

GEORGIA INSTITUTE OF TECHNOLOGY  
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

March 23, 2001

**Problem Set #9–Solutions**

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**Problem 9.1:** In the circuit below,  $v_c(0) = 8\text{V}$  and  $i_\ell(0) = 4\text{A}$ . Find  $i_\ell(t)$  for  $t > 0$ .

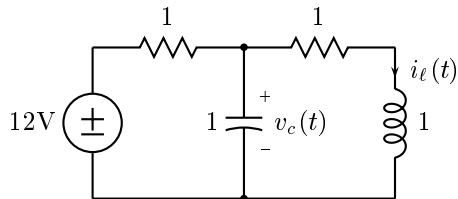
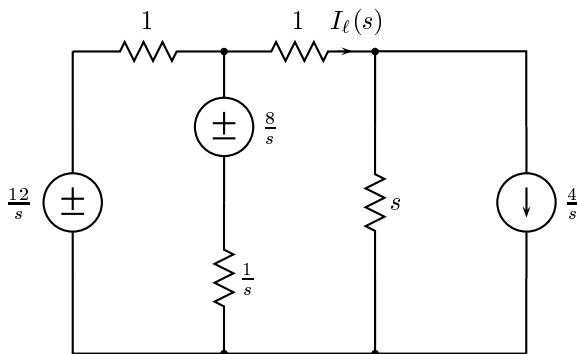


Figure 1: Circuit for Problem 9.1.

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**Solution:** First we draw the circuit in the Laplace domain



We will attack this problem using the mesh method. Let  $I(s)$  be a clockwise mesh current in the left mesh,  $I_\ell(s)$  be a clockwise mesh current in the center mesh, and  $\frac{4}{s}$  be a clockwise mesh current in the right mesh. Since the basic network has only two meshes, we write KVL equations for the left and center meshes of the complete circuit. These are:

$$-\frac{12}{s} + I(s) + \frac{8}{s} + \frac{1}{s}(I(s) - I_\ell(s)) = 0$$

$$\frac{1}{s}(I_\ell(s) - I(s)) - \frac{8}{s} + I_\ell(s) + s(I_\ell(s) - \frac{4}{s}) = 0.$$

Simplifying, these become

$$(s + 1)I(s) - I_\ell(s) = 4$$

$$-I(s) + (s^2 + s + 1)I_\ell(s) = 4s + 8.$$

After some algebra, we solve for  $I_\ell(s)$

$$I_\ell(s) = \frac{4(s^2 + 3s + 3)}{s(s^2 + 2s + 2)} = \frac{6}{s} + \frac{-1 - j}{s + 1 - j} + \frac{-1 + j}{s + 1 + j}.$$

Therefore,

$$i_\ell(t) = 6 + 2\sqrt{2}e^{-t} \cos\left(t - \frac{3\pi}{4}\right), \quad t > 0.$$


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**Problem 9.2:** The first-order circuit in Figure 2 has a constant input for  $t > 0$ .

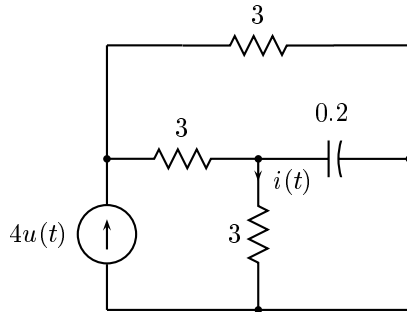


Figure 2: First-order circuit for Problem 9.2.

- Find  $i(0)$ .
- Find  $i(\infty)$ .
- Find  $i(t)$  for  $t > 0$ .

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**Solution:**

- At  $t = 0$  the capacitor looks like a short circuit. Since there is a short circuit across the vertical  $3\Omega$  resistor

$$i(0) = 0.$$

- At  $t = \infty$  the capacitor looks like an open circuit. At this point there is a  $3\Omega$  resistor connected in parallel with an effective  $6\Omega$  resistor connected in parallel with the current source. Therefore,

$$i(\infty) = \frac{2}{3} \cdot 4 = \frac{8}{3}.$$

- Turning off the current source and looking at the equivalent resistance seen from the terminals of the capacitor, we have

$$R_T = 6\Omega || 3\Omega = 2\Omega.$$

Therefore,

$$\tau = R_T C = (2)(0.4) = 0.4$$

and

$$i(t) = \frac{8}{3}(1 - e^{-2.5t}), \quad t \geq 0.$$

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**Problem 9.3:** Two capacitors with capacitance  $C_1$  and  $C_2$ , respectively, are connected in parallel. Show that the parallel connection behaves like a single capacitor and determine the impedance of the parallel connection in terms of  $C_1$  and  $C_2$ .

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**Solution:** The effective admittance of two elements connected in parallel is the sum of their admittances. For two capacitors connected in parallel we have

$$Y_{eq}(s) = C_1 s + C_2 s = (C_1 + C_2)s$$

This looks like a capacitor with an effective capacitance of

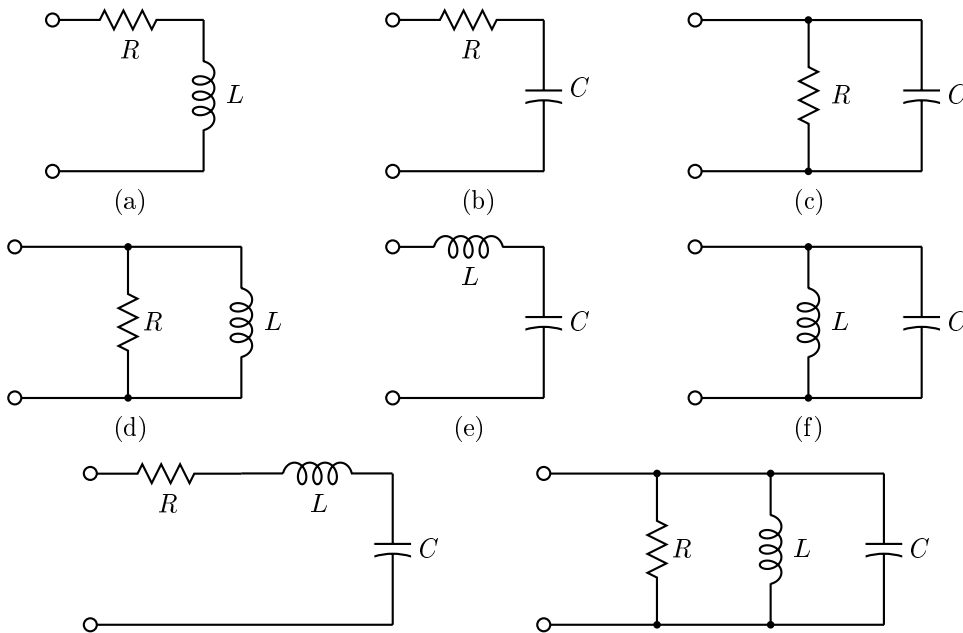
$$C_{eq} = C_1 + C_2$$

and an impedance of

$$Z_{eq}(s) = \frac{1}{(C_1 + C_2)s}.$$


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**Problem 9.4:** For each of the networks in Figure 3, determine the equivalent impedance. Express your answers as ratios of polynomials in  $s$ .



(g) Figure 3: Circuits for Problem 9.4. (h)

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**Solution:**

(a)  $Z_a(s) = R + Ls$

(b)  $Z_b(s) = R + \frac{1}{Cs} = \frac{RCs+1}{Cs}$

(c)  $Z_c(s) = \frac{\frac{R}{Cs}}{R+\frac{1}{Cs}} = \frac{R}{RCs+1}$

(d)  $Z_d(s) = \frac{RLs}{R+Ls}$

(e)  $Z_e(s) = Ls + \frac{1}{Cs} = \frac{LCs^2+1}{Cs}$

(f)  $Z_f(s) = \frac{\frac{L}{Cs}}{Ls+\frac{1}{Cs}} = \frac{LCs^2+1}{Cs}$

(g)  $Z_g(s) = R + Ls + \frac{1}{Cs} = \frac{LCs^2+RCs+1}{Cs}$

(h)  $Z_h(s) = \frac{1}{\frac{R}{L}+\frac{1}{Ls}+Cs} = \frac{RLs}{RLCs^2+Ls+R}$

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**Problem 9.5:** For each of the networks in Figure 4 determine the Laplace domain formula that relates the indicated output variable to the source variable. Your answers should be expressed in terms of the Laplace transforms of the source variables and the Laplace domain variable  $s$ .

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**Solution:**

(a)  $H_a(s) = \frac{Ls}{Ls+r}$

(b)  $H_b(s) = \frac{1}{Cs} \cdot \frac{Ls}{R+\frac{1}{Cs}+Ls} = \frac{Ls}{LCs^2+RCs+1}$

(c)  $H_c(s) = 1$

(d)  $H_d(s) = \frac{1}{C_1s} \cdot \frac{R_1}{R_1+\frac{1}{C_1s}} = \frac{R_1}{R_1C_1s+1}$

(e)  $H_e(s) = \frac{L_1s}{R+(L_1+L_2)s} \cdot R = \frac{RL_1s}{(L_1+L_2)s+R}$

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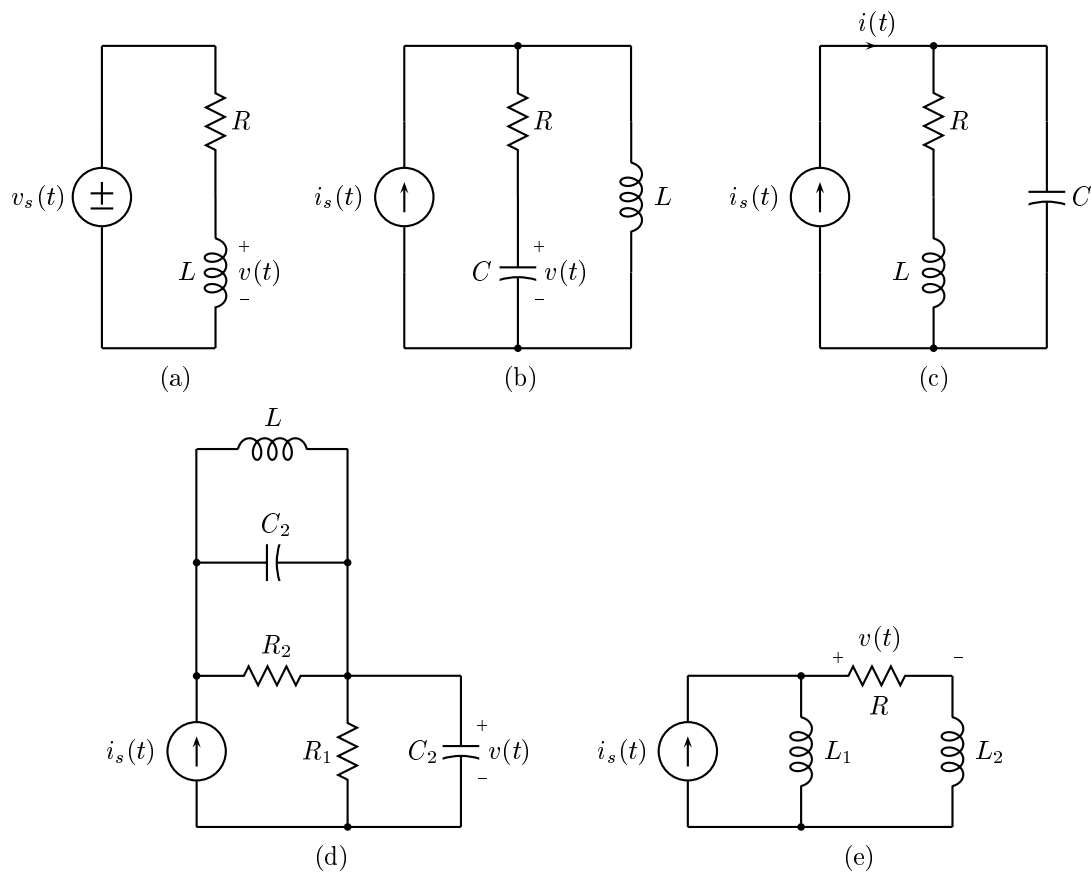


Figure 4: Networks for Problem 9.5.