

GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course EE 6416

Multidimensional Digital Signal Processing

October 2, 1998

Problem Set #1–Solutions

Problem 1.1: For each of the systems described below, determine whether the system is (1) linear and (2) shift invariant.

(a) $y_a[n_1, n_2] = x[n_1, n_2]x[n_1 - N, n_2]$

(b) $y_b[n_1, n_2] = \sum_{m_2=-\infty}^{\infty} x[n_1, m_2]$

(c) $y_c[n_1, n_2] = \sum_{m_2=-1}^1 x[n_1, m_2]$

Solution:

- (a) This system is *nonlinear*. One way to see this is to observe that doubling the input does not double the output; instead it multiplies it by four. The system is *shift invariant*, however. A shift in either n_1 or n_2 in the input sequence is passed through to the output. This will always be the case whenever the independent variables n_1 and n_2 appear only as independent variables in the input sequence.
- (b) Notice that $y[n_1, n_2]$ is constant as a function of n_2 . This system is both *linear* and *shift-invariant* as can be seen by applying the definition. The impulse response of the system is a vertical line impulse.
- (c) This system is *linear* and *shift-varying*. Linearity follows straightforwardly by applying the definition. It is straightforward to see that the system is not shift invariant for shifts that have a non-zero vertical component. The output of the system is uniform in the n_2 direction, and in the n_1 direction it is the sum of the three rows of the input at $n_2 = -1, 0$ and 1 . As the input is shifted vertically, the three rows of the original sequence that are added

will vary, producing a result that is not a simple vertical shift of the output to the unshifted input.

Problem 1.2:

Determine the 2-D convolution of the following two 2-D sequences.

$$\begin{aligned} x[n_1, n_2] &= u[n_1, n_2] \\ h[n_1, n_2] &= \delta[n_1 + n_2] \end{aligned}$$

Solution:

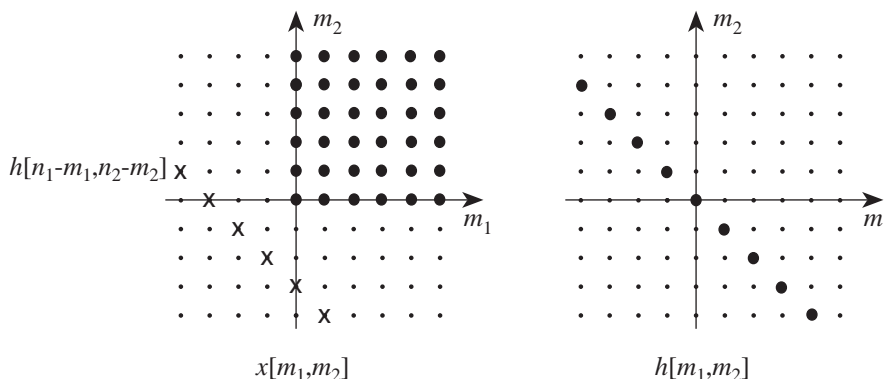


Figure 1:

From Figure 1, the value of y is constant on each NW to SE diagonal and the value at each sample on the diagonal is equal to the sum of the samples on that diagonal in the input sequence.

$$y[n_1, n_2] = \begin{cases} (n_1 + n_2 + 1), & n_1 + n_2 \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

This is shown in Figure 2.

Problem 1.3: Determine the two-dimensional convolution of the two sequences $x[n_1, n_2]$ and $h[n_1, n_2]$ sketched in Figure 3. (*Note:* Both sequences have sample values equal to 1 at the locations indicated by large black dots and are zero elsewhere.)

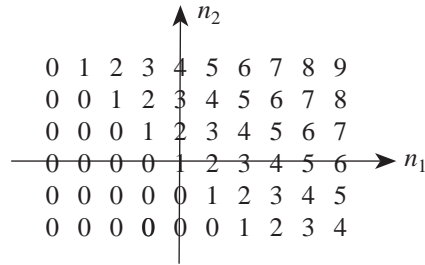


Figure 2:

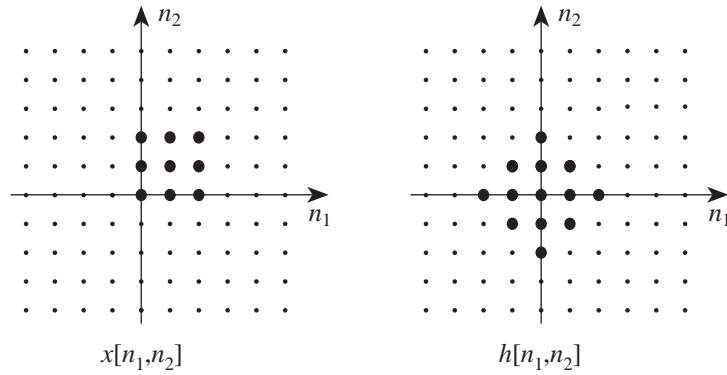


Figure 3:

Solution: The solution is shown in Figure 4.

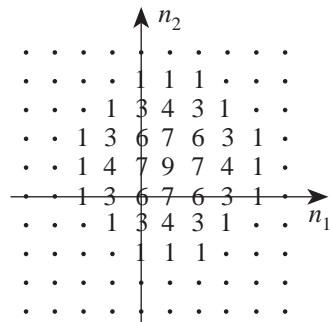


Figure 4:

Problem 1.4: Determine the convolution of the two signals

$$f[n_1, n_2] = f[n_1]\delta[n_2]$$

$$g[n_1, n_2] = \delta[n_1]g[n_2]$$

There should be no unevaluated sums in your answer.

Solution:

$$\begin{aligned}y[n_1, n_2] &= \sum_{m_1=-\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} f[m_1]\delta[m_2]\delta[n_1 - m_1]g[n_2 - m_2] \\ &= \sum_{m_1=-\infty}^{\infty} f[m_1]\delta[n_1 - m_1] \sum_{m_2=-\infty}^{\infty} \delta[m_2]g[n_2 - m_2] \\ &= f[n_1]g[n_2]\end{aligned}$$

Problem 1.5: Prove the following statement: A separable LSI system with impulse response

$$h[n_1, n_2] = f[n_1]g[n_2]$$

is stable if and only if $f[n_1]$ and $g[n_2]$ are absolutely summable sequences.

Solution: Recall that a two-dimensional system is stable if and only if its impulse response is absolutely summable.

Sufficiency: Assume that

$$\sum_{n_1} |f[n_1]| = S_f; \quad \sum_{n_2} |g[n_2]| = S_g$$

are both finite. Then

$$\begin{aligned}\sum_{n_1} \sum_{n_2} |h[n_1, n_2]| &= \sum_{n_1} \sum_{n_2} |f[n_1]| |g[n_2]| \\ &= \sum_{n_1} |f[n_1]| \sum_{n_2} |g[n_2]| \\ &= S_f S_g\end{aligned}$$

must also be finite since it is the product of two finite numbers.

Necessity: Since

$$\sum_{n_1} \sum_{n_2} |h[n_1, n_2]| = S_f S_g$$

if either S_f or S_g is not finite, then the two-dimensional impulse response is not finite either.
