GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course EE 6416

Multidimensional Digital Signal Processing

Final Exam

Wednesday, December 9, 1998

Name: ______________________________

GENERAL INSTRUCTIONS

1. This is a *open book, open notes* exam. You may use any of the materials distributed in class or any of your own notes or problem sets.
2. Please do all of your work on the exam itself. You may use the backs of the pages, if necessary.
3. Please be as neat and well organized as possible.
4. Clearly indicate your answers.

Problem	*Max*	*Score*
1	10	
2	10	
3	10	
4	10	
5	15	
6	15	
7	15	
8	15	
Total	100	

Problem F.1:

Compute and sketch the convolution of the two signals

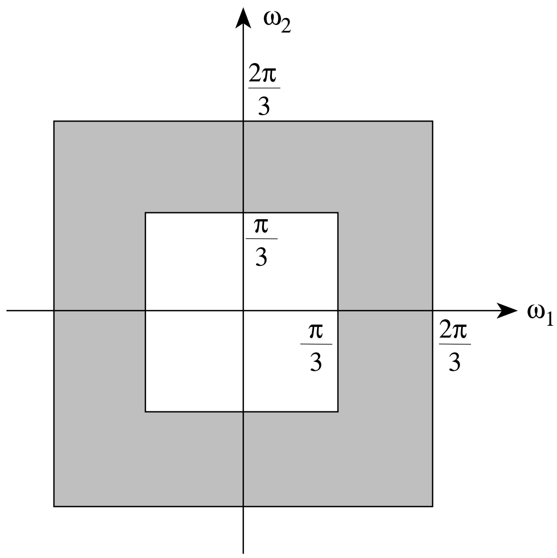
$$f[n_1, n_2] = \alpha^{n_1} \delta[n_2]$$

and

$$g[n_1, n_2] = \beta^{n_1} \delta[n_1 + n_2]$$

Problem F.2:

A two-dimensional linear, shift-invariant filter has the frequency response that is equal to one in the shaded region in the figure below and zero elsewhere (over one period).



Determine the impulse response $h[n_1, n_2]$ of the filter.

Problem F.3:

Consider the bandlimited (analog) signal whose Fourier transform occupies the band shown in Figure 1. What is the minimum number of samples per square meter that

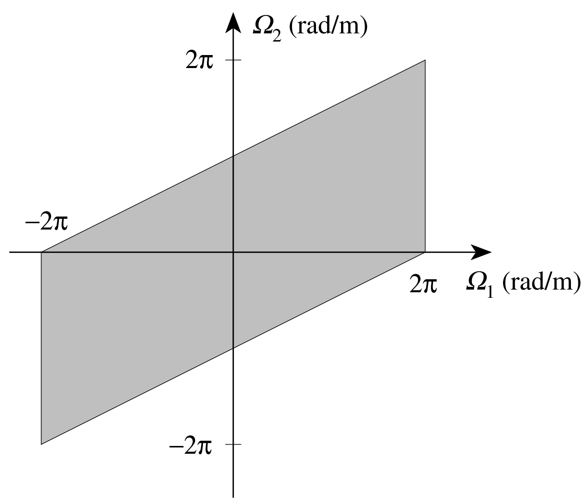


Figure 1:

is required to sample this signal without aliasing?

Problem F.4:

A two-dimensional signal is said to be *semiperiodic* if it is periodic in one variable but not in the other. Let $x[n_1, n_2]$ be semiperiodic with horizontal period N. Then

$$x[n_1, n_2] = x[n_1 + N, n_2] \quad \forall [n_1, n_2] \ .$$

Such a signal can be represented by a hybrid Fourier transform.

$$X(\omega, k) = \sum_{n_1=0}^{N-1} \sum_{n_2=-\infty}^{\infty} x[n_1, n_2] \exp[-j\frac{2\pi}{N} k n_1] \exp[-j\omega n_2]$$

(a) Derive an expression for the inverse hybrid Fourier transform that expresses $x[n_1, n_2]$ in terms of $X(\omega, k)$.

(b) Express the energy in the signal

$$E = \sum_{n_1=0}^{N-1} \sum_{n_2=-\infty}^{\infty} |x[n_1, n_2]|^2$$

in terms of $X(\omega, k)$. (This is a form of Parseval relation for these signals.)

Problem F.5:

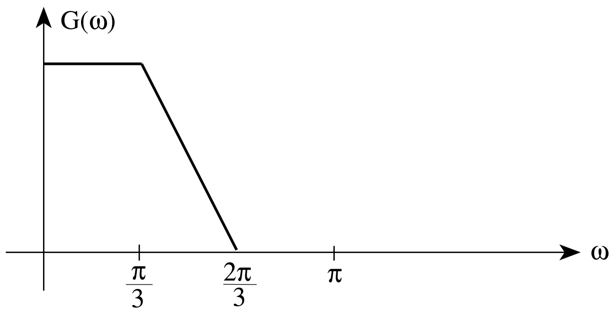
The 2-D DFT of an $(N_1 \times N_2)$ point finite-extent array is computed using the periodicity matrix

$$\boldsymbol{N} = \begin{bmatrix} N_1 & 1 \\ 0 & N_2 \end{bmatrix}$$

Where in the 2-D frequency plane (ω_1, ω_2) is sample $[k_1, k_2]$ located ?

Problem F.6:

The McClellan transformation can be used to design filters of any dimensionality, including one-dimensional filters. Let $G(\omega)$ be the prototype filter sketched below.



If the transformation function is

$$F(\omega) = 2\cos^2 \omega - 1$$

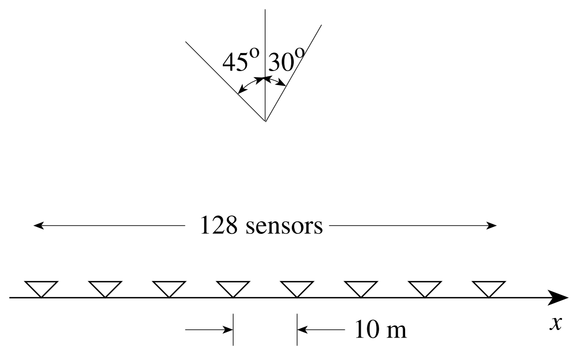
sketch the frequency response of the resulting filter, $H(\omega)$.

Problem F.7:

Determine whether or not the linear, shift-invariant system described by the following difference equation is stable and justify your answer. $x[n_1, n_2]$ and $y[n_1, n_2]$ are the system input and output, respectively.

$$y[n_1, n_2] - 0.9y[n_1 + 1, n_2] - 0.6y[n_1, n_2 - 1] + 0.6y[n_1 + 1, n_2 - 1] = x[n_1, n_2]$$

Problem F.8:



Consider a linear array of 128 sensors with an intersensor spacing of 10 m. Two plane waves, both traveling at 1000 m/s, impinge on the array.

	temporal freq	direction
wave 1	100 sec^{-1}	30°
wave 2	150 sec^{-1}	−45°

The sensor outputs are sampled simultaneously at $t = 0$ (or $n = 0$), weighted uniformly, and a DFT is computed with respect to the spatial variable to produce the sequence, $S[\ell]$. Sketch the magnitude of $S[\ell]$.