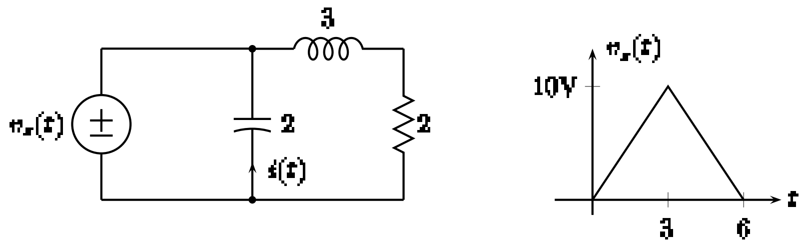
GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 2040
Circuit Analysis

Quiz #1 – Solutions

Problem Q1.1:

The source waveform $v_s(t)$ applied to the circuit in Figure 1(a) is shown in Figure 1(b). Sketch



(a) (b)

Figure 1: (a) Circuit for Problem Q1.1. (b) Voltage source waveform $v_s(t)$.

the current $i_c(t)$ flowing through the capacitor as a function of time.

Solution: The inductor and the resistor do not affect the capacitor current and can be ignored. Clearly the voltage drop across the capacitor is $v_s(t)$.

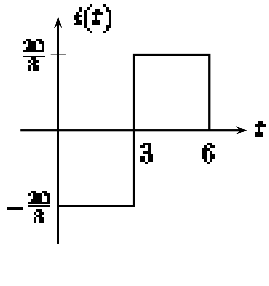
$$i_c(t) = C\frac{v_c(t)}{dt} = 2\frac{dv_s(t)}{dt}$$

But $i_c(t)$ is defined to flow into the + terminal. Thus,

$$i(t) = -i_c(t) = -2\frac{dv_s(t)}{dt}.$$

Performing the derivative amounts to measuring the slopes of the line segments that make up $v_s(t)$.

$$i(t) = \begin{cases} 0, & t < 0 \\ -20/3, & 0 < t < 3 \\ 20/3, & 3 < t < 6 \\ 0, & 6 < t \end{cases}$$



Problem Q1.2:

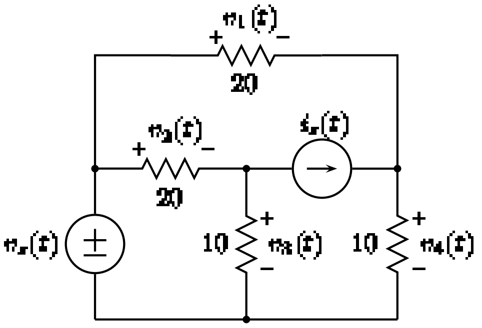


Figure 2: Circuit for Problem Q1.2.

(a) For the circuit in **Figure 2** write a minimal sufficient of **KVL** equations that will provide independent constraints on the element voltages that are labeled. Write your equations in terms of the element voltages and the source signals only.

(b) For the same circuit write a minimal sufficient set of **KCL** equations that will provide independent constraints on the same element voltages. Again, write your equations in terms the element voltages and source signals only.

Solution: The basic network contains two meshes and three nodes.

(a)

$$v_1(t) - v_2(t) - v_3(t) + v_4(t) = 0$$
$$v_2(t) + v_3(t) = v_s(t)$$

(b) Any two of the following (or their negatives) constitute a minimal set of **KCL** equations.

$$\frac{1}{20}v_2(t) - \frac{1}{10}v_3(t) = i_s(t)$$
$$\frac{1}{20}v_1(t) - \frac{1}{10}v_4(t) = -i_s(t)$$
$$\frac{1}{20}v_1(t) + \frac{1}{20}v_2(t) - \frac{1}{10}v_3(t) - \frac{1}{10}v_4(t) = 0$$

Problem Q1.3: In the circuit of Figure 3 determine $v(t)$ as a function of $v_{s_1}(t)$, $v_{s_2}(t)$ and $i_s(t)$.

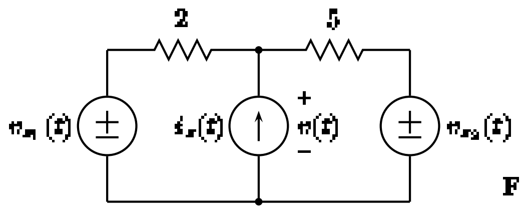


Figure 3: Circuit for Problem Q1.3.

Solution: Writing a KCL equation (incorporating Ohm's Law and KVL) at the upper node gives

$$\frac{1}{5}[v(t) - v_{s_2}(t)] + \frac{1}{2}[v(t) - v_{s_1}(t)] = i_s(t).$$

Solving for $v(t)$ gives the result

$$v(t) = \frac{5}{7}v_{s_1}(t) + \frac{2}{7}v_{s_2}(t) + \frac{10}{7}i_s(t).$$

Problem Q1.4: For the circuit in Figure 4

(a) Compute the power *dissipated* by the independent voltage source.

(b) Compute the power *dissipated* by the dependent current source.

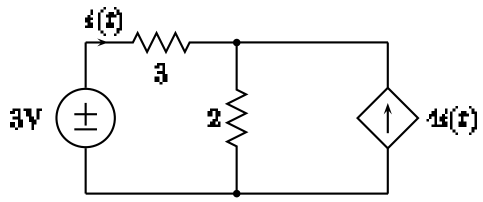


Figure 4: Circuit for Problem Q1.4.

Solution: Since the only independent source waveform is constant, all of the voltages and currents in this circuit will be constant as a function of t. Let i' denote the current flowing (downward) through the 2Ω resistor. From KCL at the top node

$$i + 4i = i' \implies i' = 5i.$$

By KVL

$$3i + 2i' = 3.$$

If we substitute for i', this gives

$$13i = 3 \longrightarrow i = \frac{3}{13}A; \quad i' = \frac{15}{13}A.$$

(a) The power dissipated by the voltage source is the product of its voltage drop (3V) and the current entering its + terminal ($-i$). This

$$P_v = -(3)(\frac{3}{13}) = -\frac{9}{13}W.$$

(b) The power dissipated by the dependent current source is the product of its voltage drop ($2i'$) and the current entering its + terminal ($-4i$). Therefore,

$$P_i = -8(\frac{15}{13})(\frac{3}{13}) = \frac{360}{169}$$
