GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 2040
Circuit Analysis

Quiz #3–Solutions

Problem Q3.1: Determine $v_{out}(t)$ as a function of $v_1(t)$ and $v_2(t)$ for the network in Figure 1.

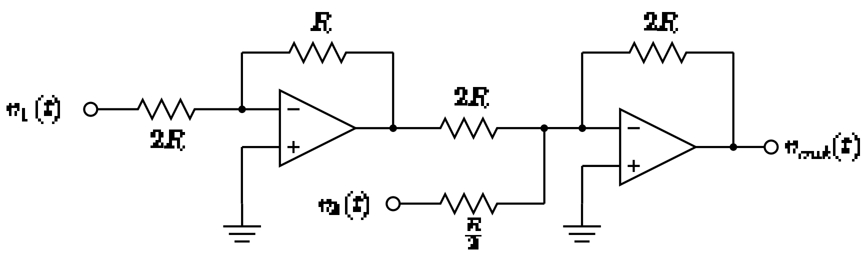


Figure 1: Circuit for Problem Q3.1.

Solution: The node potentials at the inverting inputs nodes of both of the opamps are zero (virtual ground connection). Let the node potential at the output of the left opamp be $e(t)$. We then write a KCL equations at the two nodes that are connected to the two inverting nodes. At the left one

$$\frac{v_1(t)}{2R} + \frac{e(t)}{R} = 0$$

from which we get

$$e(t) = -\frac{1}{2}v_1(t).$$

At the right inverting node, we have

$$-\frac{v_1(t)}{4R} + \frac{2v_2(t)}{R} + \frac{v_{out}(t)}{2R} = 0.$$

This gives the result

$$v_{out}(t) = \frac{1}{2}v_1(t) - 4v_2(t).$$

Problem Q3.2: A signal of the form

$$x(t) = \left(A + Be^{-t}\sin 2t + Ce^{-t}\cos 2t\right)u(t)$$

has the Laplace transform

$$X(s) = \frac{4s^2 + 11s + 15}{s^3 + 2s^2 + 5s}.$$

Determine the values of A, B, and C.

Solution: The form of the answer simplifies the partial fraction expansion. We know that

$$\frac{4s^2 + 11s + 15}{s(s^2 + 2s + 5)} = \frac{A}{s} + \frac{2B}{(s+1)^2 + 4} + \frac{C(s+1)}{(s+1)^2 + 4}.$$

By recombining the terms in the partial fraction expansion and comparing the result with the original numerator, we see

$$A(s^2 + 2s + 5) + 2Bs + C(s^2 + s) = 4s^2 + 11s + 15$$

or

$$(A + C)s^2 + (2A + 2B + C)s + 5A = 4s^2 + 11s + 15.$$

Thus, $A = 3$, $B = 2$, and $C = 1$.

Problem Q3.3: Find $i(t)$ for $t > 0$ in the circuit in Figure 2.

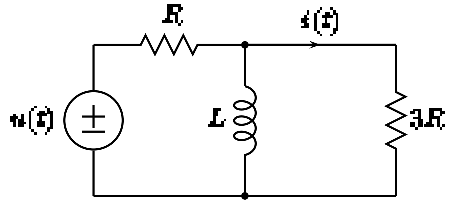


Figure 2: Circuit for Problem Q3.3.

Solution: Since this is a first-order circuit with a constant input, the simplest approach to use is the inspection method. At $t = 0$ we replace the inductor by an open circuit and observe

$$i(0) = \frac{1}{4}.$$

At $t = \infty$ we replace the inductor by a short circuit. Since all of the current will flow through that short circuit, we see

$$i(\infty) = 0.$$

To get the time constant, we first find the Thevenin equivalent resistance seen by the inductor.

$$R_T = \frac{3R}{4} \implies \tau = \frac{L}{R_T} = \frac{4L}{3R}.$$

Therefore, putting everything together we have

$$i(t) = \frac{1}{4}e^{-\frac{3Rt}{4L}}, \quad t > 0.$$

Problem Q3.4: In the circuit in Figure 3 the initial voltages on the two capacitors are known to be $i_L(0) = 0$, $v_c(0) = 1$.

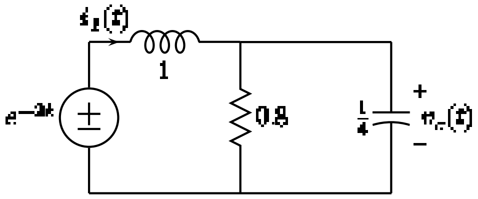


Figure 3: Circuit for Problem Q3.4.

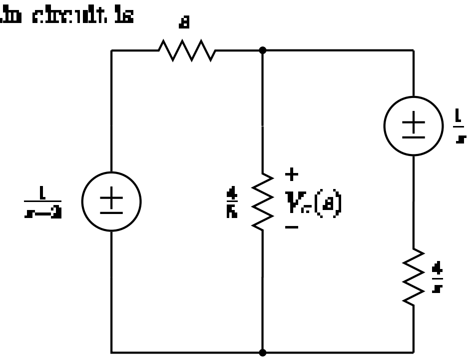
(a) Draw the Laplace-domain circuit model.

(b) Determine $V_c(s)$.

(c) Determine $v_c(t)$, $t > 0$.

Solution:

(a) The Laplace domain circuit is



(b) Using superposition of sources

$$V_c(s) = \frac{\frac{\frac{4}{5}\frac{4}{s}}{\frac{4}{5}+\frac{4}{s}}}{\frac{\frac{4}{5}\frac{4}{s}}{\frac{4}{5}+\frac{4}{s}} + s} \cdot \frac{1}{(s+2)} + \frac{\frac{\frac{4s}{5}}{\frac{4}{5}+s}}{\frac{\frac{4s}{5}}{\frac{4}{5}+s} + \frac{4}{s}} \cdot \frac{1}{s}$$

$$= \frac{4}{(s+2)(s^2+5s+4)} + \frac{s}{(s^2+5s+4)}.$$

(c) Performing a partial fraction expansion

$$V_c(s) = \frac{A}{s+2} + \frac{B}{s+1} + \frac{C}{s+4}$$

$$A = \lim_{s\to-2} \frac{4}{s^2+5s+4} = -2$$

$$B = \lim_{s\to-1} \frac{4}{(s+2)(s+4)} + \frac{s}{s+4} = 1$$

$$C = \lim_{s\to-4} \frac{4}{(s+2)(s+1)} + \frac{1}{s+1} = 2$$

Therefore,

$$v_c(t) = -2e^{-2t} + e^{-t} + 2e^{-4t}, \quad t > 0.$$