ϕ .

Problem 7.4: Find the inverse Laplace transform of

$$X(s) = \frac{s + 3}{s^3 + 3s^2 + 6s + 4}.$$

Problem 7.5: Find the inverse Laplace transform of

$$X(s) = \frac{5s - 1}{s^3 - 3s - 2}.$$

Problem 7.6: Find the signal $x(t)$ for $t \geq 0$ if its Laplace transform is

$$X(s) = \frac{s^3}{(s + 1)([s + 1]^2 + 4)}.$$