GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 2040
Circuit Analysis

Quiz #2 – Solutions

Problem Q2.1: Determine the two node potentials $e_a(t)$ and $e_b(t)$ in the circuit of Figure 1 as functions of $v_s(t)$ and $i_s(t)$.

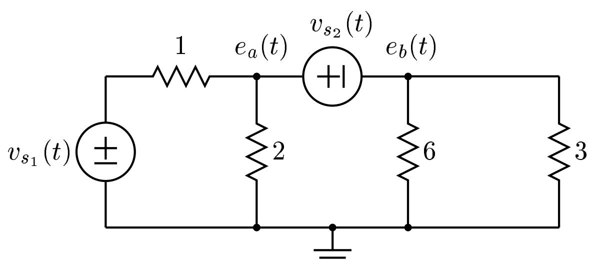


Figure 1: Circuit for Problem Q2.1.

Solution: First notice that the two node potentials are not independent since

$$e_a(t) = e_b(t) + v_{s_2}(t).$$

Notice also that the two resistors connected in parallel can be replaced by a single 2Ω resistor. It is sufficient to write a single KCL equation at the supernode that encircles the voltage source $v_{s_2}(t)$.

$$[e_b(t) + v_{s_2}(t) - v_{s_1}(t)] + \frac{1}{2}[e_b(t) + v_{s_2}(t)] + \frac{1}{2}e_b(t) = 0.$$

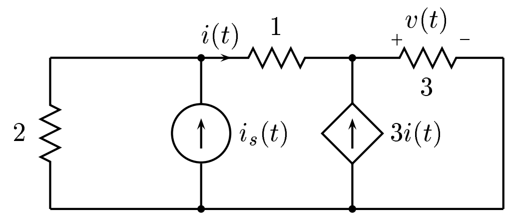
Its solution is

$$e_b(t) = \frac{1}{2}v_{s_1}(t) - \frac{3}{4}v_{s_2}(t).$$

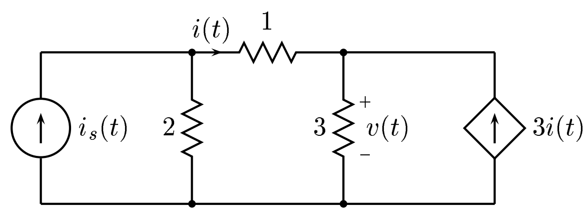
Therefore,

$$e_a(t) = \frac{1}{2}v_{s_1}(t) + \frac{1}{4}v_{s_2}(t).$$

Problem Q2.2: Determine the voltage $v(t)$ and the current $i(t)$ in the following circuit.



Solution: One way to approach this problem is to use the mesh method, which is made conceptually easier if we first redraw the circuit. With the redrawing it is clear



that we only need to write a single KVL equation over the center mesh. Furthermore, the mesh current in that equation is $i(t)$, which is one of the variables of interest. Let the mesh current in the left mesh be $i_s(t)$ and define a *counterclockwise* mesh current $3i(t)$ in the right mesh. Then the KVL equation is

$$2[i(t) - i_s(t)] + i(t) + 3[i(t) + 3i(t)] = 0,$$

which becomes

$$15i(t) = 2i_s(t)$$

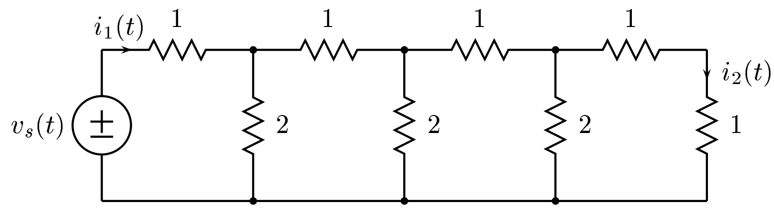
or

$$i(t) = \frac{2}{15} i_s(t).$$

Knowing this current we can find $v(t)$.

$$v(t) = 3[i(t) + 3i(t)] = 12i(t) = \frac{24}{15} i_s(t) = \frac{8}{5} i_s(t)$$

Problem Q2.3:



(a) Determine $i_1(t)$ (in terms of $v_s(t)$) for the above circuit.

(b) Determine $i_2(t)$ (in terms of $v_s(t)$).

Solution:

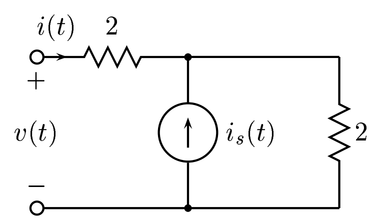
(a) By applying the series and parallel combination rules to the ladder of resistors, it is straightforward to verify that the equivalent resistance seen by the voltage source is 2Ω. Therefore,

$$i_1(t) = \frac{1}{2}v_s(t).$$

(b) By similar reasoning, the circuit to the right of each 2Ω resistor also looks like a 2Ω resistor. Thus at every node half of the current goes through the vertical 2Ω resistor and half goes farther to the right. Therefore,

$$i_2(t) = \frac{1}{8}i_1(t) = \frac{1}{16}v_s(t).$$

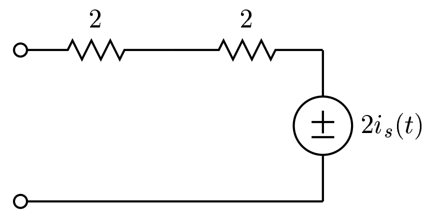
Problem Q2.4:



(a) Find the $v - i$ relation for the two-terminal network above.

(b) Find and *sketch* the Norton equivalent network that has the same $v - i$ relation.

Solution:



(a) Rather than write equations, we can perform a source substitution to give the equivalent circuit below. From this we can write the $v-i$ relation by inspection:

$$v(t) = 4i(t) + 2i_s(t).$$

(b) One more source substitution gives the Norton equivalent.